Int. j. adv. multidisc. res. stud. 2026; 6(3):31-35

Received: 10-03-2026
Accepted: 20-04-2026

International Journal of Advanced Multidisciplinary Research and Studies

ISSN: 2583-049X

Effect of Fingero-Imbibition Phenomena Arising in Oil Reservoir Through Fractured Porous Media

[1] **Darpan N Bulsara,** [2] **Shailesh S Patel**
[1] Research Scholar, Gujarat Technological University, Ahmadabad, Gujarat, India
[2] Head, Department of Applied Sciences and Humanities, GIDC Degree Engineering College, Navasari, Gujarat, India

DOI: https://doi.org/10.62225/2583049X.2026.6.3.6198
Corresponding Author: **Shailesh S Patel**

Abstract
In this paper we examine a one-dimensional nonlinear partial differential equation describing the fingero-imbibition phenomenon in double phase flow through a fractured porous medium during secondary oil recovery. The Power series method is employed to solve the equation with suitable initial and boundary conditions. The resulting solution depicts the saturation of injected water in the fingero-imbibition process. Graphical and Numerical results are presented.

Keywords: Fingero- Imbibition Phenomenon, Capillary Pressure, Fractured Porous Media, Similarity Transformation

1. Introduction
It is well known physical fact that if a porous medium filled with some fluid is brought into contact with another fluid, which preferentially wets the medium, then there is a spontaneous flow of the wetting fluid into the medium and a counterflow of the fluid from the medium. This phenomenon is called resident imbibition. Again, when a fluid contained in a porous medium is displaced by another of lesser viscosity, instead of a regular displacement of the whole front, protuberances may occur that shoot through the porous medium at relatively great speed. These protuberances are called fingers and the phenomenon occur singly or is called fingering. These phenomena of imbibition and fingering, whether simultaneously in displacement processes. The simultaneous occurrence of these phenomena is called fingero- imbibition phenomenon. This phenomenon, and the flow of two immiscible fluids through porous media, have gained considerable current interest due to their frequent occurrence in problems of petroleum technology. Many researchers have discussed the fingero imbibition phenomena from different point of view and finding its analytic as well as approximate and numerical solution. For example, Prajapati and Bhathawala [16] used Elzaki Transform decomposition method to obtain the approximate analytical solution in fingero-imbibition phenomena. Prajapati Dipak and Desai N.B [6] used optimal Homotopy analysis method to approximate analytical solution for fingero imbibition in homogeneous and Prajapati M.A and Desai N.B [12] Homotopy method for heterogenous porous medium. Tandel P.V [17] used Homotopy perturbation and Elzaki Transform method in inclined porous medium in Fingero Imbibition phenomena. also, this phenomenon was investigated by Shah and Verma [19] in a heterogeneous porous media with magnetic field effect. Fingero-Imbibition in Artificial Replenishment Ground Water through Cracked Porous Medium by Verma [4]. Mehta and Verma [15] investigated the composite expansion of finger imbibition in Heterogeneous porous media. The Fingero Imbibition phenomenon in homogeneous porous material with magnetic field influence in vertical downward direction was discussed by Parikh, A. K., Mehta *et al* [7]. Approximate analytical study of finger imbibition phenomenon of time fractional type in double phase flow through homogeneous porous media by Iyiola and Folarin [8]. Series Solution for Porous Medium Equation Arising in Fingero-Imbibition Phenomenon during Oil Recovery Process by Mehar. S.K and Mehar. R [18]. Mathematical Modelling of Fingero-Imbibition Phenomenon in Heterogeneous Porous Medium with Magnetic Field Effect by Patel M.A. and N.B. Desai [13]. Mehta M.N *et al* [9] discussed Saturation of water increases with distance and slightly decreases with time of Fingero Imbibition in homogeneous porous medium. Due to our particular interest, we have considered Fractured porous medium with capillary pressure. The mathematical formulation of governing laws and equations, and certain basic assumptions yields a nonlinear partial differential system governing Fingero Imbibition in the investigated. Purpose of the research is to measure saturation of the injected water at distance x and Time τ for fingero-imbibition phenomenon in the fractured porous media in the oil recovery process when fingero- imbibition phenomenon takes place. The mathematical

formulation leads to a one-dimensional nonlinear partial differential equation. This equation with appropriate initial and boundary conditions has been solved by Power Series Method.

