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Numerical Reuleaux Method Applications for Primary Mathematical-Computational Demonstration of Earth Rotation Axis Time-Interval Dynamical Changes. Introduction II

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Abstract

The Numerical Reuleaux Method (NMR), has multiple applications in science and engineering. Based on a previous publication, this article shows the analytic geometry and mathematical-physics demostration of the changes in the earth rotation axis (ERA) position when nutation dynamics. ERA in precession and nutation is not constant. For the ERA changes during nutation time-intervals, method-base algorithms are formulated-calculated with NRM. Results comprise the presentation of principal algorithms/equations and the 2D-3D NRM numerical database simulations from previous research. Applications in computational astrophysics, dynamics/satellite-dynamics, geophysics, gravitation forces acting on earth structure, and relativistic/non-relativistic aerospace physics-engineering are briefed.

Keywords: Numerical Reuleaux Method (NRM), Earth Rotation Axis (ERA), Computational Intelligence (CI), Dynamics, Instantaneous Rotation Axis (IAR), Computational Simulations, Voxel, Pixel, Pseudo-Rigid Body (PRB), Nonlinear Optimization, Matrix Algebra, Spherical-Spatial Analytical Geometry

1. Introduction

The Numerical Reuleaux Method [1-7], was created for deformable non-rigid/pesudo-rigid bodies dynamics [1-7]. In earth geophysics, Euler rigid body rotation equations [8-17] are not exactly functional/applicable. The earth has been extensively demonstrated as a non-rigid or pseudo-rigid solid—and not an elastic body anyway [10]. Earth dynamics is subjected to a number of pseudo-solid-body constraints [18-22]. For example, to define any solid as rigid, the most important requirement is that the body distances among molecules and its composition/space-setting remain constant during the movement. Also the total-body-volume spatial density distribution. However, for earth dynamics, these strictly classical equations are not applicable as planet is a pseudo-rigid body [8-13, 19-22]. This earth physical property has been extensively proven. A definite equation for earth as pseudo-rigid body dynamics reads,

$$TORQUE\,[\vec{L}],\ EXERTED\ ON\ EARTH\,,$$

$$\vec{L} = \vec{H} + \vec{\omega} \times \vec{H}\,;$$

$$where,$$

$$H:\ Earth\ Angular\ Momemtum\,;$$

$$\vec{H}\ :\ Earth\ Angular\ Momemtum\ time-derivative;$$

$$\vec{\omega}\ :\ Earth\ Angular\ Velocity\,;$$

$$\vec{L}\ :\ Torque\ exerted\ on\ earth\,;$$

(1)

This Equation (1) holds for earth as pseudo-rigid-body, and is approximately functional, subject to several constraints—just remark that recently [8], the changes in earth volume are proven. For Equation (1), the most important requirement is that the body distances among matter-molecules and their composition/space-setting do not remain constant during the movement [8]. Also the total-body-volume spatial density distribution [8]. Therefore, for earth dynamics, the Euler classical equations are not applicable as planet is a pseudo-rigid body [8-13, 15-22]. According to these assertions, a number of contrasted comparisons among earth dynamics and basic euler dynamics equations are.

1. If and only if the solid is perfectly rigid, basic Euler equations work for approximately exact results. Remark [18, 19]: the perfect rigid body in nature and geophysics almost does not exists.
2. For euler equations, the solid rotates around an axis or has a defined/fixed rotation center. That could be function of time [14, 18, 19]. In earth rotation, angular momentum transfers occur between the solid earth and the fluid, gas, or plasma regions (the underlying semi-liquid metallic core and the overlying hydrosphere and atmosphere). Concomitant torques are caused by hydrodynamic or magneto-hydrodynamic stresses acting at the fluid stratified solid earth interfaces. An essential euler earth rotating angular moment equation [20, pp 356] reads,

