Int. j. adv. multidisc. res. stud. 2026; 6(6):787-792

Received: 29-09-2026
Accepted: 09-11-2026

International Journal of Advanced Multidisciplinary Research and Studies

ISSN: 2583-049X

Influence of Couple Stress on the Jeffrey Fluid Flow Over a Vertical Cone in the Presence of Non-Uniform Heat Source/Sink and Thermal Radiation

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Abstract

This study investigates the steady, two-dimensional boundary-layer flow and heat transfer characteristics of a non-Newtonian Jeffrey fluid over a vertical cone, incorporating the combined effects of couple stress, thermal radiation, and a non-uniform (spatially-dependent) heat source/sink. The governing partial differential equations for mass, momentum, and energy conservation are transformed into a system of non-linear ordinary differential equations using appropriate similarity variables and solved numerically via the bvp4c solver in MATLAB. The results reveal that an increase in the Deborah number, which characterizes the elastic memory of the Jeffrey fluid, reduces the fluid velocity due to enhanced elastic resistance to deformation. Similarly, a rise in the couple stress parameter decreases the velocity profile by introducing micro-rotational resistance that effectively increases the fluid's viscosity. Conversely, the fluid temperature increases with both the thermal radiation parameter, as radiation adds extra thermal energy beyond conduction, and the spatial-dependent heat source parameter, due to additional internal heat generation. The skin friction coefficient increases with both the Deborah number and the couple stress parameter, reflecting intensified wall shear stress. The Nusselt number increases with the thermal radiation parameter, indicating enhanced heat transfer, but decreases with the spatial-dependent heat source parameter, signifying reduced cooling effectiveness. These findings provide valuable insights for optimizing thermal management systems and industrial processes involving complex fluids with microstructural and viscoelastic effects.

Keywords: Non-Newtonian Fluid, bvp4c, Prandtl Number, Nusselt Number, Skin Friction

1. Introduction

Fluid flow over a vertical cone is a canonical geometry for studying boundary-layer behaviour and buoyancy-driven transport, with significance rooted in its ability to model fundamental heat and mass transfer phenomena. Its applications span thermal management systems, such as heat exchangers and nuclear reactor cooling, as well as industrial manufacturing processes including hot rolling, wire drawing, and glass fiber production. Research on this configuration often incorporates complexities like porous media, magnetic fields, and nanofluids to optimize performance in solar thermal collectors, geothermal energy conversion, and chemical reactors. Rao *et al.* [1] developed a mathematical model for Darcy free convection of a water-based nanofluid containing gyrotactic microorganisms over an isothermal vertical cone embedded in a porous medium, incorporating Brownian motion, thermophoresis, chemical reaction, and a convective boundary condition. The results demonstrated that the rescaled microorganism density rises with buoyancy ratio and bioconvection Rayleigh number but falls as the bioconvection Lewis number and bioconvection Péclet number increase. Meena [2] developed a mathematical model for steady, two-dimensional laminar mixed convection flow of an incompressible viscous fluid over a vertical cone saturated in a porous medium, accounting for double dispersion (thermal and solutal) and a first-order homogeneous chemical reaction. The study showed that rising Prandtl number increases skin friction and Sherwood number while lowering Nusselt number, and higher Schmidt number boosts mass transfer but reduces skin friction and heat transfer. Reddy *et al.* [3] investigated the magnetohydrodynamic flow of a Ag/SWCNT–water Maxwell hybrid nanofluid over a vertical cone, incorporating the Cattaneo–Christov heat flux model, thermal radiation, chemical reaction, convective boundary condition, and suction effects. Sayed and Fathy [4] examined the laminar boundary-layer flow and heat transfer of a nanofluid along an upward vertical cone under the combined influence of a transverse magnetic field, Hall current, thermal radiation, Joule heating, and internal heat generation/absorption. Ragulkumar *et al.* [5] numerically investigated unsteady MHD free convective flow of water-based nanofluids (Al_2O_3, Ag, Cu, and TiO_2) over a vertical cone under the combined effects of viscous dissipation, thermal radiation,

chemical reaction, heat source/sink, and suction/injection, using heat and mass flux boundary conditions. Shiva Rao and Deka [6] examined the steady, two-dimensional MHD flow of a SWCNT–Al_2O_3/water hybrid nanofluid over a vertical permeable cone embedded in a porous medium, incorporating thermal radiation, chemical reaction, heat generation, viscous dissipation, Joule heating, and suction effects. Later, various researchers [7-11] studied different fluid flows over a vertical cone in the presence of various parameters including magnetic field parameter.

