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First-Order Weak Coupling Descriptions of the $P_{cc\bar{c}}(5715)$ and $P_{bb\bar{b}}(15569)$ Pentaquarks

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Abstract
A first-order pentaquark models is utilized to model the proposed $P_{cc\bar{c}}(5715)$ and $P_{bb\bar{b}}(15569)$ resonances as weakly coupled meson plus baryon clusters. A first-order model predicts possible $1/2^-$, $3/2^-$, and $5/2^-$ assignments for these pentaquark configurations. An analysis by Wu *et al.* predicts a $5/2^-$ assignment. The model mass results are within about 5% of deep-neural-network-based variational Monte Carlo calculations of Wu *et al.*

Keywords: First-Order Pentaquark Mass Formula, $P_{cc\bar{c}}(5715)$ and $P_{bb\bar{b}}(15569)$ Resonances, Cluster Model, Quark Model, and Quasimolecular Resonances

1. Introduction

The quark model proposed by Gell-Mann [1] and Zweig [2] provides a framework for classifying hadrons into mesons ($q\bar{q}$) and baryons (qqq). Multiquark states such as tetraquarks and pentaquarks were also proposed [1, 2].

In the spirit of [1, 2], Wu *et al.* [3] report deep-neural-network-based variational Monte Carlo calculations that suggest new pentaquark states. These states include combinations of heavy and light quarks. These include the $P_{cc\bar{c}}(5715)$ and $P_{bb\bar{b}}(15569)$. No bound solutions were found for $J^\pi = 1/2^-$ and $3/2^-$, but a $5/2^-$ state could exist [3].

This paper investigates the $P_{cc\bar{c}}(5715)$ and $P_{bb\bar{b}}(15569)$ structures as weakly coupled $ccdd\bar{c}$ and $bbdd\bar{b}$ pentaquarks, respectively. The meson and baryon substructures of these pentaquarks are defined utilizing the methodology of Zel'dovich and Sakharov [4, 5]. First-order models were previously used to describe other possible experimentally observed pentaquarks [6-15].

2. Formalism

The first-order pentaquark model weakly couples a meson (m) and baryon (b). Mesons and baryons are modeled using the approach of Refs. 4 and 5 to determine their respective masses:

$$\mathbf{M_m = \delta_m + m_1 + m_2 + b\,(m_0^2 / m_1 m_2)\,\sigma_1 \cdot \sigma_2} \tag{1}$$

$$\mathbf{M_b = \delta_b + m_1 + m_2 + m_3 + (b/3)\,[(m_0^2 / m_1 m_2)\,\sigma_1 \cdot \sigma_2 + (m_0^2 / m_1 m_3)\,\sigma_1 \cdot \sigma_3 + (m_0^2 / m_2 m_3)\,\sigma_2 \cdot \sigma_3]} \tag{2}$$

Where $\delta_m = 40$ MeV/c^2, $\delta_b = 230$ MeV/c^2, m_i is the mass of the quark comprising the meson (i = 1, 2) or baryon (i = 1, 2, 3), m_0 is the average mass of a first generation quark, and b = 615 MeV/c^2. In Eq. 1, $\sigma_1 \cdot \sigma_2$ = -3/4 or 1/4 for a pseudoscalar or vector meson, respectively.

In Eq. 2, the values of $\sigma_i \cdot \sigma_j$ depend on the baryon spin. For a J = 3/2 baryon, $\sigma_i \cdot \sigma_j$ has the value 1/4. If the total baryon spin is 1/2 and it has two *identical* quarks q_2 and q_3, the values of $\sigma_i \cdot \sigma_j$ are:

$$\boldsymbol{\sigma_2 \cdot \sigma_3 = 1/4} \text{ and } \boldsymbol{\sigma_1 \cdot \sigma_2 = \sigma_1 \cdot \sigma_3 = -1/2} \tag{3}$$

If the baryon contains three different quarks, then the values of $\sigma_i \cdot \sigma_j$ are defined by the methodology of Refs. 4 and 5.

The first-order mass formula assumes that the meson and baryon reside in their ground states, and there is zero angular momentum between the clusters. If excited states are involved, the difference in masses between the excited and ground states must be considered. The methodology for addressing excited states is outlined in Refs. 6-15.

