Int. j. adv. multidisc. res. stud. 2026; 6(2):1473-1485

Received: 26-02-2026
Accepted: 06-04-2026

International Journal of Advanced Multidisciplinary Research and Studies

ISSN: 2583-049X

Omega Factor Radiotherapy Wedges 3D Advanced Simulations at 18 MEV-Depth [z = 5,15 cm] with AAA Model for Breast Cancer Treatment Planning Optimization

Francisco Casesnoves

PhD Engineering, MSc Physics-Mathematics, Physician, Independent Bioengineering Laboratory Director, Baltic States, International Association of Advanced Materials, Sweden, UniScience Global Scientific Member, Wyoming, USA

Corresponding Author: **Francisco Casesnoves [Email:** casesnoves.research.emailbox@gmail.com**]**

DOI: https://doi.org/10.62225/2583049X.2026.6.2.6125

Abstract

Continuing with the Author's created Omega Factor (OF) convolution for Anisotropic Analytic Model (AAA) radiotherapy research/improvements, a number of equations with their mathematical-geometrical development are improved/reviewed with analytic geometry calculations. Subject to breast cancer radiotherapy, 3D Isodoses detailed-graphics are presented for beam-modification Treatment Planning Optimization TPO with wedge filters (WF). Results for 3D Isodose-zone graphics for [z = 5, 15 cm], based on all these algorithms/software, are for OF-corrected AAA model at 18 Mev photon-beam. 3D Isodose charts, in Type 1 [Vertical 3D Isodoses], and Type 2 [Horizontal 3D Isodoses are presented] in contrast to classical 2D Isodoses. These 3D Isodoses radiotherapy simulations software are explained through the 3D graphics series elaboration. TPO applications with WF, and beam modification devices in general, for breast tumors are shown.

Keywords: Omega Factor (OF), Radiation Dose, Attenuation Exponential Factor (AEF), Simulations, Nonlinear Optimization, Matrix Algebra, Spherical-Spatial Analytical Geometry, Organ at Risk (OAR), Multi-Leaf Collimator (MLC), Wedge Filter (WF), Conformal Wedge Filter, Anisotropic Analytic Model AAA, Intensity Modulated Radiotherapy (IMRT), Intense Modulated Protontherapy (IMPT), Fluence Factor (FF), Treatment Planning Optimization (TPO), Breast Tumor (BT), Computarized Thomography (CT)

1. Introduction and Objectives

Together with lung tumors, breast cancer shows the highest incidence/prevalence in developed countries. However, the ethiology and pathological causes for these two tumors types are different. While toxicity and tobacco plays the most important cause for lung cancer, breast one is hormone-dependent [9.1-9.4]. Additionally, survival time shows high difference in favor of breast tumors. The radiation therapy treatment for breast tumors has improved during recent clinical investigation times [1-20, 73, 74], towards getting minimum dose-highest efficiency, and minimal surgery invasion/resection. Optimal radiotherapy treatment dose delivery is usually a rather difficult conformal 3D operation, Imaging-guided and on-time controlled by CT in 3D also [74]. Therefore, the innovations of this study are two. Firstly, the mathematical demonstration of Omega Factor is proven with sharp equations. Secondly, the research presents a 3D Isodoses graphics for TPO optimization with WF made by software-engineering design. These new 3D Isodoses processing images here obtained, get the dose delivery data for several depths with WF subject to 18 Mev photon-beam. They constitute a primary example for its future practical applications possibilities in TPO. It is also possible to vary in the same chart WF dimensions, LINAC output Megavoltage, WF angles and other parameters for TPO [1-20, 73]. Consequently, the 3D Isodoses developed in this contribution involve two methodology strands. The first one is the AAA model calculations to implement data into the software [1-20, 73, 74]. The second part is the rather long programming work to obtain 3D image processing Isodoses. The type of WF dose delivery shown correspond to previous contributions types [1-20, 73]. Currently radiotherapy treatment is performed in 3D usually with Computarized Thomography (CT) imaging data before, during, and after the irradiation sessions [74]. The OARs and normal radiosensitive tissues constitute an important part of the radiotherapy plan to improve the life quality of the patient [1-20, 73, 74].