2. Mathematical Description

Certain assumptions were considered when mathematically formulating this phenomenon. The instability phenomenon in heterogeneous porous media has been investigated under essential basic assumptions: immiscible flow of two incompressible fluids, water and oil; the porous medium is isotropic, incompressible, finite in extent, with a horizontal and impermeable bottom boundary; mass is conserved, and gravitational forces are neglected. The natural oil field is substantial enough to analyze the effects of water injection (water flooding) on the oil recovery process. To examine the instability phenomenon in heterogeneous porous media, we selected a cylindrical piece of porous matrix with length L, where three sides are impermeable except for one end, from which water is injected. The stability of the water flood relies on the mobility ratio between oil and water, the heterogeneity of the porous medium, fluid segregation in the reservoir, and the dissipation of fluid fronts due to capillary pressure. These frontal instabilities are typically characterized by numerous penetrating fingers of the displacing fluid. Consequently, all oil at the initial boundary $X = 0$ (where, X is measured in the direction of displacement) is displaced a distance L due to water injection.

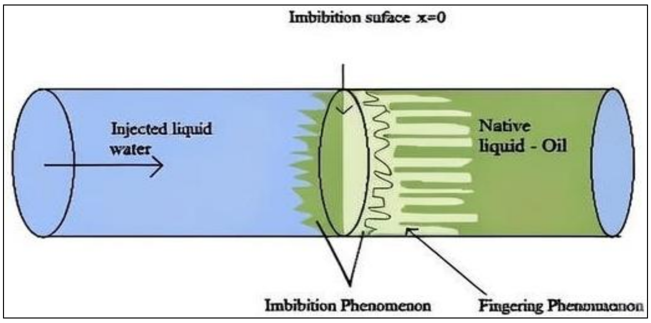

Fig 1: Representation of Fingering Phenomenon in a cylindrical piece of heterogeneous porous matrix [12]

3. Governing Laws and Standard Relations

During the injection process, the seepage velocities of injected water V_w and native oil V_o are expressed by Darcy's law as, [1]:

$$V_w = -\frac{\kappa_w}{r_w} K \frac{\partial \mathrm{P}_w}{\partial x} \tag{3.1}$$

$$V_o = -\frac{\kappa_o}{r_o} K \frac{\partial \mathrm{P}_o}{\partial x} \tag{3.2}$$

Where, K is a variable permeability of the Fractured porous medium, k_w and k_o are relative permeability of displacing fluid and native fluid. P_w and P_o are pressures; r_w and r_o are the constant kinematics viscosities of the displacing and native fluids. The equations of continuity for these two fluids are expressed as,

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \tag{3.3}$$

$$\varphi \frac{\partial S_0}{\partial t} + \frac{\partial V_0}{\partial x} = 0 \tag{3.4}$$

Where, $\varphi = \varphi(x)$ is the variable porosity of the fractured porous media. From the definition of phase saturation.

$$S_w + S_o = 1 \tag{3.5}$$

$$\mathrm{P}_c(S_w) = P_o - P_w \tag{3.6}$$

We consider the capillary pressure and relative permeability function for water as, [21]:

$$\mathrm{P}_c = \beta(S_w^{-1} - C_o) \tag{3.7}$$

Where, β and C_o are constant of proportionality.
The standard relationship between the relative permeability and phase saturation was, [2, 21]:

$$k_w = S_w^3$$

$$k_o = 1 - \alpha S_w \tag{3.8}$$

Where, α is constant and its value is $\alpha = 1.11$ Since, The porosity and permeability as function of x only, [3]:

$$\varphi(x) = \frac{1}{a_1 - a_2 x} \tag{3.9}$$

Where, $a_1 - a_2 x \geq 1$; where, a and b are constants, Since, $\varphi(x)$ can't exceed unity $0 \leq x \leq L$, For Simplicity, we consider, [5]:

$$K \propto \varphi(x) \tag{3.10}$$

$$K = K_c \varphi(x) \tag{3.11}$$

For saturation, the equation of motion can be obtained by substituting the Eq. (3.1) and Eq. (3.2) in continuity of the equations respectively, we get:

$$\varphi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\kappa_w}{r_w} K \frac{\partial \mathrm{P}_w}{\partial x}\right) \tag{3.12}$$

$$\varphi \frac{\partial S_o}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\kappa_o}{r_o} K \frac{\partial \mathrm{P}_o}{\partial x}\right) \tag{3.13}$$