Eqs. 1 and 2 utilize effective quark masses. These masses were determined by Griffiths [16] for d, u, s, c, b, and t quarks that have the values 340, 336, 486, 1550, 4730, and 177000 MeV/c^2, respectively. Using the convention of the Standard Model [16, 17], the quarks are grouped into three generations: [d(-1/3), u(+2/3)], [s(-1/3), c(+2/3)], and [b(-1/3), t(+2/3)] [16, 17]. Quark charges are given within the parentheses in terms of the unit charge e.

3. Results and Discussion

The $P_{cc\bar{c}}(5715)$ and $P_{bb\bar{b}}(15569)$ are modeled as $ccdd\bar{c}$ and $bbdd\bar{b}$ pentaquarks, respectively. The spin and parity assignments are provided by the primitive coupling structure of the assumed pentaquark structures. Masses are obtained from the application of Eqs. 1 and 2.

3.1 Predicted Spin and Parity Assignments

Given these $ccdd\bar{c}$ and $bbdd\bar{b}$ cluster assignments, the total spin and parity of the assumed configurations, within the scope of the first-order model, are obtained by coupling the baryon spin and the meson spin. The baryon spin couples the three quarks each with $J^\pi = 1/2^+$ that yields $1/2^+$ and $3/2^+$ values. In a similar manner, the meson J^π values are 0^- and 1^-. Therefore, the $P_{cc\bar{c}}(5715)$ and $P_{bb\bar{b}}(15569)$ pentaquarks can assume the values:

$$\mathbf{J^\pi(P_{cc\bar{c}}(5715)\ \text{and}\ P_{bb\bar{b}}(15569)) = (1/2^+,\ 3/2^+) \otimes 0 \otimes (0^-, 1^-)}$$

$$\mathbf{= 1/2^-, 3/2^-, 5/2^-} \tag{4}$$

Specific coupling combinations are provided in Tables 1 and 2.

3.2 Predicted Mass Values

The $P_{cc\bar{c}}(5715)$ and $P_{bb\bar{b}}(15569)$ pentaquark mass values, as predicted by the first-order model, are given by the following relationships:

$$\mathbf{M(P_{cc\bar{c}}(5715)) = M(ccd) + M(d\bar{c}) + \Phi} \tag{5}$$

$$\mathbf{M(P_{bb\bar{b}}(15569)) = M(bbd) + M(d\bar{b}) + \Phi} \tag{6}$$

Where $\mathbf{\Phi}$ is the interaction between the clusters. In the first-order pentaquark model $\mathbf{\Phi}$ is assumed to be much smaller than the cluster masses and is ignored. The meson (baryon) mass is obtained from Eq. 1(2). Using Eqs. 5 and 6 and the first order methodology of Eqs. 1 and 2, leads to the values summarized in Tables 1 and 2.

Table 1: Model Results for the Proposed $P_{cc\bar{c}}(5715)$

Baryon Cluster - 1			**Meson Cluster – 2**			**T_{bc}**	
Configuration	**J^π**	**Mass (MeV/c^2)**	**Configuration**	**J^π**	**Mass (MeV/c^2)**	**Mass (MeV/c^2)**	**J^π**
ccd	$3/2^+$	3.686	$d\bar{c}$	0^-	1830	5516	$3/2^-$
ccd	$3/2^+$	3.686	$d\bar{c}$	1^-	1963	5649	$1/2^-,3/2^-,5/2^-$
ccd	$1/2^+$	3.682	$d\bar{c}$	0^-	1830	5458	$1/2^-$
ccd	$1/2^+$	3.682	$d\bar{c}$	1^-	1963	5591	$1/2^-,3/2^-$
Calculations [3]						5715	$5/2^-$

Table 2: Model Results for the Proposed $P_{bb\bar{b}}(15569)$

Baryon Cluster - 1			**Meson Cluster – 2**			**T_{bc}**	
Configuration	**J^π**	**Mass (MeV/c^2)**	**Configuration**	**J^π**	**Mass (MeV/c^2)**	**Mass (MeV/c^2)**	**J^π**
bbd	$3/2^+$	10034	$d\bar{b}$	0^-	5077	15111	$3/2^-$
bbd	$3/2^+$	10034	$d\bar{b}$	1^-	5121	15155	$1/2^-,3/2^-,5/2^-$
bbd	$1/2^+$	10016	$d\bar{b}$	0^-	5077	15093	$1/2^-$
bbd	$1/2^+$	10016	$d\bar{b}$	1^-	5121	15137	$1/2^-,3/2^-$
Calculations [3]						15569	$5/2^-$