$$
\begin{aligned}
&ANGULAR\ MOMEMTUM,\ EXERTED\ ON\ EARTH\ ,\\
&\vec{H}(t) = \vec{I}(t)\,\vec{\omega}(t) \times \int \vec{x}(t) \times \dot{\vec{x}}\ dm;\\
&because\ \vec{H}(t)\ is\ given\ by\ the\ sum\ of\ terms,\\
&the\ part\ caused\ by\ changes\ in\ the\ earth\ inertia\ tensor\\
&by\ variations\ in\ internal\ mass\ distribution\ \vec{I}(t)\,\vec{\omega}(t)\\
&the\ every\ dm\ motion\ relative\ to\ the\ rotating\ reference\ frame,\\
&such\ as\ :\ \int \vec{x}(t) \times \dot{\vec{x}}\ dm\ ;\\
&where,\\
&\vec{I}\,(t):\ Earth\ Inertia\ Tensor\ in\ function\ of\ time\ ;\\
&\vec{H}\ :\ Earth\ Angular\ Momemtum\ ;\\
&\vec{\omega}(t)\ :\ Earth\ Angular\ Velocity\ ;\\
&\vec{x}\,(t)\ :\ Vector\ position\ of\ dm\ ;\\
&\dot{\vec{x}}\,(t)\ :\ Vector\ velocity\ of\ dm\ ;
\end{aligned}
\qquad (2)
$$

This Equation (2) shows the internal instantaneous changes of mass distribution within the earth [20, 21]—it comprises internal core torque and external one [11]. In general, there is a number of torques acting on earth globe. Essentially one external and other internal. Namely, inner core and external core, which are opposite.

3. For euler equations, the mass and gravity center is fixed, and does not change with movement [6-17]. This does not happen with the earth planet [19-22].
4. From (2), the radii distances, r, from the rotation axis, center, or mass center are not constant during earth movements [20, pp 356].
5. The radii distances are essential for determination of clue physics parameters. Namely, linear and angular momentums, forces, and other [17-22]. In relativistic conditions are significant.
6. The earth mass center position is not constant. That is caused by tectonic plates movement, gas/water underground deposits changes, re-distribution and chemical transformations of materials, and other [17-22]. That is,

$$
\begin{aligned}
&MASS\ CENTER\ FORMULATION\\
&FOR\ DISCRETE\ VOXELS,\\
&\vec{M}_C \backsimeq \frac{\sum_{i=1}^{i=N} m_i\,\vec{r}_i}{\sum_{i=1}^{i=N} m_i}\ ,\\
&FOR\ CONTINUOUS\ MASS\ DISTRIBUTION\ VOXELS,\\
&\vec{M}_C \backsimeq \frac{1}{M_{TOTAL}} \sum_{i=1}^{i=N} \int_{V_i} \rho(\vec{r}_i)\ \vec{r}_i\, d(v_i)\ ,
\end{aligned}
\qquad (3)
$$

Therefore, the complexity of the research problem is considered sharp. It is important remark two types of ERA changes. First, the earth axis rotation precession changes related to sun, moon, and planets gravity which are not the objective of this study. Instead, the nutation, that is, the variations of ERA related to symmetry axis caused mainly by inhomogeneus mass distribution within, and volume/shape variations of earth, is the concomitant subject of this research—and these internal/surfactal changes are increasing recently [8]. The principal reasons are: (1) Earth is a pseudo-rigid body, (2) Further, earth is a pseudo-rigid body whose internal structure shows at least one additional torque while rotating, (3) The internal earth structure, its volume, and geometrical shape changes. As said, earth axis of symmetry in general does not coincide with the ERA. A demonstrated base of the study utility, it is important to illustrate the subject with one determined fact [8]. Namely, a change of 1 mm for a earth spherical/ellipsoid sector, multiplied by the sector circular-length/length, causes a significant volume/shape/mass mutation-distribution in the earth.

Objectives of the Study

The objectives of the study are mainly:

1. Confirm/demonstrate that the ERA position changes according to literature [8-19], and the validity for applied Numerical Reuleaux Method [1-7] in this earth nutation research field.
2. Once the objective (1) is proven, develop a numerical optimization method based on NRM for ERA nutation calculus within a time-interval. Complementary, begin to analyze the interactive, internal and external, geophysical factors that cause the ERA nutation changes. That is, from the applied NRM to obtain the ERA position, guess inversely the angular momemtum wobbles of the mantle, the fluid outer core angular momemtum, and the solid inner core one. That is a rather complicated inverse problem—getting dynamical-geometrical information of their changes/fluctuations.
3. Present a briefing of the findings applications for geophysics, earth dynamics, aerospace engineering and astronomy in general.

In summary, an introductory research to apply NRM for ERA time-interval nutation-calculus was done/presented at introduction stage. Geophysics and dynamics applications are briefed.