Eliminating $\frac{\partial \mathrm{P}_w}{\partial x}$ from Eq. (3.12) and Eq. (3.6), we have:

$$\varphi\left(\frac{\partial S_w}{\partial t}\right) = \frac{\partial}{\partial x}\left[\frac{\kappa_w}{r_w} K \left(\frac{\partial \mathrm{P}_o}{\partial x} - \frac{\partial \mathrm{P}_c}{\partial x}\right)\right] \tag{3.14}$$

Now From, Eq. (3.13) and using Eq. (3.14) we derive,

$$\frac{\partial}{\partial x}\left[\left(\frac{\kappa_w}{r_w} K + \frac{\kappa_o}{r_o} K\right)\frac{\partial \mathrm{P}_o}{\partial x} - \frac{\kappa_w}{r_w} K \frac{\partial \mathrm{P}_c}{\partial x}\right] = 0 \tag{3.15}$$

Integrating Eq. (3.15) we get,

$$\left[\frac{\kappa_w}{r_w}K\frac{\partial P_c}{\partial x}-\left(\frac{\kappa_w}{r_w}K+\frac{\kappa_o}{r_o}K\right)\frac{\partial P_o}{\partial x}\right]=V \tag{3.16}$$

Where, V is the constant of Integration. From Eq. (3.16),

$$\frac{\partial P_o}{\partial x}=\frac{V}{\left[\frac{\kappa_w}{r_w}K+\frac{\kappa_o}{r_o}K\right]}+\frac{\left(\frac{\kappa_w}{r_w}K\right)}{\left[\frac{\kappa_w}{r_w}K+\frac{\kappa_o}{r_o}K\right]}\frac{\partial P_c}{\partial x} \tag{3.17}$$

By putting the values of $\frac{\partial P_o}{\partial x}$ in Eq. (3.17) it gives,

$$\varphi\left(\frac{\partial S_w}{\partial t}\right)+\frac{\partial}{\partial x}\left[\frac{\left(\frac{\kappa_o}{r_o}\right)K}{\left(1+\frac{\kappa_o r_w}{r_o\kappa_w}\right)}\frac{\partial P_c}{\partial x}-\frac{V}{\left(1+\frac{\kappa_o r_w}{r_o\kappa_w}\right)}\right]=0 \tag{3.18}$$

The value of the pressure of oil (P_o) can be written as, [20]:

$$P_o=\bar{P}+\frac{1}{2}(P_c),\ \bar{P}=\frac{P_o+P_w}{2} \tag{3.19}$$

Where, $\bar{P}$ is the mean pressure and is constant, Now

$$\frac{\partial P_o}{\partial x}=\frac{1}{2}\left(\frac{\partial P_c}{\partial x}\right) \tag{3.20}$$

Substituting the value of $\frac{\partial P_o}{\partial x}$ in Eq. (3.15) it gives,

$$V=\frac{1}{2}K\left[\frac{\kappa_w}{r_w}-\frac{\kappa_o}{r_o}\right]\left(\frac{\partial P_c}{\partial x}\right) \tag{3.21}$$

Substituting the Eq. (3.20) into Eq. (3.17), We obtain,

$$\varphi\left(\frac{\partial S_w}{\partial t}\right)+\frac{1}{2}\frac{\partial}{\partial x}\left[\frac{\kappa_w}{r_w}K\frac{dP_c}{dx}\frac{\partial S_w}{\partial x}\right]=0 \tag{3.22}$$

Substituting Eq. (3.7), (3.8) and Eq. (3.10) into Eq. (3.21), We get:

$$\varphi\left(\frac{\partial S_w}{\partial t}\right)-\frac{\beta K_c}{2r_w}\frac{\partial}{\partial x}\left[\varphi S_w\frac{\partial S_w}{\partial x}\right]=0 \tag{3.23}$$

This is non-linear partial differential equation expressing the fingero-imbibition phenomena in fractured porous matrix during secondary recovery process which gives saturation of injected water for any distance x for $t\geq 0$.