For specific breast cancer medical physics, it is recommended further reading precision-details in several directions. The main radiotherapy dosimetry optimization physics, [12-20, 29, 48, 50, 51, 58, 59, 66, 72, 84]. The biomodels for breast cancer are explained in [22-25, 83-86]. Some related informative modern breast tumor publications can be found at [9.1-9.4, 52, 70, 75]. In the radioprotection field, when radiation therapy treatment is given, standard general and breast cancer norms for radioprotection, [79-81]. Mathematical inverse problems theory/algorithms to be applied with/for Tikhonov dosimetry optimization, [32, 61, 62, 67, 68, 73, 74], Equations 1-3. In imaging and imaging-guided applications for tumors, [24, 26, 74] are useful. The 2D-AAA classical model publications along the literature are extensive, for example, [28, 29, 30, 33, 34, 36, 46]. Concepts about adjuvant radio-chemo-inmmuno therapy, are included in [41, 55, 69, 70].

Specifically in breast cancer, [9.1-9.4, 24-26, 38, 39, 40, 74, 82], the breathing movement, [9.3], of thorax cavity creates a precision delivery problem. Furthermore, the breast cancer patients in no few circumstances present breath difficulties that make even more complicated that movement control. Biological models, [1, 21-25], are used to modulate the dose delivery magnitude according to specific tissue radiobiological parameters. Normal tissue dose complications probability biological models (NTCP) are summed to the radiotherapy plan [1, 21-25, 74], to minimize the OARs radiation damage also. The mathematical optimization basis, form the primary step that was applied to develop modern TPO at both planning system and LINAC apparatus [11, 12]. Therefore, the AAA model step forward from 2D to 3D for all planning systems and radiation dataset constitutes a precision requirement at present to improve the efficacy of results in radiotherapy [1-20, 73, 74].

In consequence, the innovation of this study is to present a 3D Isodoses graphics to pass on from 2D Isodoses to 3D Isodoses imaging acquisition TPO work data. In Proton therapy, [31, 57, 74], 3D Isodoses could be also used. However, IMPT models are different from photon-dose ones.

When Hyper-fractionated radiation protocol is used for breast tumors cure, [23, 24, 73, 74], 3D Isodoses are useful. The motives among several others, are to avoid OARs doses and increase radioprotection, and give the patient a better quality of life during radiation treatment, [25]. It is not the focus of this contribution to discuss about the hyper-hypo fractionated radiation delivery polemic, [25]. Several ideas about recent advances in cancer research, [1-20, 73, 74], comprise pre-hypothesis about future radiation oncology. Namely, [Casesnoves, 2007], Radiation Therapy will remain/continue in clinical oncology future mainly for the primary/secondary attack to eliminate the tumor volume, and set/open the field for subsequent surgery, Chemo-Inmuno Therapy, or Nano-Inmunotherapy stages. Preventive Medicine in early stage diagnosis and prevention (for example, elimination of smoking, alcohol abuse, modern tendency in women for pregnancy, etc), has proven be useful/significant for cancer incidence reduction [1-20]. In summary, this study reports a software programming series for 3D Isodoses engineering software methods, mathematical algorithms, and graphics processing techniques, focused to be applied on breast cancer dosimetry with static wedges [1, 25, 38, 39, 74]. Software development methods are explained according to image processing. Medical Physics applications for breast tumors, and general radiation oncology TPO are described and analyzed.

1.1 Objectives of the Research

The objectives of the study are mainly two. First, to develop and explain sharply/detailed the 3D Omega Factor algorithm-correction for the AAA model, Equations 1-3, Figures 1,2. Secondly, to implement these calculations within the model for getting 3D imaging-processing graphics. These charts show the Isodose zones, Figures 2-7, for photon-dose delivery at [z= 5,15] cm, 18 MeV, for static wedge filters beam modification.

2. Omega Factor AAA Model Foundations Review

Omega Factor, [Casesnoves, 2015, Philadelphia], was developed for AAA model mathematical-geometrically, to improve the precision of photon-beam path through static wedges and in general triangular-shaped beam modification devices. In the foundations of AAA model, the path through static wedges was calculated in 2D, and Attenuation Exponential Factor (AEF) is set within the standard superposition-convolution integral to get dose delivery precision. However, if it is precisely determined for 3D, there is a factor, conceived as Omega Factor, that sets a dose-delivery higher precision in AAA model. Precisely, OF is not the unique analytic method to obtain 3D-accuracy for AAA improved dose delivery. Here the most important convolution formulas/development is reviewed and improved with further analytic geometry sketchs, Figures 1-2.

