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Predicted Stability of Z=121 Systems

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Abstract
Single-particle energies of Z=121 nuclei are calculated to determine their decay properties. The Rost interaction is used as the basis for these calculations. A = 301 – 307 systems are less stable than Z=119 and 120 predictions. The most stable isotope occurs at A = 305, and it has an alpha decay half-life of 37.5 ns. These model results are consistent with the observed properties of Z = 110 – 118 nuclei and calculations for Z=119 and 120 systems.

Keywords: Z=121 Energy Levels, Superheavy Nuclei, Alpha and Beta Decay Half-Lives, Nuclear Structure, and Single-Particle Energies

1. Introduction

Superheavy nuclei and their associated stability have been active experimental and theoretical areas of research [1-31]. Calculations in the superheavy mass region require the selection of an appropriate nuclear interaction. The focus on Z = 119 and neighboring systems has been an area of active investigation [14, 22-27]. Additional effort predicted Z =120 values [28, 30, 31]. Experimental and theoretical calculations for the Z=121 have also been published [32-35].

Calculations for the Z = 121 systems are presented in this paper. These calculations determine the nuclear structure and predict the decay characteristics of the Z = 121 systems.

Z = 121 system stability is evaluated using the methodology Lukasiak and Sobiczewski [3] and Petrovich *et al.* [4]. The calculations are based on the unmodified Rost interaction [2]. Single-particle model calculations facilitate the investigation of superheavy systems that have yet to be experimentally detected and have received limited theoretical study [22-35]. These studies provide insight into binding energy systematics and nuclear stability beyond the previously established mass regions.

Using a more sophisticated method than the single-particle approach is not warranted in view of the uncertainties encountered in these initial calculations. These uncertainties include extrapolations of the nuclear interaction into the Z > 118 mass region. Since there are limited experimental data to guide the calculations, single-particle models are a reasonable approach for initial calculations of Z = 121 nuclei [3, 5, 21, 29-31].

The stability of Z = 121 systems is determined by evaluating the various decay modes (i.e., alpha decay, beta decay, positron decay, electron capture, and spontaneous fission). Based on previous calculations [1, 3, 5, 21-31], stability in the Z = 121 mass region is expected to be dominated by alpha decay.

2. Calculational Methodology

The model used to describe the particle (i) plus core (c) system represents an application of the standard method of Lukasiak and Sobiczewski [3] and Petrovich *et al.* [4]. The calculational method used to generate the single-particle level spectrum determines the binding energy E_{NLSJ} of a particle in the field of a nuclear core by solving the radial Schrödinger Equation:

$$\left[\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{L(L+1)}{r^2}\right) - E_{NLSJ} - V_{LSJ}(r)\right] U_{NLSJ}(r) = 0 \quad (1)$$

Where r is the radial coordinate defining the relative motion of the nuclear core and the particle; $V_{LSJ}(r)$ is the model interaction; E_{NLSJ} is the core plus particle binding energy; $U_{NLSJ}(r)$ is the radial wave function; and L, S, and J are the orbital, spin, and total angular momentum quantum numbers, respectively. N is the radial quantum number and μ is the reduced mass. For the present application, V_{LSJ} is defined as:

$$V_{LSJ}(r) = -\frac{V_0}{1+exp\left(\frac{r-R_O}{a_O}\right)} - V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{a_{so}r}\frac{exp\left(\frac{r-R_{so}}{a_{so}}\right)}{\left[1+exp\left(\frac{r-R_{so}}{a_{so}}\right)\right]^2}F(L,S,J) + Z_i Z_C e^2 C(r) \quad (2)$$

Where

$$R_0 = r_0 A^{\frac{1}{3}} = R_C \quad (3)$$

And

$$R_{so} = r_{so} A^{1/3} \quad (4)$$

The parameters V_0, r_0, and a_0 are the strength, radius parameter, and diffuseness for the central potential. Similarly, V_{so}, r_{so}, and a_{so} are the corresponding parameters for the spin-orbit potential. To complete the specification of (2), we define:

$$F(L,S,J) = J(J+1) - L(L+1) - S(S+1) \quad (5)$$

And

$$C(r) = \frac{1}{2R_C}\left(3 - \left(\frac{r}{R_C}\right)^2\right) \; for \;\; r < R_C \quad (6)$$

$$C(r) = \frac{1}{r} \;\; for \;\; r \geq R_C \quad (7)$$

For the Coulomb potential, it is assumed that the particle is a point charge of magnitude $z_i e$. The core has a charge $Z_C e$ uniformly distributed through a sphere of radius R_C. Since the potential is not a function of the spherical coordinates, the solution of the angular equation is most easily expressed in terms of spherical harmonics $Y_{LM}(\theta,\varphi)$.