2. Mathematical-Geometrical Computational Method

Most generally, the calculation of ERA involves a rather large number of factors, Table 1. The NRM first stage at

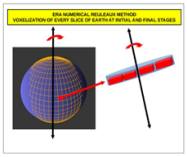
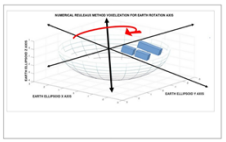
present is empirical. It determines ERA nutation for a defined time-interval, taking exclusively the movement of the earth voxels at initial and final interval with rather large software engineering. Therefore, based on inverse theory, the interaction of gravitational forces, internal and deformation forces that cause precession may (migth) be inversely determined, given the variation of ERA nutation for that interval, Table 1 [1-17].

Table 1: Briefing of External and Internal Factors that determine the position/variations of ERA [1-15]

INTERACTIVE FACTORS FOR ERA DETERMINATION WITH NUMERICAL REULEAUX METHOD [Casesnoves, Intellectual Property creation, 2006-7]	
EARTH INTERNAL STRUCTURE FACTORS	
GEODYNAMICS FACTOR	CONSEQUENCES
Tectonic plates movement/changes	Changes in density distribution, gravitation forces, and earth kinetic and angular moment
Gas internal zones	Changes in density distribution and gravitation forces
Water internal zones	Changes in density distribution and gravitation forces
Ice melting at poles	Changes in earth volume and geometry density distribution and gravitation forces
Earth volume changes	For example, recent publications show a variation of [0.5 – 1] mm/year in earth volume
EARTH EXTERNAL FACTORS	
GEODYNAMICS FACTOR	CONSEQUENCES
Sun gravitation forces	High magnitude forces
Earth and moon mass center	Significant influence in both orbit and rotation forces
Other solar system planets	Significant influence in both orbit and rotation forces
GEOPHYSICS CONSEQUENCES; ALL THESE FACTORS CONVOLUTE/INTERACT ONE ANOTHER	

The method involves initial stage and final stage (inner final changes) mainly, with a software engineering series for all the steps.

Table 2: Interactive-convoluted External and Internal Factors that determine the position/variations of ERA [1-17]

ERA DETERMINATION SIMULATION PRIMARY COMPUTATIONAL NUMERICAL REULEAUX METHOD	
GENERAL STAGES	
STAGE	COMPUTATIONAL RESULT
Divide the earth globe in slices	The higher number of slices, the better results
Initial stage: divide every slice in voxels/ pixels according to experimental literature	The higher number of voxels/pixels, the better results
Final stage: apply NRM for every voxel/ pixel which according to experimental literature that has changed in volume/distribution	Computational-analytic methods. The higher number of voxels/pixels, the better results
Obtain for every slice a point corresponding to slice-IRC of ERA, for 2D is ERA point, for 3D is optimal polygon [see NRM main references]	It is got a number of points from north pole to south pole
Apply a numerical method to obtain a straight line that is the ERA for that time-interval	Usually polynomial software programming. There are more options. The higher number of slices and voxels/pixels, the better results
SOFTWARE PROGRAMMING	
STAGE	COMPUTATIONAL RESULT
Set matrices for initial and final stages	Software arrays and patterns
Program to calculate the geometrical intersections of NRM for every slice, getting a number of slice-IRCs	That is rather complicated
Make software for a linear fit with those points and obtain equation of straight line which is the ERA for that time-interval	Linear optimization usually with a one-degree polynomial and inverse method
GEOPHYSICS CONSEQUENCES; THE HIGHER NUMBER OF SLICES AND VOXELS/PIXELS, THE BETTER NUMERICAL PRECISION FOR ERA, DECREASE OF ERRORS AND GETTING HIGHER PRECISION	
 PIXELIZATION-VOXELIZATION GRAPHICAL ABSTRACT NUMERICAL REULEAUX METHOD FOR EARTH ROTATION AXIS [ERA] 	

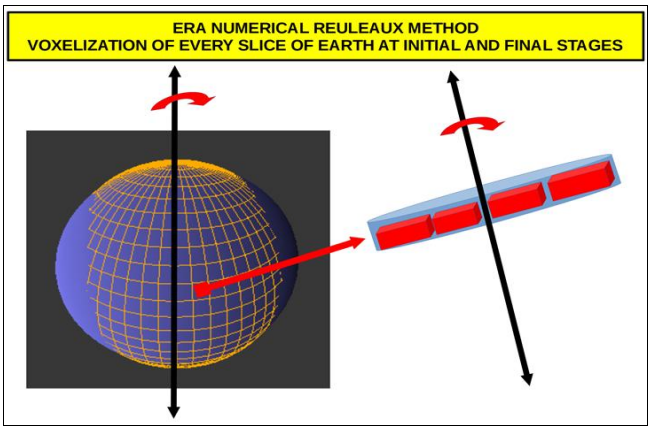


Fig 1: Saggital slice of earth globe showing voxelization to calculate that slice IRC. GNU-Octave software