Definition 1: Geometrical Omega Factor, namely, [Ω]F, is defined as a numerical coefficient that transforms the 2D approximated integral attenuation factor, [AEF]-Approximated, into an exact attenuation factor, [AEF]-Exact. Equation 1.

Proposition 1: Geometrical Omega Factor, namely, $[\Omega]_F$, can be expressed in multiple geometrical-algebraic forms, and one efficacious for integration as Equation (1).

The geometry of WF and other beam modification devices make rather difficult to get precision for the dose delivery. Therefore, the first calculations were limited to 2D, [30]. Gaussians are applied for radiotherapy models in convolution integrals, and also for probabilistic models. However, to obtain an exact integration involves appropiate changes, to get usually an Erf functions solution. In [30], the quadratic change was implemented to make dose delivery formula, Equation 3 . The analytic geometry of this study was designed to reach a 3D algorithm and at the same time to preserve the exact integral solution with Erf functions.

$OMEGA\ FACTOR\ [\Omega]_F\ FUNDAMENTAL\ FORMULA\ [Casesnoves\ 2015-2016]$

$Definition: Omega\ Factor,\ [\Omega]_F,\ is\ a\ convolution\ factor$
$set\ exponentially\ to\ add\ 3D\ precision\ to$
$the\ photon-beam\ pathway\ length-magnitude$
$throughout\ the\ radiotherapy\ wedge-filter$
$beam\ modification\ device;$

$The\ AEF\ in\ 2D\ for\ the\ thin$
$part\ of\ the\ wedge, according\ AAA\ model\ foundation\ reads,$

$$f(u,z,\phi,\alpha,L)=e^{-\mu_w\left[\left(L\pm\frac{c\,u}{F+z}\right)\times\left(\frac{sin\alpha}{cos(\alpha-\phi)}\right)\right]}\ ;$$

$hence,\ for\ 3D,$

$[\Omega]_F\ fundamental\ equation\ added\ factor\ at\ exponent\ is,$

$$[\Omega]_F=\left[1+\frac{tan^2\phi_2}{1+tan^2\phi_1}\right]^{\frac{1}{2}}\ ;$$

$where,$

$\phi_1\ :\ Coronal\ plane\ angle\ of\ the\ photon-beam\ pathway\ ;$
$\phi_2\ :\ Sagital\ plane\ angle\ of\ the\ photon-beam\ pathway\ ;$

$Note\ :\ If\ \phi_2\ angle\ is\ computated,\ the\ 3D\ precision\ is\ reached\ ;$

(1, Casesnoves Bioengineering Laboratory, 2015)

According to Equation 1, the numerical dose-delivery consequences of the Omega Factor convolution show maxima and minima, Figure 1. The maximum happens when φ1 is minimum and φ2 is maximum. On the opposite, the minimum, that is the 2D integral; convolution remains the same as it is in 2D, occurs when φ2 is minimum (≈ 0) and φ1 is maximum, Figure 1. This has implications in dose-delivery magnitude differences between 2D and 3D. Figure 1 shows a half-wedge part with the saggital plane for coordinate V1, and the coronal plane for coordinate U1. At Figure 2 it is detailed all the mathematical proof for the Omega Factor with Equations 2-3.

2.1 Geometrical Justification for Angles φ1 and φ2 Settings

Angle φ1 is set at coronal plane to preserve the initial 2D calculations, Angle φ2 is set at saggital plane to get rectangular triangles for calculations. If these angles were set between the 3D path-line and the coronal and saggital planes, instead at the plane-projections of 3D path-line, calculations would get rather more complicated, [1-20, 33, 34, 36, 46, 73].

