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B-Spline Surface Approximation for CAD Model Reconstruction

[1] Vu Thi Lien, [2] Ngo Ngoc Vu

[1] Faculty of Mechanical Engineering and Mechatronics, Phenikaa School of Engineering, Phenikaa University, Ha Noi, Vietnam

[2] Faculty of Mechanical Engineering, Thai Nguyen University of Technology, Thai Nguyen, Vietnam

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Corresponding Author: **Ngo Ngoc Vu**

Abstract

In this study, a proposed method of B-spline surface approximation has been developed to reconstruct free-form surfaces from measured points that were obtained from physical objects. An automatic reverse process from data points to manufacture was applied for CAD/CAM/CNC to obtain accurate and smooth surfaces. These criteria were solved by minimizing sum of squared distance errors and generating knot vectors. Finally, the smooth model could be achieved a mean of distance error of 0.0291 mm with a standard deviation of 0.0661 mm from the given data. The finding results indicate that the developed method can overcome the surface quality problem for manufacturing mould on CNC machining center from measured point data.

Keywords: B-Spline, Surface Fitting, Reverse Engineering, Free-Form, CAD/CAM/CNC

1. Introduction

In recently years, the development of CAD/CAM technology allows the users not only to design new products but also to reconstruct 3-D models from physical parts. The process of obtaining a geometric CAD model from 3-D points acquired by scanning or digitizing physical objects is known as reverse engineering [1]. 3-D geometric representation of physical parts is a process that contains three main steps. First, physical parts are measured by a contact device such as a coordinate measuring machine (CMM) or various type of non-contact techniques such as a laser scanner or ATOS. Depending on the complexity of the digitized data, segmentation process is done to partition the 3-D data into smaller regions. Second, after segmentation, CAD system is used to fit curves or surfaces to the segmented data points. Finally, 3-D models are obtained after manufacturing by CNC machines or rapid prototyping machines.

There is some available commercial CAD software for transforming measured data to CAD models directly such as Catia, Pro/E or MasterCam etc. Although they work very fast and effectively in the step two, one problem will be appeared that is non-smoothness of tool paths. It effects on the quality of surfaces. For example: in Fig 1, from 28 measured points, the fitted curve is built by many consecutive short lines that across all these points. Therefore, there are many edges on the finished surface after milling process.

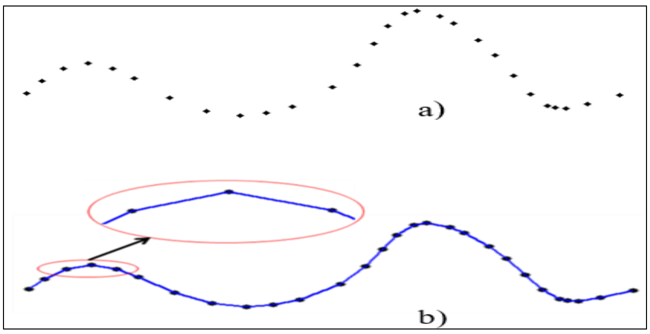

Fig 1: a) 28 data points; b) Curve fitting by CAD software

Many researches have been done to increase smoothness of fitting surfaces by different techniques [2, 3]. In [1], some available reverse engineering software is also introduced to convert directly from 3-D objects to smooth CAD models. However, almost CAD models are reconstructed in triangular mesh format; they are more suitable used for fabrication by using rapid prototyping than conventional material removal methods such as milling.
To meet the practical conditions, a flexible method is needed to be developed to convert a grid of data points into accurate and smooth CAD models in DXF format. B-spline offers flexibility and precision for handling both analytic and freeform shapes. Then, B-spline approximation [5] will be integrated with CAD/CAM system to improve the smoothness of tool paths and the quality of products. The detail of the proposed method is shown in Fig 2.

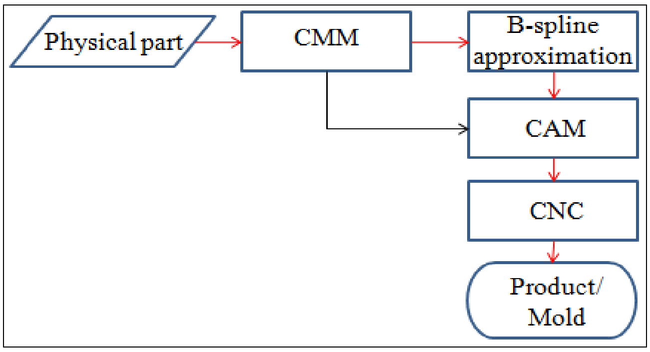

Fig 2: 3-D geometric representation flowchart

The rest of the paper is organized as follows: In Section 2, the literature review about B-spline. In Section 3, detail on the B-spline approximation method is presented. In Section 4, results and discussion are given to demonstrate the usefulness and quality of the proposed method. In Section 5, some conclusions are obtained from this study.