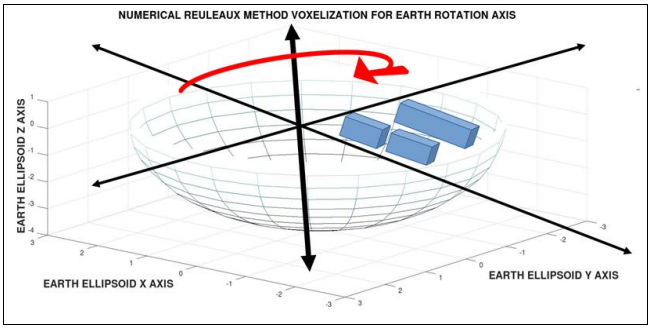


Fig 2: Saggital slice of earth globe showing voxelization to calculate that slice IRC. GNU-Octave software. [Casesnoves Bioengineering Laboratory Software 113-A]

3. Equations-Method Results and Physics Dynamics Consequences

Fig 3 shows the simulation technique to prove the NRM, [see NRM algorithms, 1-7]. Firstly, the IRC is calculated without deformation. Secondly, the IRC is calculated with deformations applied of points with arrows.

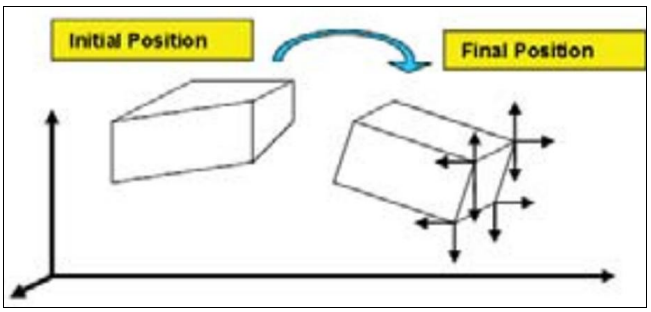


Fig 3: [From 1, Figures 1-7 for 2D, and Fig 8 for 3D]. 3D deformation of the pseudo-rigid solid involves the arrow vertices of the pseudo-rigid body after rotation or rotation-translation movement. It is recommended to study the [1, Fig 1] for sharp understanding of NRM method with comparison to classical-simple Reuleaux Method

At appendix, [1-7], numerical results for 1 and 2 in 2D pixels(2D)/voxels(3D) are demonstrated. That is, the error for IRC decreases when the number of pixels/voxels increase. At [1-7], a series of numerical results tables is shown. These numerical results are applicable for this study in upcoming further simulations.

4. Geophysics Dynamics Applications

Table 3 shows a number of geodynamics NRM in ERA determination usages. Just to remark a few important points. Namely, the relativistic velocity influence when distance to the ERA increases. Also, when any aircraft big mass increase happens, a similar effect occurs. It is important to remark the interaction of dynamical and geophysics essential parameters/concepts in this ERA precise determination problem. There is a convergence of torques, mass center, precession, symmetry axis, ERA, interior earth structure, and gravitation forces, among others. Fig 4, from [7], shows an example of NRM aerospace application for an aircraft.

Table 3: Dynamics and geophysical applications of study method results [1-7]

EARTH ROTATION AXIS DETERMINATION DYNAMICS APPLICATIONS WITH NUMERICAL REULEAUX METHOD BRIEFING	
GENERAL APPLICATIONS	
APPLICATION	**CONSEQUENCES**
Relativistic earth dynamics	If the distance to ERA and earth mass center, when a surrounding aerospace aircrafts fly at relativistic velocity, creates variations in the momemtums, among other parameters.
Changes in earth mass center	When ERA varies, the mass center also changes position, with geodynamics consequences
Aerospace aircrafts with high mass variations	Provided any aircraft surrouding earth has a high mass, the physical constants, such as momemtums, change if ERA varies.
Earth geophysical changes database	If ERA and earth volume/shape change, the prediction for life evolution at planet provides with anticipation survival precautions.
Medical devices	There is a large number of medical rotating prostheses/devices which are rotating pseudo-rigid bodies with a rotation axis
PHYSICS DYNAMICS APPLICATIONS	
APPLICATION	**CONSEQUENCES**
Pseudo-rigid bodies dynamical rotation algorithms	Modified Euler formulation
Pseudo-rigid bodies dynamical rotation algorithms subject to internal additional torque(s)	Not exclusive of earth planet, also useful for mechanical and aerospace engineering, among other areas of geophysics
Pseudo-rigid bodies dynamical rotation algorithms subject to volume and/or geometrical shape changes in time-intervals	New equations/algorithms frame for these type of solids

