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A Simple Model to Calculate Visibility Reduction by Fog

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Abstract
Generally, fog is assumed to occur when visibility is less than 1,000 m. (Srisarakham and Exell page 75 – Thai Journal of Mathematics) The derivation for fog visibility is a simple one. (Jacobson page 315) Srisarakham and Exell page 77 have this equation as the starting equation from Jacobson, setting $k = \pi r^2 n$, because k is the product of an effective across section of fog droplets πr^2 and a number concentration of droplets n. k = total extinction coefficient.

$$d I'/d x' = \pi r^2 n (Io - I')$$

The integral is the following, using (CRC Handbook of Chemistry and Physics 1985-1986), page A-21 Eq. 27.

$$(Io – I) / Io = \exp (- \pi r^2 n x)$$

This is where Io is the intensity of light emanating from a black object observed at distance x, where the light intensity is I. When the object cannot be discerned, that is the visibility limit. The contrast ratio C_{ratio} is the following. The Koschmeider equation is Eq. (6).

$$C_{ratio} = (Io – I) / Io$$

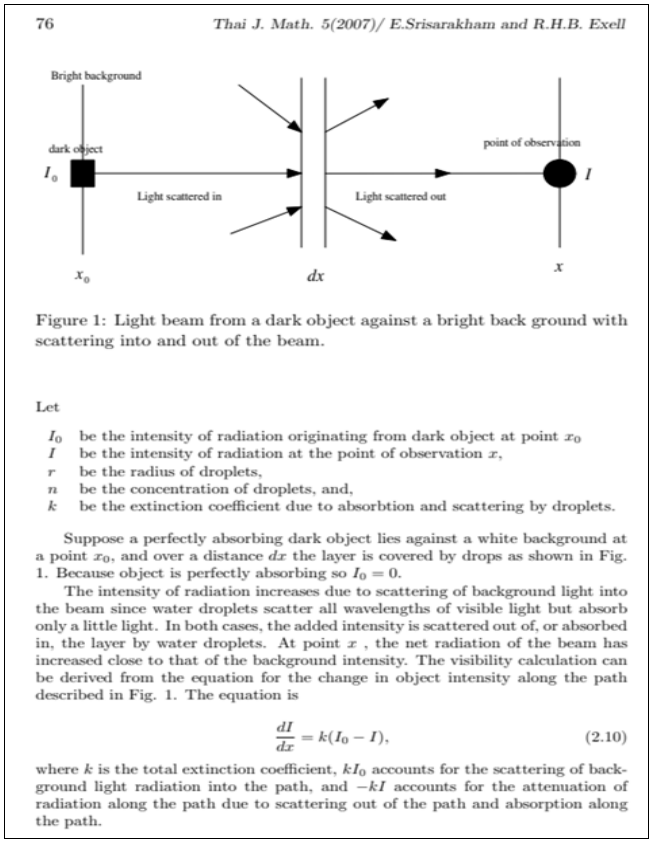
It is known from human experimentation that $C_{ratio} = 0.02$. (See pages 314-315 of Jacobson.) There is meteorological data from the Don Muang Airport in Bangkok in the Thai article of which does not relate to this paper and it will not be included.

Keywords: Fog Visibility, Extinction Coefficient, Contrast Ratio

Introduction

The Thai mathematicians use the Jacobson derivation to set up a thermodynamic model where an observer at distance x from a black object makes out its outline. Jacobson cites papers where experimentation has been done on humans, including (Koschmieder 1924) [4]. Koschmeider selected 0.02 as the threshold contrast ratio or the lowest visually perceptible brightness contrast a person can see: $C_{ratio} = 0.02$. Thus we have.

$$0.02 = \exp (- \pi r^2 n x) \quad (1)$$



76 Thai J. Math. 5(2007)/ E.Srisarakham and R.H.B. Exell

Figure 1: Light beam from a dark object against a bright back ground with scattering into and out of the beam.