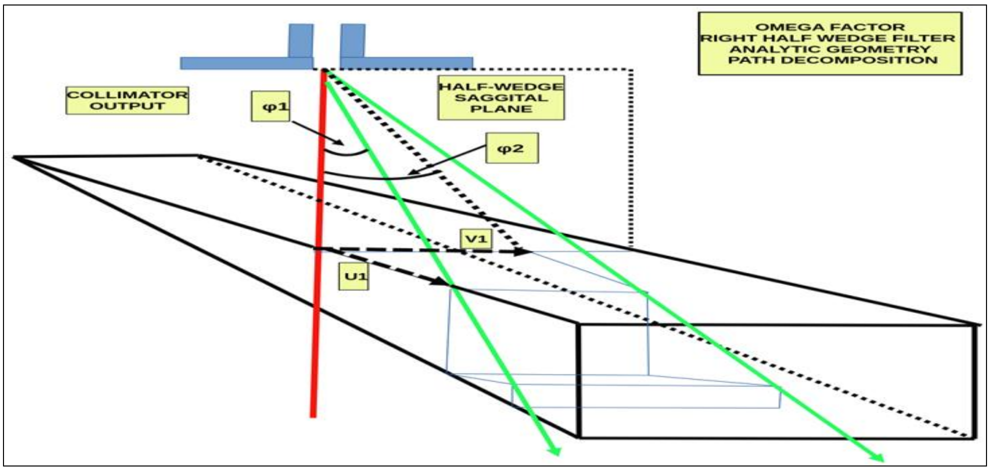

Fig 1: The analytic geometry sketch of the spatial-3D geometric al analysis carried out to determine Omega Factor $[\Omega]_F$ for complete/exact resolution of the integral of [30, 33, 34, 36]. The wedge is divided into two parts. This right one has the positive coordinates for V1 and X according to [limited 2D-AAA model, Ulmer, 1996]. It was intended a sharp simplification of the geometry in order to get a caption of the proportional segments of the pyramid corresponding to coordinates U1 and V1. The WF is divided into two halves to show the coordinates center better, showing here the half part and in Figure 2 the section of right half-part to get geometrical 3D Omega Factor precision

2.1 Briefing of Omega Factor Mathematical-Geometrical Proof

This Subsection deals with the analytic geometry OF proof, according to Figure 1. From previous contributions, Figure 1 shows the geometrical-parameters, the geometrical-analytical equations with solutions for Omega Factor are demonstrated in

Equations 2-3. The first stage is to set the proportional triangles distances and angles. That is a rather complicated analytic geometry analysis, Equation 2. Hence, the mathematical starts as follows,

OMEGA FACTOR ANALYTIC GEOMETRY

According to Figure,

The proportional segments, (Figure), quotients,

$$\frac{Y_1}{M_1} = \frac{X_1}{M} ; \frac{Y_2}{Y_1} = \frac{X_2}{X_1} ; \frac{Y_2}{M_1} = \frac{X_2}{M} ;$$

hence,

Attenuation 3D − path exponential along wedge material,

$$[AEF]_{Exact} = e^{-\mu_w \times [X_1 + X_2]} = e^{-\mu_w \times [Y_1 + Y_2] \times \left[\frac{M}{M_1}\right]} = \ldots$$

$$\ldots = \left[[AEF]_{Approximated}\right]^{[\Omega]_F} ;$$

hence,

$$Omega\ Factor\ [\Omega]_F = \left[\frac{M}{M_1}\right] ;$$

Then, forgettingapracticalalgorithm, bysettingcoordinates uand v (Figure),

$$\left[\frac{M^2}{M_1^2}\right] = \frac{s^2+u_1^2+v_1^2}{s^2+u_1^2} = \left[1 + \frac{v_1^2}{s^2+u_1^2}\right] ; or,$$

$$[\Omega]_F = \left(\left[\frac{M^2}{M_1^2}\right]\right)^{\frac{1}{2}} = \left[1 + \frac{v_1^2}{s^2+u_1^2}\right]^{\frac{1}{2}} = \left[1 + \frac{S^2\, tan^2\phi_2}{S^2 + S^2\, tan^2\phi_1}\right]^{\frac{1}{2}} = \ldots$$

$$\ldots = \left[1 + \frac{tan^2\phi_2}{1+ tan^2\phi_1}\right]^{\frac{1}{2}} \cong (series) \cong 1 + \frac{1}{2} \times \frac{tan^2\phi_2}{1+ tan^2\phi_1} ;$$

Thus, the complete exponential factor for integrand ,

$$f(u, z, \phi_1, \phi_2\alpha, L) = e^{-\mu_w \left[\left(L \pm \frac{c\,u}{F+z}\right) \times \left(\frac{sin\alpha}{cos(\alpha-\phi)}\right)\right] \times [\Omega]_F} ;$$

and the previous series approximation is applicable ;

(2, Casesnoves Bioengineering Laboratory, 2015)

