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Derivation of Maxwell-Faraday Equation

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Abstract
Michael Faraday is regarded as a great experimentalist. He noticed that a coiled wire created a field (when a current passed through it) that would cause another coiled wire (not in physical contact) to have an electric current measured by a current meter. The current flow in the first coil (when the current was changing) induced another current in the second coil. James Clerk Maxwell is known for the four equations of electromagnetism and derived the following formula based on Faraday's induction experiment. See Feynman (1964):

$$\nabla x \mathbf{E} = - \partial \mathbf{B}/\partial t$$

Maxwell realized there is a field created by the changing current in the first coil. This is the phenomenon known as electromagnetic induction. Here in this paper the author will go through the mathematical steps Maxwell went through to come up with this formula. Maxwell used Stokes theorem which is. Feynman (1964):

$$\oint_\Gamma \mathbf{E} \cdot d\mathbf{s} = \int_s (\nabla x \mathbf{E}) \cdot \mathbf{n}\, da = - \int_s \partial \mathbf{B}/\partial t \cdot \mathbf{n}\, da$$

Γ is any closed curve and S is any surface bound by it. Γ is a *mathematical* curve fixed in space, and S is a fixed surface. Examining this gives the former formula. The author notes that the four Maxwell equations describe light as an electromagnetic procession of waves.

Keywords: James Clerk Maxwell, Michael Faraday, Maxwell-Faraday Formula, Induction

Introduction

Maxwell realized that in the Faraday induction experiment there was an invisible field between the two coils. See Fig from Internet for the setup, so there is an induced emf ε in the outer circuit 2 from the change in the magnetic flux $\int \mathbf{B} \cdot d\mathbf{S} = \mathbf{\Phi}$ in the inner circuit 1 where the current was changing. Thus we have.

$$\varepsilon = - d\mathbf{\Phi}/dt \text{ volts} \quad (1)$$

Φ is the magnetic flux and the integral is over the area **S** surrounded by the circuit. **B** is the vector quantity for the magnetic field. Ampére proved this equation (2) where there is no motion. Jennings (2026) [3]:

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_o i \quad (2)$$

Results

Eq. (2) is for the line integral around the closed path. The magnetic field B is given by Eq. (3).

$$B = \mu_o i/2\pi r \quad (3)$$

Maxwell added a term when there is motion to get the following equation, by Maxwell-Ampere (4). Jennings (2026) [3] Eq, (5) is from Kip (1969) [2]. page 443.

$$\nabla \text{ x } \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \partial \mathbf{E}/\partial t \quad (4)$$

$$i = \int \mathbf{j} \cdot d\mathbf{S} \quad (5)$$

j in Eq. (4) is the current density and **E** is the vector of the electric field. μ_0 is the permeability constant and ϵ_0 is the permittivity constant. Using Eq. (1) and the following formula from Kip (1969) [2], page 440.

$$\oint \mathbf{E} \cdot d\mathbf{l} = \varepsilon \quad (6)$$

We may write, by using $\int \mathbf{B} \cdot d\mathbf{S} = \mathbf{\Phi}$ and (1).

$$\varepsilon = - d\mathbf{\Phi}/dt \quad (1)$$

So, we have, by putting (1) and (6) together.

$$\oint \mathbf{E} \cdot d\mathbf{l} = - \int d\mathbf{B}/dt \cdot d\mathbf{S} \quad (7)$$

Eq. (7) is the integral form of the Maxwell-Faraday equation. According to Kip (1969) [2], page 440 there is the differential form, Eq. (8), of the Maxwell-Faraday equation as in the Abstract.

$$\nabla x\mathbf{E} = - \partial\mathbf{B}/\partial t \quad (8)$$

"In a region where the magnetic field changes with time, an electric field is set up." Kip (1969) [2], page 440.

Conclusion

What followed from Maxwell putting these four laws for electromagnetism together? These things followed: wireless communication, electric motors, power generation, optics and radar. When asked whose shoulders he stood on, Einstein said, Maxwell's. James Clerk Maxwell was a believing Christian.

Nomenclature

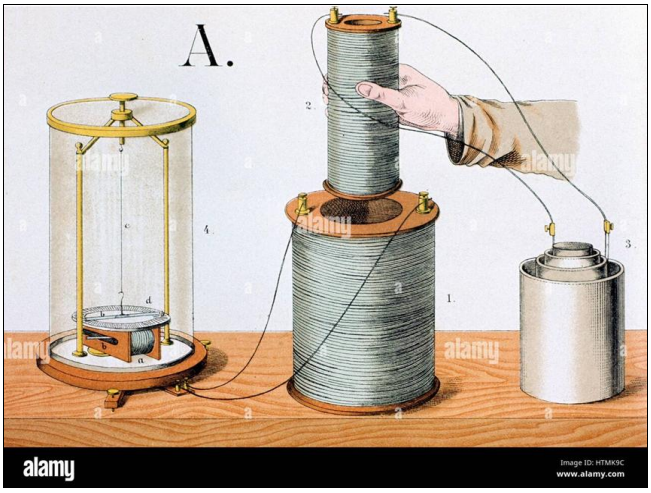
B magnetic field (vector)
E electric field (vector)
i electric current
j current density
l length (for line integral)
n unit normal vector to the surface element da
r radius
S area (surrounded by the circuit)
t time
ε induced electromotive force
μ_o permeability constant
Φ magnetic flux
ϵ_o permittivity constant
Γ symbol indicating closed curve through space

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References

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2. Kip AF. Fundamentals of Electricity and Magnetism. McGraw-Hill, Inc, 1969.
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LEGEND for Faraday's electromagnetic induction experiment
1) Outer coil of wire has induced emf
2) Inner coil of wire is powered by battery on right
3) Battery connected to the inner wire coil
4) Current meter connected to outer coil