Couple stress effects become significant when the characteristic geometric length scale is comparable to the material's intrinsic length, which is particularly relevant in narrow gaps and microscale systems. The inclusion of couple stresses fundamentally alters flow behaviour by introducing a size-dependent effect absent in classical Newtonian fluids, typically leading to enhanced pressure buildup, greater load-carrying capacity, and prolonged squeeze-film times in lubrication applications. These characteristics make the couple stress model indispensable for accurately predicting the performance of journal bearings, squeeze film dampers, and MEMS devices where conventional theory falls short. Beyond tribology, the model finds utility in characterizing biological fluids and complex industrial processes such as roll coating, where the couple stress parameter moderates velocity gradients and pressure distribution. Palaiah *et al.* [12] numerically investigated unsteady, incompressible free convective flow of a magnetized couple stress fluid past a vertical cylinder embedded in a porous medium, incorporating thermal radiation, viscous dissipation, and chemical reaction effects. Sneha *et al.* [13] analytically examined the unsteady flow and heat transfer of a couple stress ternary hybrid nanofluid (Al_2O_3, single-wall CNT, and graphene in water) over a porous stretching/shrinking sheet, incorporating thermal radiation, mass transpiration, and Biot number effects. Higher couple stress values reduced the temperature distribution and skin friction coefficient, while axial velocity profiles rose with the couple stress parameter in both stretching and shrinking cases, with stronger mass transpiration further thinning the thermal layer. Nasir *et al.* [14] examined the unsteady magnetohydrodynamic nonlinear convective flow of a CuO –Cu/blood couple stress Casson hybrid nanofluid in the stagnation region of a rotating sphere, incorporating thermal radiation, Brownian motion, thermophoresis, and chemical reaction effects. Later, several authors [15-18] discussed the impact of couple stress on various fluid flows over different geometries.

Despite extensive research on flow over vertical cones involving various fluids and parameters, the combined influence of couple stress and Jeffrey fluid viscoelasticity on boundary-layer flow over a vertical cone with non-uniform heat source/sink and thermal radiation has not been systematically examined. The Jeffrey fluid model captures both elastic and viscous behavior, making it suitable for polymer solutions and biological fluids, while couple stress effects become significant in microscale systems and narrow gaps where the intrinsic material length scale is comparable to the geometric length scale. The inclusion of a spatially-dependent heat source/sink further reflects realistic thermal conditions in industrial processes such as heat exchangers, nuclear reactor cooling, and glass fibre production. This study addresses this gap by developing a comprehensive mathematical model and providing quantitative results for velocity, temperature, skin friction, and Nusselt number under the combined effects of these parameters. The findings offer valuable guidance for designing and optimizing thermal management systems and manufacturing processes involving complex non-Newtonian fluids with microstructural characteristics.

2. Mathematical Formulation

In this study, steady flow of the Jeffrey fluid over a vertical cone in the presence of thermal radiation, couple stress and non-uniform heat source is considered.

- The x-coordinate is measured along the surface of the cone starting from the vertex O, while the y-coordinate is measured normal to the surface into the fluid domain.
- The corresponding velocity components along the x- and y-axes are denoted by u and v, respectively.
- The semi-vertical angle of the cone is represented by α, and r denotes the local radius of the cone measured from the axis of symmetry to the surface.

Governing equations and boundary conditions for the present study are:

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial y}(rv)=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{1}{1+\lambda_0}\left(\frac{\partial^2 u}{\partial y^2}+\lambda_1\left(\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\,\partial y}+u\frac{\partial^3 u}{\partial x\,\partial y^2}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right)\right)-\frac{\gamma}{\rho}\frac{\partial^4 u}{\partial y^4}+g\left(\beta_T(T_w-T_\infty)\right)Cos\,\alpha \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{(\rho C_p)}\left(\left(k+\frac{16}{3}\frac{\sigma^* T_\infty^3}{k^*}\right)\frac{\partial^2 T}{\partial y^2}\right)+\frac{1}{(\rho C_p)}\frac{kGr^{\frac{1}{2}}}{x^2}\left(A(T_w-T_\infty)e^{-\frac{y}{x}(Gr_x)^{\frac{1}{4}}}+B(T-T_\infty)\right) \tag{3}$$

$$\left.\begin{aligned}&\text{at } y=0: u=0, v=0, T=T_w,\\&\text{as } y\to\infty: T\to T_\infty, u\to 0.\end{aligned}\right\} \tag{4}$$