4. Solution of the Problem

Dimensionless Variable are chosen as follow,

$$\chi=\frac{x}{L}\quad \tau=\frac{\beta K_c}{2L^2 r_w}t \tag{4.1}$$

Substituting this Value in Equation Eq. (3.22), we have,

$$\frac{\partial S_w}{\partial\tau}-\left(\frac{\partial S_w}{\partial\chi}\right)^2-\left(\frac{1}{\varphi}\frac{\partial\varphi}{\partial\chi}\right)S_w\left(\frac{\partial S_w}{\partial\chi}\right)-S_w\left(\frac{\partial^2 S_w}{\partial\chi^2}\right)=0 \tag{4.2}$$

Equation Eq. (4.2) is non-linear partial differential equation, for the sake of simplicity of the problem here the term $\frac{1}{\varphi}\frac{\partial\varphi}{\partial\chi}$ has been simplified as,

$$\frac{1}{\varphi}\frac{\partial\varphi}{\partial\chi}=\frac{\partial}{\partial\chi}(\log\varphi)=L\left(\frac{a_2}{a_1}\right)=\frac{L}{2\sqrt{\tau}}(neglating\ the\ higher\ term\ of\ \chi) \tag{4.3}$$

Where, $\frac{a_2}{a_1}=\frac{1}{2\sqrt{\tau}}$ for any $0<\tau\leq 1$ Hence, Eq. (4.2) will be,

$$\frac{\partial S_w}{\partial\tau}-\left(\frac{\partial S_w}{\partial\chi}\right)^2-\left(\frac{L}{2\sqrt{\tau}}\right)S_w\left(\frac{\partial S_w}{\partial\chi}\right)-S_w\left(\frac{\partial^2 S_w}{\partial\chi^2}\right)=0 \tag{4.4}$$

The Appropriate Boundary Condition to Solve Eq. (4.4) as:

$$S_w(\chi,0)=0=S_{wc}\quad for\quad \chi>0 \tag{4.5}$$

$$S_w(0,\tau)=1=S_{wo}\quad for\quad \tau>0 \tag{4.6}$$

$$S_w(1,\tau)=0=S_{wi}\quad for\quad \tau>0 \tag{4.7}$$

Choose the Similarity Transformation, [14]:

$$S_w(\chi,\tau)=f(\zeta)\quad Where,\quad \zeta=\frac{\chi}{\sqrt{\tau}} \tag{4.8}$$

The governing Eq. (4.4) reduce to an ordinary differential equation together with boundary condition Eq. (4.5) and Eq. (4.6) we have,

$$6f(\zeta)f''(\zeta)+6f'^2(\zeta)+2Lf(\zeta)f'(\zeta)+3\zeta f'(\zeta)=0 \tag{4.9}$$

With the Boundary Conditions Eq. (4.5) and Eq. (4.6),

$$f(0)=S_{wo},\quad \chi=0,\quad \tau>0 \tag{4.10}$$

$$f(\infty)=S_{wc},\quad \tau=0,\quad \chi>0 \tag{4.11}$$

$$f'(0)=\omega\neq 0,\qquad \tau>0, \tag{4.12}$$

To find successive coefficients of Maclaurin's series at $\zeta=0$. By taking nth derivative of Eq. (4.9), solving for $f^{n+2}(\zeta)$ and evaluating at $\zeta=0$, we have:

$$6\sum_{k=0}^{n}\binom{n}{k}f(\zeta)^k f^{(n-k+2)}(\zeta)=-2L\sum_{k=0}^{n}\binom{n}{k}f(\zeta)^k f(\zeta)^{n-k+1}$$
$$-6\sum_{k=0}^{n}\binom{n}{k}f(\zeta)^{(k+1)}f^{(n-k+1)}(\zeta)+3\left[\zeta f^{(n+1)}(\zeta)+nf^{(n)}(\zeta)\right] \tag{4.13}$$