2. B-Spline Definition

B-spline is a polynomial parametric form. B-spline is a flexible form of mathematical function very commonly used in CAD applications. The B-spline curve of degree p defined [5, 6, 7]:

$$C(u) = \sum_{i=0}^{h} N_{i,p}(u) P_i \,,\ 0 \le u \le 1 \qquad (1)$$

Where $P_i = (x_i, y_i, z_i)$, $i = 0,\dots, h$ denote control points and $N_{i,p}(u)$, $i = 0,\dots, h$ denote the B-spline basis functions defined over a knot vector:

$$U = \{\underbrace{0,\dots,0}_{p+1},\ u_{p+1},\dots,u_{e-p-1},\ \underbrace{1,\dots,1}_{p+1}\},$$

and $e = h+p+1$ denotes number of knots.
The i-th B-spline basis function of degree p, written as $N_{i,p}(u)$, is defined recursively as follows:

$$N_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \le u \le u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u)$$

Where n is a similar way. B-spline surface is defined:

$$S(u,v) = \sum_{i=0}^{h} \sum_{j=0}^{t} N_{i,p}(u) N_{j,q}(v) P_{ij}$$

where $\{P_{i,j} = (x_{i,j}, y_{i,j}, z_{i,j})\}$ denote control points, and the $\{N_{i,p}(u)\}$ and $\{N_{j,q}(v)\}$ are B-spline basis functions of degree p and q in the bi-parametric u and v directions, respectively, and two knot vectors U, V.

3. B-Spline Curve Approximation

Two types of fitting are distinguished interpolation and approximation [6, 7]. In interpolation, a B-spline curve or surface passes all data points in the given order. Thus, the distance between a data point and its corresponding point on the curve is zero. Since the parametric curve or surface has to pass through all data points, it may wiggle through all data points. To overcome this problem, approximation method is used in this research. The B-spline curve does not have to contain any point except for the first and last data points. Thus, it can follow the data polygon closely and wiggle problem is solved.
Formulation of B-spline curve fitting is considered as two main factors including knots and control points. For fitting process, they are unknowns and needed to be solved.
Suppose there are $n+1$ data points D_i ($0 \le i \le n$ and wish to find a B-spline curve of degree (p) that approximates all of them. The degree p is selected according to the requirements. In the least square technique, the fitting procedure includes the follow main steps:

3.1 Parameterization

A set of parameters that can be fixed data points at certain values is needed to find. In this research, chord length method [7] is chosen to determine the parameters $T = (t_0, t_1, \dots, t_n)$. The length between D_{i-1} and D_i is $|D_i - D_{i-1}|$, and the length of the data polygon is the sum of the lengths of these chords:

$$L = \sum_{i=1}^{n} |D_i - D_{i-1}|$$

Since the domain is [0,1], then parameter t_k should be located at the value of L_k:

$$t_0 = 0;\ t_n = 1$$

$$t_k = \frac{1}{L} \sum_{i=1}^{k} |D_i - D_{i-1}| \quad \text{for } 1 \le k \le n\text{-}1$$

3.2 Knot generation

After the parameters are determined, knot vectors $U = \{0,\dots, 0, u_{p+1},\dots, u_{e-p-1}, 1,\dots, 1\}$ are generated by knot placement method. The number of control points (h) is chosen to save computing time and guarantee smoothness of B-spline curve. The placement of the knots [7] should reflect the distribution of the T. There are ($h-p$) internal knots, and ($h-p+1$) internal knot spans.

Set: $d = \dfrac{n+1}{h-p+1}$

Let: $i = int(jd)$ and $\alpha = jd - i$

Then: $u_{p+j} = (1-\alpha)t_{i-1} + t_i$ for $j = 1,\dots, h-p$

3.3 Least squares B-spline approximation

To evaluate how well a curve can approximate the given data polygon, the concept of error distance is used [4, 6, 7, 8]. The distance error is the distance between a data point and its corresponding point on the curve. Thus, if the sum of these error distances is minimized, the curve should follow the shape of the data polygon closely.