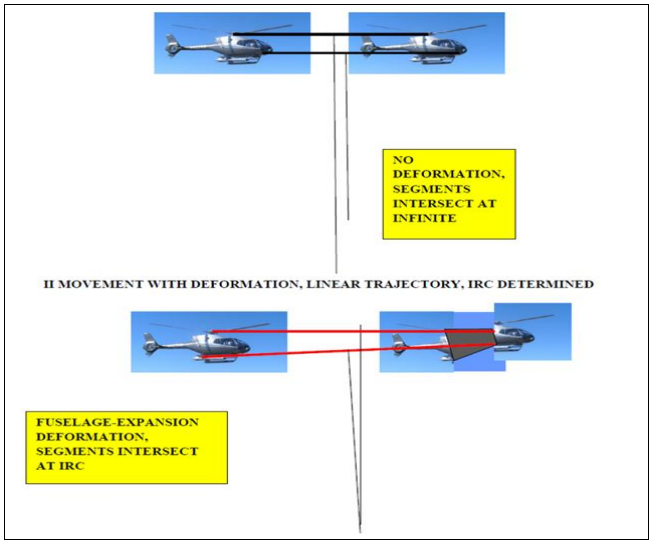


Fig 4: Figure of dynamics and geophysical applications of study method results [1-7]

5. Discussion and Conclusions

The objective of the study was to obtain an initial numerical-computational method to determine the changes in ERA with NRM. The proofs based on recent and previous publications are incorporated within Numerical Reuleaux Method framework.

Explicitly, the purposes of the study were mainly two. Namely, first to prove that earth is not a rigid body in motion. Secondly, to show the application of NRM to obtain earth planet ERA during a defined time-interval for nutation. The differences and limitations of the Euler theoretical method for rigid body dynamics in earth movement, specifically rotation, are proven and explained. The Numerical Reuleaux Method was applied, based on large number of previous publications, to approximate time-interval variations of the ERA mainly during nutation, not precession. This NRM application is based on a large number of previous publications [1-7].

Results comprise the base of the mathematical-dynamical method to get numerically the ERA for a defined time interval. The previous contributions 3D numerical-geometrical demonstrations which show that the rotation center of a pseudo-rigid body precision increases directly proportionaly to the number of 2D pixels or 3D voxels are explained. This numerical-geometrical fact is directly applicable for ERA determination.

The method is empirical for a time-interval of earth rotation. It is not in function of gravitation forces from moon, sun and planets that create the earth rotation variations and day-cycle duration changes. That is the current study stage. Dynamics for earth geophysics applications are briefed and justified. Dynamics fundamental equations with these findings applications are explained. Definitely, a new possibly important application of Nmerical Reuleaux Method, [Casesnoves, 2006-7] was discovered.

In summary, results comprise a geometrical-numerical demonstration of NRM validity for determination of time-interval of ERA by using approximated 2D-3D pixelization-voxelization. Geophysics earth dynamics consequences are briefed.

6. Scientific Ethics Standards

According to European Union and International Scientific Ethics. All the software/algorithms were done by Author. Numerical reuleaux Method invention and intellectual property belongs to Dr Francisco Casesnoves PhD since [2006-2007] [1-7]. No artificial intelligence (AI) was used for programming or any article-part. If were so, it is declared, both in tools/systems for software anyway, and AI browsers-direct-information. When any mathematical statement, algorithm/proposition/theorem is presented, proofs and parameters/units are detailed/referred. Earlier research database, algorithms/images, are selected from prior publications in order to sharp reading with further references included. If any inconsistency is found, it is clarified in subsequent publications. Any citation such as [Casesnoves, 'year'], is intended to clarify intellectual property at current times, without intention to brag. Article is exclusively scientific, without persuasive commercial, institutional, academic influences, deliberate obfuscation, religious/religious-influences, non-scientific theories, personal opinions, political ideas, or economical-conflict links. Author has no conflict of interest.

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