To solve this, we must get the derivatives of $f^n(0)$ for all $n=1,2,3,\ldots$, $f'(0)$ can be obtained from Eq. (4.11) and formula (4.9) can be used to find the derivative $f''(0)$. By entering $n=1,2,3,\ldots$, all additional higher derivatives can also be found using Eq. (4.13). Consequently, the intended worth of $f(\zeta)$ can be computed by Maclaurin's series,

$$f(\zeta) = \Sigma_{k=0}^{\infty} f^k(0)\frac{\zeta^k}{k!} \tag{4.14}$$

Therefore, we have,

$$f(\zeta) = f(0) + \zeta f'(0) + \frac{\zeta^2}{2!}f''(0) + \frac{\zeta^3}{3!}f'''(0) + \frac{\zeta^4}{4!}f^{iv}(0) + \frac{\zeta^5}{5!}f^{v}(0) + \cdots \tag{4.15}$$

Now From Eq. (4.7),

$$S_w(X,T) = f(0) + \frac{X}{\sqrt{T}}f'(0) + \frac{X^2}{2(\sqrt{T})^2}f''(0) + \frac{X^3}{6(\sqrt{T})^3}f'''(0) \tag{4.16}$$

$$+ \frac{X^4}{24(\sqrt{T})^4}f^{iv}(0) + \frac{X^5}{120(\sqrt{T})^5}f^{v}(0) \ldots$$

Where,

$$f(0) = S_{wo}$$

$$f'(0) = \omega$$

$$f''(0) = -\frac{1}{S_{wo}}\left[\omega^2 + \frac{1}{3}L\omega S_{wo}\right]$$

$$f'''(0) = \frac{1}{S_{wo}^2}\left[3\omega^3 + L\omega^2 S_{wo} + \frac{1}{9}L^2\omega S_{wo}^2 - \frac{1}{2}\omega S_{wo}\right]$$

$$f^{iv}(0) = -\frac{1}{S_{wo}^3}\left[\begin{matrix}15\omega^4 + 7L\omega^3 S_{wo} - 3\omega^2 S_{wo} + \frac{10}{9}L^2\omega^2 S_{wo}^2 + \\ \frac{2}{27}L^3\omega S_{wo}^3 - \frac{2}{3}L\omega S_{wo}^2\end{matrix}\right]$$

$$f^{v}(0) = \frac{1}{S_{wo}^4}\left[\begin{matrix}105\omega^5 - 60L\omega^4 S_{wo} - \frac{49}{2}\omega^3 S_{wo} + \frac{38}{3}L^2\omega^3 S_{wo}^2 + \\ \frac{11}{9}L^3\omega^2 S_{wo}^3 + \frac{17}{6}L\omega^2 S_{wo}^2 - \frac{11}{8}L^2\omega S_{wo}^3 + \\ \frac{4}{81}L^4\omega S_{wo}^4 + \frac{3}{4}\omega S_{wo}^2\end{matrix}\right] \tag{4.17}$$

When water is injected at a common interface during the secondary oil recovery process, Eq. (4.15) depicts the saturation of the injected fluid during the Fingero - Imbibition phenomenon in fractured porous media.

5. Analysis of Convergence

The convergence of Maclaurin's series is sufficient to examine the convergence of Eq. (4.16), since Eq. (4.15) reflects Maclaurin's series in the form of ζ and Eq. (4.16) represents the initial values of X and τ. From Eq. (4.15),

$$f_k = f^j(0)\frac{\zeta^j}{j!} \tag{5.1}$$

Now, from using Cauchy Root Test for convergence,

$$\lim_{k\to\infty}|f_k| = \lim_{k\to\infty}\left|\frac{f^j(0)x^j}{j!}\right|^{\frac{1}{j}} = 0 < 1 \tag{5.2}$$

Assuming $f^j(0)$ is a real number and considering the absolute value for convergence and $\zeta = \frac{X}{\sqrt{\tau}}$ for $\tau \neq 0$. Hence the Eq. (4.15) is absolutely convergent series then it is convergent series.