Because, the curve passes the first and last data point, $P_0 = D_0$ and $P_h = D_n$. The sum of all squared error distances is:

$$f(P_1,\dots,P_{h-1}) = \sum_{k=1}^{n-1} |D_k - C(t_k)|^2$$

To minimize f, we can compute the partial derivatives and set them to zero. Finally, the solution of control points is computed from following equation:

$$(N^T N)P = Q$$

Where,

$$P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_{h-1} \end{bmatrix}; \quad Q = \begin{bmatrix} \sum_{k=1}^{n-1} N_{1,p}(t_k)Q_k \\ \sum_{k=1}^{n-1} N_{2,p}(t_k)Q_k \\ \vdots \\ \sum_{k=1}^{n-1} N_{h-1,p}(t_k)Q_k \end{bmatrix}$$

$$Q_k = D_k - N_{0,p}(t_k)D_0 - N_{h,p}(t_k)D_n$$

$$N = \begin{bmatrix} N_{1,p}(t_1) & N_{2,p}(t_1) & \dots & N_{h-1,p}(t_1) \\ N_{1,p}(t_2) & N_{2,p}(t_2) & \dots & N_{h-1,p}(t_2) \\ \vdots & \vdots & \ddots & \\ N_{1,p}(t_{n-1}) & N_{2,p}(t_{n-1}) & \dots & N_{h-1,p}(t_{n-1}) \end{bmatrix}$$

Finally, the knots u_i, degree p and control points P_i are utilized to construct B-spline curve $C(u)$. A B-spline curve with degree 3 and 4 control points that fits with 8 data points is shown in Fig 4.

In a similar way, in B-spline surface approximation, control points are computed, as follows:

$$P = (B^T B)^{-1} B^T D$$

Where, [B] denotes an $(m+1)*(n+1)\times(h+1)*(t+1)$ matrix of products of the B-spline basic functions.

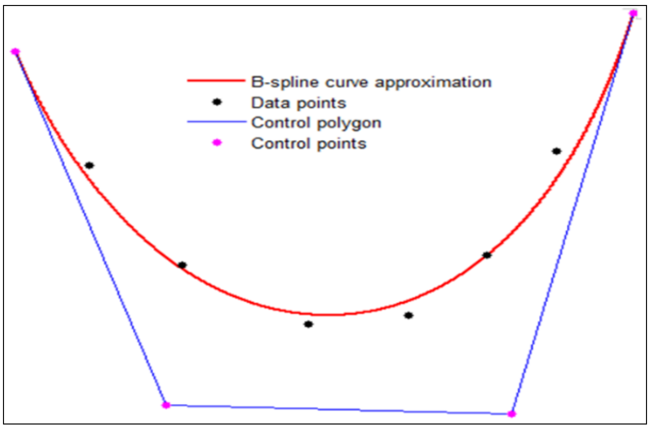

Fig 4: A B-spline curve approximation

The B-spline surface approximation integrated with CAD/CAM system is described by a flowchart in Fig 5.

4. Results and Discussion

Two fitting examples are done for a free-form curve and surface by both available reverse CAD software and the proposed method. For curve fitting, from 28 data points shown in Figure 6(a), a CAD software is used to reconstruct a curve in Figure 6(b). The fitted curve is built by many consecutive short lines that across all these points. In surface fitting case, from the 16x18 data points shown in Figure 7(a), a surface is generated from a set of data points by the CAD software, shown in Figure 7(b). Observing by human eyes and manufacturing some models in CNC machine, the quality of machined surface is failed with much roughness (many edges on the finished surface). Because the tool paths in milling process are follow the reconstructed curve and surface, tool paths will be non-smoothness curves in these cases.

They are actually broken lines. That is reason to explain why the requirement about quality of the machined surface can not achieved. This problem may be solved if the density of data points in each planar contour is large enough. It means that the distance between two succeed points is very small.

However, these fitted curve and surface are not built based on any mathematics form, it is very hard to evaluate the smoothness of the curve and surface.

In Figure 6(c), B-spline approximation shows that it can solve above problem very well. The proposed method creates a smoothness B-spline curve with degree 3 and 24 control points from the discrete points. It means that tool path is also is smooth curve. The advantage of this method is expressed more clearly in surface fitting case. The smooth model is achieved a mean of distance error of 0.0291 mm with a standard deviation of 0.0661 mm from the 16x18 data points, shown in Figure 7(c). Therefore, the proposed method satisfies both criteria: accuracy and smoothness. For known physical model such as circle and sphere, how smoothness of CAD models can be evaluated by curvature values.