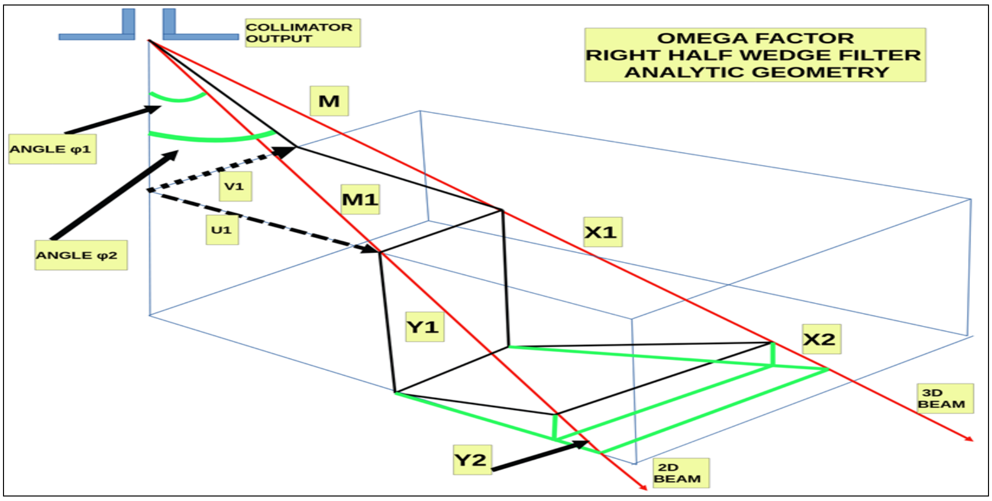

Fig 2: The sketch of the spatial-3D geometric al analysis carried out to determine Omega Factor $[\Omega]_F$ for complete/exact resolution of the integral equation of [28, 29, 30, 33, 34, 36]. It was intended a sharp simplification of the geometry in order to get a caption of the proportional segments of the pyramid corresponding to coordinates U1 and V1. The WF is divided into two halves to show the coordinates center better. Note that the coordinates system for isocenter are two: the coordinates of convolution integral U, V; and the coordinates set for 2D wedge filter calculations, X, Y, [29, 30]

3. Mathematical and Computational Methods

This section is divided into two sub-sections. First the AAA model and data mathematical framework is presented. Secondly, the algorithms computational software and program implementation is explained. Basic mathematical formulation and dataset along a series of papers, [1-20, 73, 76], the equations of the AAA original model photon-dose with WF were calculated. AAA model was named Superposition-Convolution for its mathematical formulation. That is, it is a photon-dose Cumulative Dose-distribution Integral Operator with a Gaussian, that convolutes with a photon-dose distribution obtained from several Gaussian-optimized steps starting with the Yukawa Function [29]. In other words, the term 'Superposition' comes from the sum of three

Gaussians into the integral operator. The term 'Convolution' describes the mathematical transformation carried out into the Dose-Deposition Kernel at the Integral Operator. As said, based on this number of previous articles, [1-20, 73], the WF 3D exact dose delivery in water without tissue-attenuation corrections is developed in Equation 3, [Casesnoves, 2015].

3.1 The Omega Factor Integral Equation Exact Solution

From 2.1 subsection, here the OF-corrected AAA integral equation analytical solution is reviewed. Once the OF is determined, implemented within the exponential, the analytical solution of the photon-beam dose can be calculated exactly, or with a simple series development of OF. The starting point and the final analytic solution is shown in Equation 3.

$$OMEGA\ FACTOR\ [\Omega]_F\ EXACT\ ANALYTICALINTEGRATION\ [Casesnoves\ 2015-2016]$$
$$Definition: Omega\ Factor,\ [\Omega]_F,\ is\ a\ convolution\ factor$$
$$set\ exponentially\ to\ add\ 3D\ precision\ to$$
$$the\ photon-beam\ pathway\ length-magnitude$$
$$throughout\ the\ radiotherapy\ wedge-filter,$$
$$as\ beam\ modification\ device;$$
$$The\ AEF\ in\ 3D\ for\ the\ thin$$
$$part\ of\ the\ wedge, according\ AAA\ model\ foundation\ reads,$$

$$f(u,z,\phi,\alpha,L)=e^{-\mu_w\left[\left(L\pm\frac{c\,u}{F+z}\right)\times\left(\frac{sin\alpha}{cos(\alpha-\phi_1)}\right)\right]\times[\Omega]_F};$$