$S_w(\chi,\tau)$ *Vs Distance* χ *and Time* τ

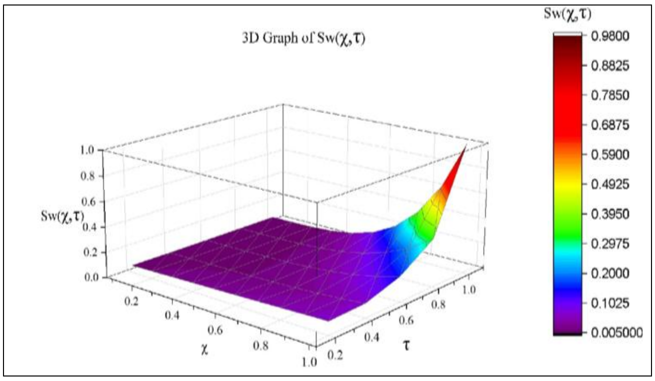

Fig 2: $S_w(\chi,\tau)$ when $\tau = 0.1 - 1$ and $\omega = 0.01$, $S_{wo} = 0.0051$, $L = 3$ fixed

Table 1: Numerical assessment of the Fingering-Imbibition Phenomenon in Fractured Porous Media

χ	τ	τ	τ	τ	τ	τ	τ	τ	τ	τ
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.0101	0.0076	0.0067	0.0063	0.0060	0.0059	0.0057	0.0056	0.0055	0.0054
0.2	0.0150	0.0100	0.0083	0.0075	0.0070	0.0066	0.0064	0.0062	0.0061	0.0060
0.3	0.0200	0.0125	0.0101	0.0089	0.0084	0.0081	0.0081	0.0083	0.0086	0.0092
0.4	0.0250	0.0152	0.0122	0.0112	0.0111	0.0116	0.0126	0.0143	0.0166	0.0198
0.5	0.0301	0.0182	0.0153	0.0150	0.0164	0.0189	0.0228	0.0283	0.0357	0.0458
0.6	0.0353	0.0218	0.0197	0.0214	0.0258	0.0327	0.0424	0.0559	0.0742	0.0989
0.7	0.0407	0.0263	0.0262	0.0316	0.0415	0.0561	0.0764	0.1045	0.1429	0.1951
0.8	0.0464	0.0321	0.0355	0.0471	0.0659	0.0931	0.1311	0.1837	0.2561	0.3553
0.9	0.0525	0.0395	0.0486	0.0696	0.1021	0.1486	0.2140	0.3051	0.4316	0.6059
1	0.0591	0.0489	0.0666	0.1012	0.1535	0.2283	0.3341	0.4828	0.6909	0.9794

6. Quantitative Results and Analysis

During the secondary oil recovery process, when water is injected, the solution (4.16) illustrates the saturation of the injected fluid, which is an ascending power series of X with time $\tau > 0$. The answer (4.15) which is also in terms of power series in ζ, satisfies constraints (4.10) and (4.11) as well. As we have just examined the first five terms in the power series, it provides a rough explanation for the fingero-imbibition phenomenon and also shows that the convergent power series. We saw water saturation from the graph at $\chi = 0$ when the Imbibition phenomena happen at the common interface for a certain value of the parameter. Fig 2 depicts the graph of $S_w(\chi,\tau)$ vs distance measured from the common interface in dimensionless variables. At time $\tau = 0.1$, the saturation graph continuously increases until $\chi = 1$, where it abruptly increases due to the initial unsaturation of the fractured porous medium. For various values of X, the saturation of water increases consistently while time τ grows to $\tau = 0.1 - 1$; The Numerical Solution for the governing equation (4.4) representing fingero-imbibition phenomenon in fractured porous media is obtained by using Maclaurin's series given by the equation (4.15) and (4.16). The term $\frac{L}{2\sqrt{\tau}}$ that appears in equation (4.4) is referred to as the heterogeneity coefficient for the governing equation, if $\frac{L}{2\sqrt{\tau}} = 0$ It describes the phenomenon of fingero-imbibition in a homogeneous porous medium. The Numerical values representing the saturation of water at different time intervals are displayed in Table 1 In conjunction with its graphical depiction from Fig 2. It is evident that the saturation of water rises over time and also increases with distance X. Therefore, the physical reality of the issue is maintained in the scenario of a fractured porous medium, allowing for a significant quantity of oil to be displaced.