The results for the $P_{cc\bar{c}}(5715)$ pentaquark masses and their spin and parity assignments are summarized in Table 1. As noted in Table 1, the suggested $5/2^-$ value is obtained as well as $1/2^-$ and $3/2^-$ values. Given the accuracy of the first-order model, all assumed configurations are within 5% of the experimental value.

The results for the $P_{bb\bar{b}}(15569)$ pentaquark masses and their spin and parity assignments are summarized in Table 2. As noted in Table 2, the suggested $5/2^-$ value is obtained as well as $1/2^-$ and $3/2^-$ values. All assumed model configurations are within 3% of the experimental value.

4. First-Order Tetraquark Model Uncertainties and Limitations

There are a number of uncertainties and limitations that affect the model results. The limited angular momentum coupling structure restricts the available J^π values that can be evaluated. In addition, assuming zero angular momentum between the clusters also limits the evaluation of possible states.

The values for the effective quark masses [16] are not definitive. Although the weak coupling assumption appears to be reasonable, the exact magnitude for the interaction strength between the clusters is unknown [6-15]. The coupling strength will likely depend on the physical properties of the interacting systems. In spite of these uncertainties, the model continues to provide reasonably credible results [6-15] for candidate pentaquark systems.

5. Justification for the Weak Coupling Assumption

The quark charges are related to the number of colors (N_c) incorporated into the fundamental quantum chromodynamics (QCD) formulation [18, 19]. For example, the first generation quark charges within SU(N_c) are:

$$\mathbf{Q_d} = \frac{1}{2}\left(\frac{1}{N_c} - 1\right)\mathbf{e} \quad (7)$$

$$\mathbf{Q_u} = \frac{1}{2}\left(\frac{1}{N_c} + 1\right)\mathbf{e} \quad (8)$$

For conventional QCD using 3 colors, the expected d and u electric charges are obtained. The importance of QCD expansions involving $1/N_c$ illustrates the weak coupling assumption [18, 19].

A key assumption of the first-order mass formula of Eqs. 5 and 6 is weak coupling between the two clusters. In particular, the model utilized in this paper assumes the pentaquark is partitioned into two clusters with the interaction between the clusters providing a minimal contribution to the pentaquark mass. Within the scope of this mass formula, the baryon-meson cluster interaction is assumed in Eqs. 5 and 6 to be weak and sufficiently small to be ignored.

This assumption is justified because QCD can be investigated as an expansion in $1/N_c$ [20, 21]. The large N_c limit reduces to a field theory of weakly interacting meson-like objects. The physical situation with $N_c = 3$ retains many of the characteristics of the $N_c \to \infty$ limit, and further justifies the weak coupling approximation.

The $1/N_c$ expansion [20, 21] is well accepted in elementary particle physics and leads to the Okubo-Zweig-Iizuka (OZI) rule [22-24] and the Skyrme model [25, 26]. In fact, in the $1/N_c$ expansion, QCD is reduced to a weakly interacting meson theory, and the meson-meson interaction is regarded to be small [20, 21]. This situation is also a characteristic of Eqs. 5 and 6.

6. Conclusions

A first-order pentaquark models is utilized to model the proposed $P_{cc\bar{c}}(5715)$ and $P_{bb\bar{b}}(15569)$ resonances as weakly coupled meson plus baryon clusters. A first-order model predicts possible $1/2^-$, $3/2^-$, and $5/2^-$ assignment for these pentaquark configurations. An analysis by Wu *et al.* predicts a $5/2^-$ assignment. The model mass results are within about 5% of deep-neural-network-based variational Monte Carlo calculations of Wu *et al.* for the proposed $P_{cc\bar{c}}(5715)$ and $P_{bb\bar{b}}(15569)$ resonances. However, the first-order model is not sufficiently accurate to provide more definitive mass and spin assignments.

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