The total bound-state wave function $\left(\Psi_{NLSJM}(\vec{r})\right)$ for the relative motion of the core plus particle, interacting through a spherically symmetric potential, is given by the product of space and spin wave functions:

$$\Psi_{NLSJM}(\vec{r}) = \frac{1}{r}U_{NLSJ}(r)\sum_{M_L M_S} C(L,M_L,S,M_S;JM)Y_{LM_L}(\theta,\varphi)\chi_{SM_S} \quad (8)$$

Where M_L and M_S are the projections of angular momentum and spin, and χ is the spin wave function. For the calculation of single-particle energy levels, N, L, S, and J specify the quantum numbers of the single-particle level.

The binding energy of a single-particle level is obtained by rewriting the radial Schrödinger equation in the form:

$$\left(\frac{d^2}{dr^2} - k(p,r)\right)U(p,r) = 0 \quad (9)$$

Where

$$U(p,r) = U_{NLSJ} \quad (10)$$

And

$$k(p,r) = \frac{L(L+1)}{r^2} + \frac{2\mu}{\hbar^2}\left(E_{NLSJ} + V_{LSJ}(r)\right) \quad (11)$$

The model searches for values of the parameter p in order to obtain the binding energy E_{NLSJ} for a given potential. The method of searching for p is provided by Brown, Gunn, and Gould [36] using the methodology of Ref. 37 to obtain a converged solution. Refs. 2 - 4, 21, 29, and 31 provide additional details of the model, numerical methods, and associated interactions.

2.1 Determination of Q Values and Half-Lives

Alpha decay Q values are calculated using the relationship:

$$Q_\alpha = 28.3 \text{ MeV} - 2\,S_n - 2\,S_p \quad (12)$$

Where S_n and S_p are the binding energies of the last occupied neutron and proton single-particle energy levels, respectively [1]. The alpha decay, beta decay, positron decay, and electron capture half-lives were determined following the methodology noted

in Ref. 3. Spontaneous fission half-lives were qualitatively evaluated using the methodology of Ref. 3 and were not limiting [4, 21].

The log ft methodology of Wong [1] is used to determine the beta decay half-lives. Allowed (first-forbidden) beta decay half-lives are obtained from the values of log ft = 5 (8) [1]. In view of the uncertainties in the calculated level energies, second and higher forbidden transitions were not determined. The beta half-lives summarized in Table 1 listed as *stable* are either beta stable or decay by these higher order forbidden transitions.

3. Nuclear Interaction

The model considers the single-particle spectrum generated by a Woods-Saxon potential with parameters optimized for extrapolation into the region of superheavy nuclei [4]. The parameters of the Rost potential were obtained from a fit to the particle levels in ^{208}Pb and ^{209}Bi [2].

The single-particle level spectrum is generated using a Woods-Saxon potential of the Rost interaction [2]. The Rost central interaction strength is given by the relationship:

$$V_0 = 51.6 \left[1 \pm 0.73 \frac{N-A}{Z}\right] MeV \quad (13)$$

The potential for protons (neutrons) is given by the upper (lower) sign. The remaining parameters were held constant and are given by Rost [2]: r_0 = 1.262 (1.295) fm, rso = 0.908 (1.194) fm, a = 0.70 (0.70) fm, and λ = 17.5 (28.2) for protons (neutrons). Vso is related toλby the relationship [2]:

$$V_{so} = \frac{\lambda V_0}{180} \quad (14)$$

The scaling relationship of Eq. 13 yields reasonable fits to observed single-particles levels in ^{120}Sn and ^{138}Ba [4]. The pairing correction term of Blomqvist and Wahlborn [38] is used in the calculations presented herein. The pairing correction improves the predicted energies of occupied levels in ^{120}Sn, ^{138}Ba, and ^{208}Pb [4]. A Woods-Saxon, potential which reproduces the single-particle levels in the lead region, provides the basis for a discussion of the stability properties of Z=121 superheavy nuclei [2, 4].