7. Conclusion

An approximate analytical solution is obtained the fingero-imbibition phenomenon in fractured porous medium by Power Series solution which satisfies both the initial and boundary conditions. The solution represents the saturation of injected water for fingero- imbibition phenomenon from the starting initial value $S_w(\chi,\tau)$. Table 1 indicates the numerical values of the saturation of injected water. Fig 2 shows that the saturation of injected water v/s distance χ for $\tau > 0$. We conclude that the saturation of injected water increase when the distance χ and time τ increses. Fig 2 shows that different parameter choices have a more substantial effect on Saturation increasing in the domain and are more suited for physically consistent results.

8. Acknowledgement

I am deeply grateful to my guide, Shailesh Patel Sir, for their Guidance and support. Their cooperation and encouragement have been invaluable throughout this work.

9. Competing Interest Declaration

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

10. References

1. Scheidegger AE. The physics of flow through porous media. University of Toronto press, 1957.
2. Scheidegger AE. The statistical behavior of instabilities in displacement processes in porous media. Canadian Journal of physics. 1961; 39(2):326-334.
3. Verma AP. Statistical behavior of fingering in a displacement process in heterogeneous porous medium with pressure. Rev. Roum. Sci. Tech., Ser. Mec. Appl.; (Romania). 1969; 14(3).
4. Verma AP. Fingero imbibition in artificial replenishment of ground water through cracked porous medium. Water Resources Research. 1970; 6(3):906-911.
5. Zhangxin Chen. Reservoir simulation: Mathematical techniques in oil recovery. SIAM, 2007.
6. Prajapati DJ, Desai NB. Approximate analytical solution for the fingero-imbibition phenomenon by optimal Homotopy analysis method. International Journal of Computational and Applied Mathematics. 2017; 12(3):751-761.
7. Parikh AK, *et al*. Mathematical modeling and analysis of fingero-imbibition phenomenon in Homogeneous porous medium with magnetic field effect in vertical downward direction. Int. J. Latest Technol. Eng. Manag. Appl. Sci. 2014; 3(10):17-23.
8. Iyiola, *et al*. Approximate analytical study of fingero-imbibition phenomena of time-fractional type in double phase flowthrough porous media. European Journal of Pure and Applied Mathematics. 2014; 7(2):210-229.
9. Meher R, *et al*. Adomian decomposition approach to fingero-imbibition phenomena in double phase flow through porous media. International Journal of Applied Mathematics and Mechanics. 2012; 6(9):34-46.
10. Bear J. Dynamics of fluids in porous media. Courier Corporation, 2013.
11. Braun M, Golubitsky M. Differential equations and their applications, volume 4. Springer, 1983.
12. Patel MA, Desai NB. Homotopy analysis method for fingero-imbibition phenomenon in Heterogeneous porous medium. Nonlinear Science Letters A: Mathematics, Physics and Mechanics. 2017; 8(1):90-100.
13. Patel MA, Desai NB. Mathematical modelling of fingero-imbibition phenomenon in heterogeneous porous medium with magnetic field effect. PRAJNA-Journal of Pure and Applied Sciences. 2017; 24(25):15-22.
14. Mehta MN. Asymptotic expansion of fluid flow through porous media. South Gujarat University, Surat, India, 1977.
15. Mehta MN, Verma AP. Composite expansion solution of fingero-imbibition in double phase flow through underground porous medium. In Proceedings of the Indian Academy of Sciences-Mathematical Sciences, volume 87. Springer, 1978, 65-70.
16. Prajapati MS, Bhathawala PH. Numerical solution of fingero-imbibition phenomena of time fractional type in double phase flow through porous media.
17. Tandel PV. Homotopy perturbation and Elzaki-Transform for solving fingero imbibition phenomenon in zero inclined porous media.
18. Meher R, Meher S. Series solution for porous medium equation arising in fingero-imbibition phenomenon during oil recovery process. Applied Mathematics and Information Science Letters, 2013.
19. Shah RC, Verma AP. Fingero-imbibition phenomenon through porous media with magnetic fluid. Indian Journal of Engineering and Materials Sciences. 1998; 5:411-415.
20. Pathak S, Singh T. Study on fingero imbibition phenomena in inclined porous media by optimal Homotopy analysis method. Ain Shams Engineering Journal. 2018; 9(4):1181-1187.
21. Patel SS. An approximate solution of displacement processes in porous media. PhD Thesis, South Gujarat University, Surat, 1997.