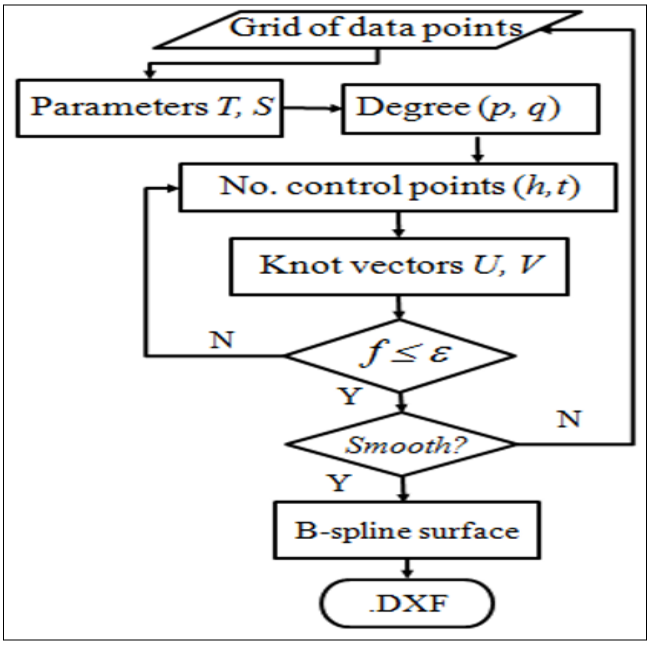

Fig 5: B-spline surface approximation flowchart

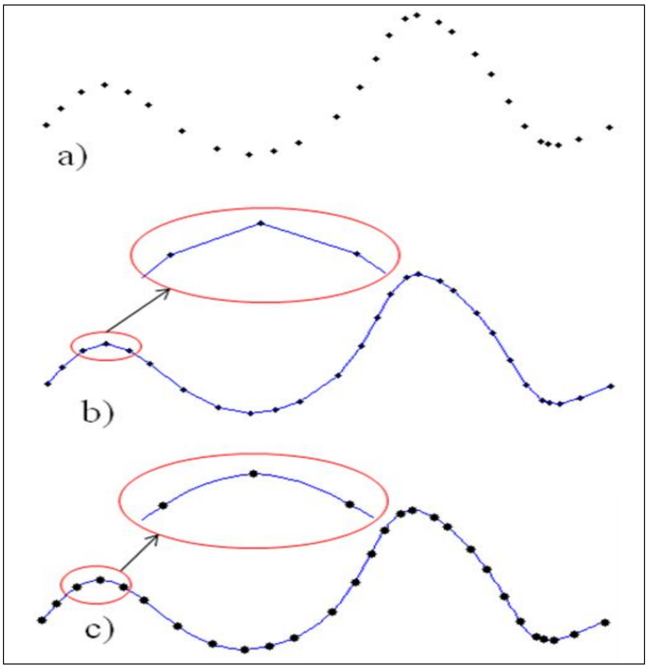

Fig 6: Curve fitting with 28 data points: a) Original curve; b) 28 data points; c) Curve fitting by CAD software

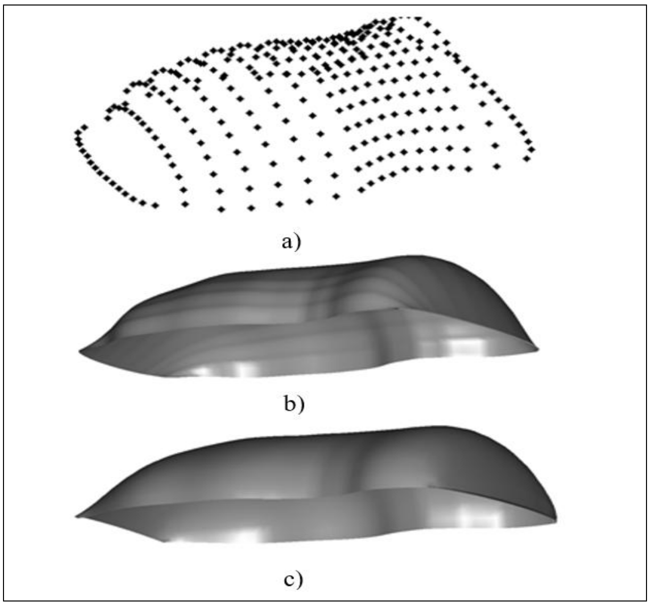

Fig 7: Surface fitting: a) 16x18 data points: b) Surface fitting by CAD software; c) B-spline surface approximation

5. Conclusions

In this study, the B-spline surface approximation was integrated with the CAD system to improve the accuracy and smoothness of CAD models and increase quality of machined surfaces made by tool path in CAD/CAM/CNC technology. This proposed method can be applied to support CAD software in reverse engineering process. It is easy to expand this method to reconstruct complex objects.

6. References

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