4. Results and Discussion

Table 1 summarizes the results of Z = 121 single-particle level calculations. This table provides the alpha decay Q value and the alpha and positron decay half-lives for A = 301 – 307, Z = 121 systems. This mass range is consistent with the calculations summarized in Ref. 27 that reviewed the possible reactions that would theoretically produce the various Z = 120 isotopes.

The results suggest that A = 301 – 307 systems have predicted half-lives that are shorter than the Z= 119 and 120 systems. Alpha emission is predicted to be the limiting decay mode. Table 1 also provides the valance neutron and proton single-particle levels for each of the calculated Z = 121 systems. The most stable Z = 121 isotope is predicted to occur at A = 305 with an alpha decay half-life of 37.5 ns. Following Ref. 3, shell corrections are included in the alpha decay half-life and Q-value calculations.

Table 1: Z=121 Q_α and Half Life Values

A (p, n single-particle level)	Q_α (MeV)	$T_{1/2}^{\alpha}$	$T_{1/2}^{\beta}$
301 ($3p_{3/2}$, $4s_{1/2}$)	14.2	0.272 ns	24.4 min[a]
302 ($3p_{3/2}$, $3d_{3/2}$)	14.0	0.696 ns	26.8 min[a]
303 ($3p_{3/2}$, $3d_{3/2}$)	13.7	2.53 ns	29.6 min[a]
304 ($3p_{3/2}$, $3d_{3/2}$)	13.4	9.49 ns	32.8 min[a]
305 ($3p_{3/2}$, $3d_{3/2}$)	13.1	37.5 ns	9.48 min[b]
306 ($3p_{3/2}$, $2h_{11/2}$)	17.6	1.94×10^{-15} s	3.65 min[b]
307 ($3p_{3/2}$, $2h_{11/2}$)	17.3	4.84×10^{-15} s	1.74 min[b]

[a] First forbidden $3p_{3/2}$(p) to $3d_{3/2}$ (n) positron transition.
[b] Allowed $4p_{1/2}$(n) to $3p_{3/2}$(p) beta transition.

The A = 305 single-particle level diagram for Z = 121 is provided in Fig 1. The last occupied neutron (proton) single-particle level is $3d_{3/2}$ ($3p_{3/2}$). Half-life values for the Z = 121 systems are in ns range, but rapidly decline beyond A = 305. Nuclear stability rapidly decreases beyond A = 305 as the valence neutron transitions from the $3d_{3/2}$ to $2h_{11/2}$ single-particle level. The decrease in the $2h_{11/2}$ neutron single-particle binding energy increases the Q_α value that leads to a shorter alpha decay half-life.

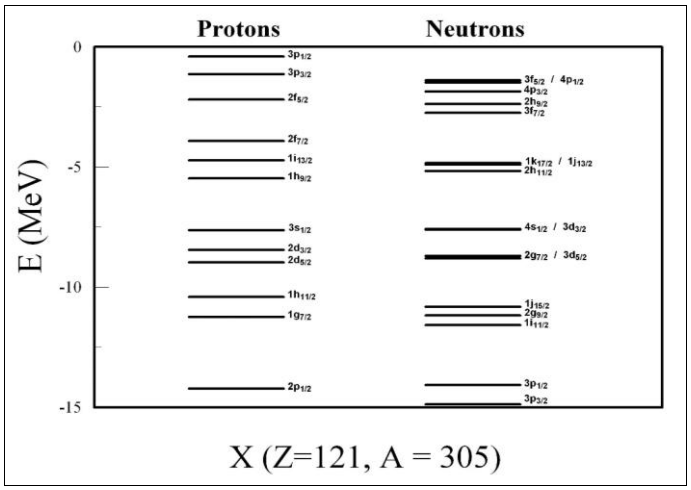

Fig 1: Calculated single-particle levels for the most stable Z = 121 nucleus (A = 305). The notation a/b is used to indicate adjacent energy levels with level a more tightly bound than level b

Imrad *et al*. [34] provide a set of Z = 121 calculations for A = 291 – 341. This range of A values exceeds the expected number of known A = 110 – 118 nuclear systems and the theoretical number of A = 120 systems predicted by Wan *et al*. [28], Sobiczewski [30], and Bevelacqua in A = 119 and 120 systems [29, 31].