Let

I_0 be the intensity of radiation originating from dark object at point x_0
I be the intensity of radiation at the point of observation x,
r be the radius of droplets,
n be the concentration of droplets, and,
k be the extinction coefficient due to absorbtion and scattering by droplets.

Suppose a perfectly absorbing dark object lies against a white background at a point x_0, and over a distance dx the layer is covered by drops as shown in Fig. 1. Because object is perfectly absorbing so $I_0 = 0$.

The intensity of radiation increases due to scattering of background light into the beam since water droplets scatter all wavelengths of visible light but absorb only a little light. In both cases, the added intensity is scattered out of, or absorbed in, the layer by water droplets. At point x , the net radiation of the beam has increased close to that of the background intensity. The visibility calculation can be derived from the equation for the change in object intensity along the path described in Fig. 1. The equation is

$$\frac{dI}{dx} = k(I_0 - I), \qquad (2.10)$$

where k is the total extinction coefficient, kI_0 accounts for the scattering of background light radiation into the path, and $-kI$ accounts for the attenuation of radiation along the path due to scattering out of the path and absorption along the path.

There is Fig 1, in page 76 of the Thai paper, that shows the setup that tests a person's ability to discern the outline of a black object at a distance x. See the last page of this paper for Fig 1 with explanation. This has to do with visibilty during fog, which poses a hazard for high speed fighter planes.

Results

We need to review the integral that gives the solution to the first equation in the ABSTRACT. In the CRC HANDBOOK, we have.

$$\int dx / (a + bx) = (1/b) \ln (a + bx)$$

So, a = Io b = -1 and Io – I = a + bI.

$$\int d I'/(Io - I'): \text{ from } I' = 0 \text{ to } I' = I$$

$$= - \ln (Io - I'): \text{ from } I' = 0 \text{ to } I' = I \qquad (3)$$

Thus we have.

$$\int d I'/(Io - I') = - \ln ((Io - I) / Io) = \pi r^2 n x \qquad (4)$$

So this gives Eq. (1), using Koschmieder's estimate for C_{ratio}:

$$0.02 = \exp (- \pi r^2 n x) \qquad (5)$$

When C_{ratio} = 0.02 at a wavelength of 0.55 μm (green light), the resulting distance x is the meteorological range (also called the Koschmieder equation), here as (6).

$$x = 3.912 / (\pi r^2 n) \qquad (6)$$

Here are the steps to get (6). From (5) ln (0.02) = - π r^2 n x. ln (0.02) = - 3.912, so – 3.912 = - π r^2 n x. Rearrange to (6), which relates the extinction coefficient π r^2 n to x, the visibility, in meters. At sea level, the Rayleigh amosphere has an extinction coefficient of approximately 13.2 × 10−6 m−1 at a wavelength of 520 nm. This means that in the cleanest possible atmosphere, visibility is limited to about 296 km.

$$x = 3.912 / (13.2 \times 10{-}6 \text{ m}{-}1) = 296400 \text{ m} = 296.4 \text{ km} \qquad (7)$$

Discussion

To arrive at C_{ratio} = 0.02, Koschmieder made a judgment where others got slightly different values. 0.02 is an average.

If the contrast ratio is unity, then an object is perfectly visible. If it is zero, then the object cannot be differentiated from background light. In Eq. (6) we have to know what r and n are to get x. In sum, the meteorological range is the distance from an ideal dark object at which the object has a 0.02 contrast ratio against a white background.

It is assumed that all light in this visual process has the same frequency spectrum and the temperature is uniform. (See Extinction Coefficient – Reference 5) That means that the extinction coefficient is the same, k, for I and Io. That validates the first equation in the Abstract.

Conclusion

This study clarifies what Srisarakham and Exell did in Thai J Math, taking the math that Jacobson came up with. Visibility is essential for aircraft, expecially high speed fighter planes. There was meteorological data from a Thai airport, but that is not the focus here.

Acknowledgments

The End is near. We hope in the mercy of St. Mary. The eternal salvation of all humans is spoken of in two Bible passages: GENESIS 5:1 and I TIMOTHY 2:1-6.

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6. Much use was made of the Internet in writing this paper.