The following similarity transformations are used to transform the governing partial differential equations into the ordinary differential equations:

$$\left.\begin{aligned} &u=(Gr_x)^{0.5}\frac{\upsilon_f}{x}f'(\eta),\eta=(Gr_x)^{0.25}\frac{y}{x},\ v=(Gr_x)^{0.25}\frac{\upsilon_f}{4x}(\eta f'(\eta)-7f'(\eta)),\\ &T=T_\infty+(T_w-T_\infty)\theta(\eta). \end{aligned}\right\} \tag{5}$$

With the aid of the terms in (5), equations (2)-(3) along with the boundary conditions in (4) are transformed as below:

$$\frac{1}{1+\lambda_0}\left(f'''+De\left(0.25(f'')^2-\frac{7}{4}ff''''-0.5f'f''\right)\right)+\theta-Csf''''-\frac{1}{2}(f')^2+\frac{7}{4}ff''=0 \tag{6}$$

$$\left(\left(\frac{1}{\mathrm{Pr}}+\frac{R}{\mathrm{Pr}}\right)\right)\theta''+\left(\frac{7}{4}f\theta'+\frac{1}{\mathrm{Pr}}\left(Ae^{-\eta}+B\theta\right)\right)=0 \tag{7}$$

$$\left.\begin{aligned} &\text{at } \eta=0: f'(\eta)=0,\theta(\eta)=1,f(\eta)=0,\\ &\text{as } \eta\to\infty: f'(\eta)\to 0,\theta(\eta)\to 0. \end{aligned}\right\} \tag{8}$$

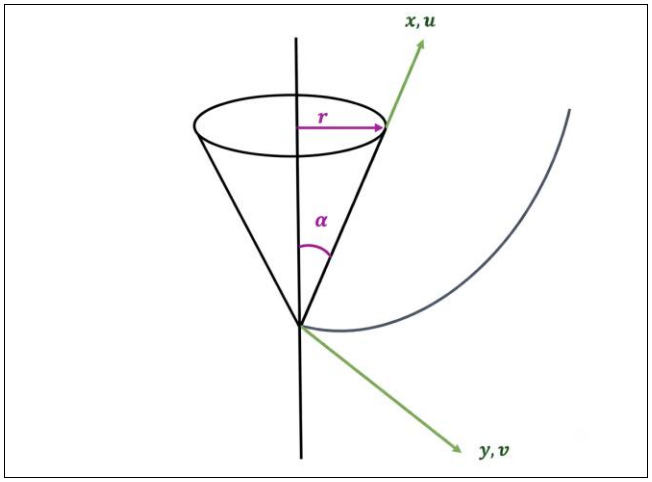


Fig 1: Graphical depiction of the current research

Note that;

$$\left.\begin{aligned} &Gr=\frac{g\beta(T_w-T_\infty)x^3\cos\alpha}{\upsilon_f^2},Cs=\frac{1}{\rho\upsilon}\frac{\gamma}{x^2}(Gr)^{0.5}\\ &r=x\sin\alpha,De=\frac{\lambda_1}{x^2}(Gr)^{0.5},\mathrm{Pr}=\frac{\mu C_p}{k},\ R=\frac{16\sigma^*T_\infty^3}{3kk^*}. \end{aligned}\right\}$$

The following are the formulae to find the skin friction coefficient and Nusselt number in this study:

