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A Continuation Proof of Fermat's Last Theorem When X = Y, X ≠ Y was Already Solved

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Abstract
This is untrivial continued solution to Fermat's theorem. X is not equal Y is already solved in a previous paper using gap or difference analysis. We transpose the next lower number to the highest number and evaluate the difference when changing the initial values and their exponents. Each number for x is evaluated not be the difference and is eliminated starting from 1.

Keywords: 11-xx, Number Theory, Difference Analysis or Gap Analysis, Fermat's Last Theorem, Simple Solution,Table Analysis

Introduction

In number theory, Fermat's Last Theorem (sometimes called Fermat's conjecture, especially in older texts) states that no three positive integers *a*, *b*, and *c* satisfy the equation $a^n + b^n = c^n$ for any integer value of *n* greater than 2. The cases *n* = 1 and *n* = 2 have been known since antiquity to have infinitely many solutions. (Anonymous 2021) [1]

> Only one relevant proof by Fermat has survived, in which he uses the technique of infinite descent to show that the area of a right triangle with integer sides can never equal the square of an integer. His proof is equivalent to demonstrating that the equation
>
> $$x^4 - y^4 = z^2$$
>
> has no primitive solutions in integers (no pairwise coprime solutions). In turn, this proves Fermat's Last Theorem for the case $n = 4$, since the equation $a^4 + b^4 = c^4$ can be written as $c^4 - b^4 = (a^2)^2$.
> (Freeman, 2005) [2]

This a continuation to the proof.

X ≠ Y is solved. (Fortunado, 2021) [3]

That caters any (positive) integer that results from Z – X or Z – Y.

0^3 = 0					
1^3 = 1	11^3 = 1331	21^3 = 9261	31^3 = 29,791	41^3 = 68,921	51^3 = 132,651
2^3 = 8	12^3 = 1728	22^3 = 10,648	32^3 = 32,768	42^3 = 74,088	52^3 = 140,608
3^3 = 27	13^3 = 2197	23^3 = 12,167	33^3 = 35,937	43^3 = 79,507	53^3 = 148,877
4^3 = 64	14^3 = 2744	24^3 = 13,824	34^3 = 39,304	44^3 = 85,184	54^3 = 157,464
5^3 = 125	15^3 = 3375	25^3 = 15,625	35^3 = 42,875	45^3 = 91,125	55^3 = 166,375
6^3 = 216	16^3 = 4096	26^3 = 17,576	36^3 = 46,656	46^3 = 97,336	56^3 = 175,616
7^3 = 343	17^3 = 4913	27^3 = 19,683	37^3 = 50,653	47^3 = 103,823	57^3 = 185,193
8^3 = 512	18^3 = 5832	28^3 = 21,952	38^3 = 54,872	48^3 = 110,592	58^3 = 195,112
9^3 = 729	19^3 = 6859	29^3 = 24,389	39^3 = 59,319	49^3 = 117,649	59^3 = 205,379
10^3 = 1000	20^3 = 8000	30^3 = 27,000	40^3 = 64,000	50^3 = 125,000	60^3 = 216,000

Fig 1: Partial Table for Cubes

Analysis
We could look for counterexamples.

$$X^3 + Y^3 \neq Z^3$$

Which is also equal to $X^3 \neq Z^3 - Y^3$
Let X be the smallest positive integer among the three.
Let Y be an positive integer. Or second smallest positive integer among the three.
Ley Z be the greatest positive integer among the three.

Substitute 1 for X, 2 for Y and 3 for Z.

$1 \neq 27 – 8$, difference of 19

As we observe in the table as Z increases and Y increases, the gap or difference is more. Or the equation is not equal if we vary Z and Y independently.
Substitute 1 for X, 3 for Y and 4 for Z.

$1 \neq 64 – 27$, difference of 37

So, it is not possible for 1 to be X.

Let us try 2
Substitute 2 for X, 3 for Y and 4 for Z.

$8 \neq 64 – 27$, difference of 37

As we observe in the table as Z increases and Y increases, the gap or difference is more. Or the equation is not equal if we vary Z and Y independently.
Substitute 2 for X, 4 for Y and 5 for Z.

$8 \neq 125 – 64$, difference of 61

So, it is not possible for 2 to be X.

Let us try 3
Substitute 3 for X, 4 for Y and 5 for Z.

$27 \neq 125 – 64$, difference of 61

As we observe in the table as Z increases and Y increases, the gap or difference is more. Or the equation is not equal if we vary Z and Y independently.
Substitute 3 for X, 5 for Y and 6 for Z.

$27 \neq 216 – 125$, difference of 91

So, it is not possible for 3 to be X.

…

It is not also possible for 4, 5, 6, ... also to be X.

The table could be extended to any number so is the analysis.
$N_1 = X$ = lowest positive integer among the three.
$N_2 = Y$ = second lowest integer among the three.
$N_3 = Z$ = the largest positive integer among the three.

$$N_1^3 \neq (N_3\uparrow)^3 – (N_2\uparrow)^3$$

As we increase N_3 and N_2 (independently), the result or difference is higher or not equal to N_1. Starting from $N_1 = 1$, each number will be eliminated.
The analysis for higher superscripts or power is the same (use gap analysis or difference analysis).

Use another table then perform the analysis.

$$N_1^x \neq (N_3\uparrow)^x – (N_2\uparrow)^x$$

Note: Each number is eliminated starting from 1. X could not be 1 or 2. What more for higher exponents. Exponent 3 has the lowest difference.

Solution
This is solving X = Y

Let n = 3

$$X^3 + Y^3 \neq Z^3$$

Since X = Y, substitute X for Y
The equation becomes,

$$X^3 + X^3 \neq Z^3$$

Or

(2) $X^3 \neq Z^3$

For the values to be an integer,

$2^1 \neq n^3$, It should be 2^3, a codivisor or a proper positive divisor.

$$2^1 \neq Z^3 / X^3$$

Examples:
Let Z = 4 and X = 2
$8 = 4^3 / 2^3$
$64/8 = 8$
Let Z = 8 and X = 4
$8 = 8^3 / 4^3$
$512 / 64 = 8$

So, with n^4…

$$X^4 + Y^4 \neq Z^4$$

$$2X^4 \neq Z^4$$

$2^1 \neq n^4$ It should be 2^4, a codivisor or a proper positive divisor.

Examples:

Let Z = 4 and X = 2

$16 = 4^4 / 2^4$

256 / 16 = 16

Let Z = 8 and X = 4

$16 = 8^4 / 4^4$

4096/256 = 16

…

$2^1 \neq n^3$, it should be 2^3.

$2^1 \neq n^4$, it should be 2^4.

$2^1 \neq n^5$, it should be 2^5.

$2^1 \neq n^6$, it should be 2^6.

$2^1 \neq n^7$, it should be 2^7.

…

Or the equation becomes,

$$X^n + Y^n = Z^n/2^{(n-1)}$$

Where X = Y and for some integers to satisfy the conjecture.

Conclusion

This is a continuation to the solution. The solutions are correct.

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