In general, the Ref. 34 model predicts significantly longer half-lives than the proposed approach. However, the model results for the A = 305 system is much closer to the Ref. 34 value, but is still a factor of about 3.5 smaller. The model (Ref. 34) predicts that the most stable Z = 121 system occurs at A = 305 (309).

Table 2 compares the Z=119-121 model results to measured Z = 110 – 118 system half-life values [39]. The most stable system for each A value is also provided. The model results appear to be consistent with the trends in Z = 110 - 118 mass and half-life values. Alpha decay is the dominant decay mode in most of these systems [39], and that is also consistent with the model results.

Table 2: Comparison of Z = 110 – 118 Data[a] and Z = 119 - 121 Model Results

Z	Mass Range	Half-Life Range	Most Stable
110	267 – 282	3.0 µs – 1.1 min	^{282}Ds
111	272 – 282	3.8 ms – 100 s	^{282}Rg
112	277 – 286	0.61 ms – 11.0 min	^{286}Cn
113	282 – 290	70 ms – 9.5 s	^{286}Nh
114	284 – 290	25 ms – 19 s	^{290}Fl
115	287 – 290	37 ms -0.65 s	^{290}Mc
116	290 – 293	8.3 ms – 95 ms	^{293}Lv
117	293 – 294	21 ms – 51 ms	^{294}Tn
118	294	0.58 ms	^{294}Og
119	290 – 304	2.48 ps – 0.112 s	$^{297}_{119}X^{b}$ [b]
120	292 – 305	2.27 ps – 77.5 ms	$^{298}_{120}X^{c}$ [c]
121	301 - 307	0.272 – 37.5 ns	$^{305}_{121}X^{d}$

[a]Derived from NuDat3 [39].
[b]Ref. 29.
[c]Ref. 31.
[d]This work.

Within the scope of the model, the effective half-life of the Z = 121 systems includes the combined effect of the alpha and positron decay modes, and is defined as:

$$T_{1/2}^{eff} = (T^{\alpha}_{1/2}\, T^{\beta}_{1/2}) / (T^{\alpha}_{1/2} + T^{\beta}_{1/2}) \quad (15)$$

Since the positron decay half-life is much larger than the alpha decay half-life, the effective half-life is essentially the alpha decay half-life.

Based on previous calculations [3, 4, 21, 29, 31], spontaneous fission stability is expected to be enhanced near closed shells. These spontaneous fission calculations utilized the Wentzel-Kramers-Brillouin (WKB) approximation methodology and the phenomenological parameter values of Ref. 3. The calculations suggest that fission half-lives near closed shells are greater than the effective decay half-lives calculated using Eq. 15.

5. Model Weaknesses

Model limitations include uncertainties in the nuclear interaction [2], exclusion of nonconventional decay modes that could exist in superheavy systems, and treating all system as spherically symmetric nuclei. In spite of these uncertainties, the Rost interaction yields results that are consistent with the trends observed in Z = 110 – 118 systems and previous Z=119 and 121 model results [29, 31].

The accuracy of the proposed model can be partially addressed by comparing the (Z, A) values of calculated system properties to the predictions of Adler's relationship [40, 41]. The Alder relationship provides the most stable nucleus Z value for a given A:

$$\mathbf{Z = (0.487\ A) / (1 + A^{2/3} /166)} \quad (16)$$

This relationship can be compared to the model predictions for the most stable A = 305 system. The Adler relationship predicts that the most stable A = 305 value occurs at about Z = 117. This is about 3% smaller than the Z =121, A = 305 result obtained from the spherical model outlined in this paper. This comparison between the model and predictions of the Adler relationship of Eq. 16 suggests at least the qualitative usefulness of the proposed model.

6. Conclusions

Single-particle energies of Z=121 nuclei are calculated to determine their decay properties. The Rost interaction is used as the basis for these calculations. A = 301 – 307 systems have predicted stability in the ns range. The most stable isotope Z = 121 is predicted to occur at A = 305 with an alpha decay half-life of 37.5 ns. These results are consistent with the observed properties of Z = 110 – 118 nuclei and Z=119=120 model calculations.

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