$$(Gr)^{1/4}\,Cf=2f''(0),(Gr)^{-1/4}\,Nu=-(1+R)\theta'(0)$$

3. Discussion of the Outcomes

The equations (6) to (7) along ith the boundary conditions in (8) are solved using the bvp4c solver. Fig 2 shows that there is a reduction in the velocity of the fluid with the rise in the Deborah number. When this number rises, the fluid behaves more like a solid elastic material: its polymer-like microstructures resist stretching and rearrange slowly, storing energy rather than dissipating it quickly. This elastic resistance and slower relaxation oppose the driving flow, adding an effective resistance that dampens motion, so the fluid velocity drops. It is evident from Fig 3 that the rise in the couple stress parameter leads to the decrement in the fluid velocity. A rise in the couple stress parameter enhances the fluid's resistance to rotational motion at the micro-scale—essentially making the fluid "stiffer" or more viscous-like in how it resists internal spin. This increased internal resistance (higher effective viscosity) creates greater opposition to flow, so for the same applied pressure gradient, the fluid velocity decreases. Fig 4 demonstrates that the rise in the thermal radiation parameter causes an enhancement in the temperature of the fluid. A higher thermal radiation parameter means radiation contributes more strongly to heat transfer within the fluid. Since radiation transports energy through electromagnetic waves rather than relying on molecular contact, it adds extra heat energy to the fluid beyond what conduction alone could supply. This additional energy input raises the overall thermal energy content, causing the fluid temperature to increase. It is clear from Fig 5 that there is an increase in the fluid temperature with the increase in the spatial-dependent heat source parameter.

From Table 1, it is clear that there is a rise in the skin friction coefficient with the rise in the Deborah number and couple stress parameters. A higher Deborah number signals stronger elastic (memory) effects, so the fluid resists deformation more

stubbornly and transmits greater stress to the wall, raising skin friction. Likewise, a larger couple stress parameter boosts the fluid's internal micro-rotational resistance, effectively thickening its resistance to shear near the surface. Both effects intensify the velocity gradient and wall shear stress, so the skin friction coefficient grows. Table 2 demonstrates that there is an enhancement in the Nusselt number with the rise in the thermal radiation parameter and the same decreases with the increase in the spatial-dependent heat source parameter. A higher thermal radiation parameter boosts the radiative contribution to heat transfer, which steepens the temperature gradient at the wall and thereby raises the Nusselt number (enhanced cooling). In contrast, a larger spatial-dependent heat source parameter adds more heat generation within the fluid, which flattens the near-wall temperature gradient and lowers the Nusselt number, indicating reduced heat transfer effectiveness.

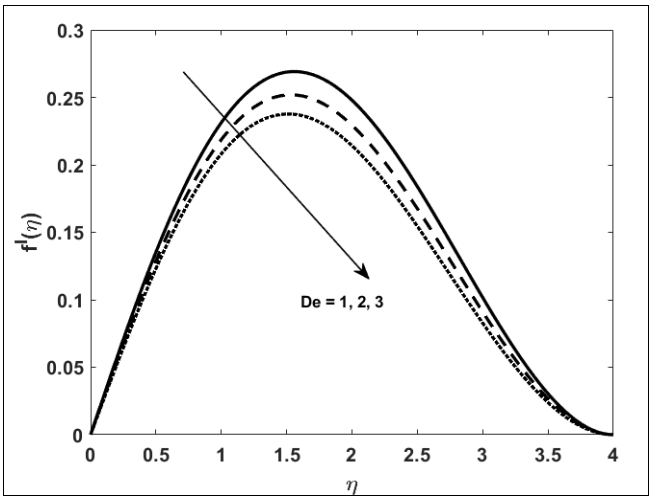


Fig 2: Impact of Deborah number on velocity profile

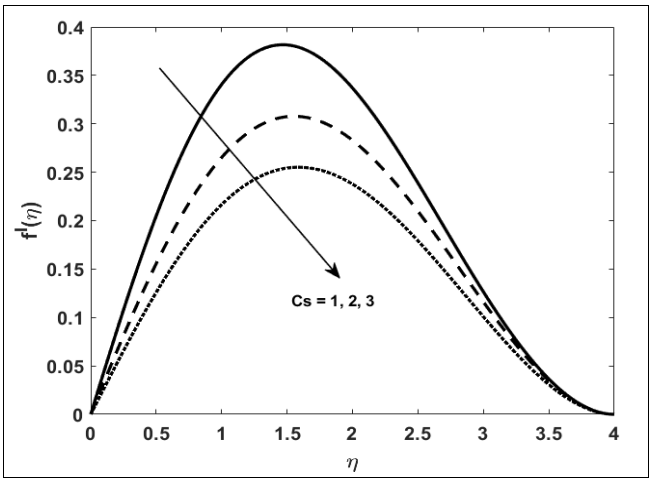


Fig 3: Impact of couple stress parameter on velocity profile

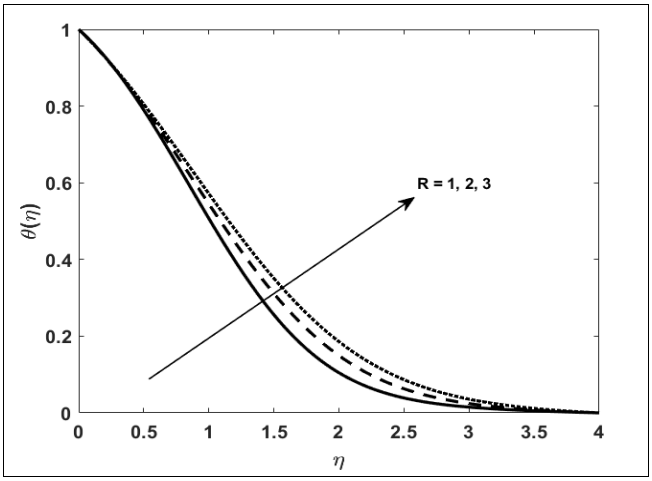


Fig 4: Impact of thermal radiation parameter on temperature profile

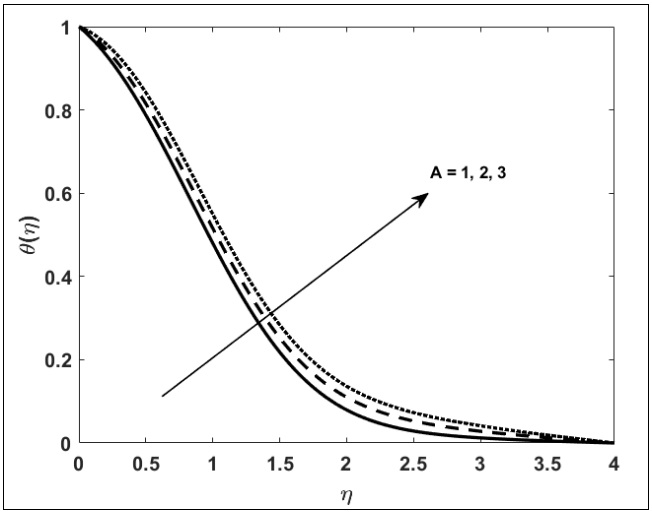


Fig 5: Impact of spatial-dependent heat source parameter on temperature profile

Table 1: Influence of couple stress parameter and Deborah number on skin friction coefficient

Cs	De	$Cf)$
0.5		0.208041154
1		0.235681679
1.5		0.272173611
2		0.323574063
	0.5	0.192977679
	1	0.196357939
	1.5	0.199976465
	2	0.203862489

Table 2: Influence of thermal radiation and spatial-dependent heat source parameters on Nusselt number

R	A	Nu
0.5		0.264626273
1		0.300612799
1.5		0.317548843
2		0.3257913
	0.3	0.290534663
	0.6	0.279408847
	0.9	0.268316506
	1.2	0.2572565

4. Conclusions

This study numerically investigated the steady, two-dimensional boundary-layer flow and heat transfer characteristics of a non-Newtonian Jeffrey fluid over a vertical cone, incorporating the combined effects of couple stress, thermal radiation, and a non-uniform heat source/sink. The governing partial differential equations were transformed into a system of non-linear ordinary differential equations using similarity transformations and solved using the bvp4c solver in MATLAB. The following are the major conclusions drawn from this investigation:

- An increase in the Deborah number leads to a decrease in fluid velocity.
- An increase in the couple stress parameter decreases the fluid velocity.
- An increase in the thermal radiation parameter raises the fluid temperature.
- An increase in the spatial-dependent heat source parameter increases the fluid temperature due to additional internal heat generation within the fluid domain.
- The skin friction coefficient increases with both the Deborah number and the couple stress parameter. Stronger elastic effects and greater micro-rotational resistance both intensify the velocity gradient and wall shear stress near the surface.
- The Nusselt number increases with the thermal radiation parameter, indicating enhanced heat transfer effectiveness due to a steeper temperature gradient at the wall, which promotes cooling.
- The Nusselt number decreases with the spatial-dependent heat source parameter, as internal heat generation flattens the near-wall temperature gradient and reduces heat transfer effectiveness.

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