$$hence,\ for\ 2D,\ the\ AAA\ Dose-deposition\ integral\ was,$$

$$D(x,y,z)=I(z)\int_{-a'}^{a'}\int_{-b'}^{b'}\sum_{K=1}^{K=3}\Phi_U(u,v,z)\frac{C_K}{\pi\sigma_K^2}e^{\left[-\frac{(x-u)^2+(u-v)^2}{\sigma_K^2}\right]}dudv;$$

$$thus,\ modified\ for\ 3D\ with\ Omega\ Factor,$$
$$D(x,y,z)=I(z)\int_{-a'}^{a'}\int_{-b'}^{b'}\sum_{K=1}^{K=3}\Phi_U(u,v,z)\frac{C_K}{\pi\sigma_K^2}e^{\left[-\frac{(x-u)^2+(u-v)^2}{\sigma_K^2}\right]}\times...$$

$$...\times\ e^{-\mu_w\left[\left(L\pm\frac{c\,u}{F+z}\right)\times\left(\frac{sin\alpha}{cos(\alpha-\phi_1)}\right)\right]\times[\Omega]_F;}dudv;$$

$$Separating\,the\,part\,of\,3D\,wedge-path\,independent\,from\,integral\,variables,$$
$$it\ is\ defined,$$
$$A=A([\Omega]_F)=e^{-\mu_w\left[L\times\left(\frac{sin\alpha}{cos(\alpha-\phi_1)}\right)\right]\times[\Omega]_F;}, corrected\,with\ [\Omega]_F\ for\,3D-path;$$
$$For\ the\ second\ part\ depending\ on\ variable\ u,\ [Ulmer, 1996],$$
$$it\ is\ necessary\ make\ a\ new\ quadratic\ change,\ which\ results,$$
$$defining\ S=S(\,[\Omega]_F\,),\ thus,$$
$$S=S([\Omega]_F)=\pm\frac{1}{2}\times\mu_w\ \frac{c\,u}{F+z}\times[\Omega]_F\ ;$$
$$Hence,\ complete\ analytical\ dose-delivery\ integration,$$
$$D(x,y,z)=\frac{\Phi_0\ I(z)\ A([\Omega]_F)}{4\left(1+\frac{z}{F}\right)^2}\sum_{K=3}^{K=1}C_K\ e^{\left[\sigma_K^2(S([\Omega]_F))^2-2S([\Omega]_F)x\right]}\times...$$

$$...\times\left[\left(erf\frac{y+b'}{\sigma_K}\right)-\left(erf\frac{y-b'}{\sigma_K}\right)\right]\times\left[\left(erf\frac{x+a'-\sigma_K^2S([\Omega]_F)}{\sigma_K}\right)-\left(erf\frac{x-a'-\sigma_K^2S([\Omega]_F)}{\sigma_K}\right)\right];$$

$$[\Omega]_F\ fundamental\ equation\ added\ factor\ at\ exponent\ is,$$

$$[\Omega]_F=\left[1+\frac{tan^2\phi_2}{1+tan^2\phi_1}\right]^{\frac{1}{2}};$$

$$where,$$
$$\phi_1:\ Coronal\ plane\ angle\ of\ the\ photon-beam\ pathway;$$
$$\phi_2:\ Sagital\ plane\ angle\ of\ the\ photon-beam\ pathway;$$
$$Note:\ If\ \phi_2\ angle\ is\ computated,\ the\ 3D\ precision\ is\ reached;$$

(3, Casesnoves Bioengineering Laboratory, 2015)

Where A (OF) and S (OF) parameters are defined at formulas according to a series of parameters. I(z) is the area integral of the dose over a plane perpendicular to the z-axis at depth z, normalized to one incident electron, σ (z) is the depth-dependent mean square radial displacement, x, y, z are the coordinates of the dose-delivery point at beam-output coordinates system, u, v, z, are the coordinates of photon-fluence at depth z, a, b, are the field-size magnitudes at depth z, CK are optimization parameters resulting for a triple Gaussian function setting, ΦU [28, 29, 30, 33, 34, 36] is the photon fluence modified for WF, a'(z) =a(1 + z/F), b'(z) =b(1 + z/F) are the halfside lengths projected into depth z, with F as the source-surface distance (SSD). Parameters μw

(WF material parameter), α (WF angle) and φ (photon-beam divergence angle) are defined at [1-10, 55, 56], Equations 1-3. The WF manufacturing angle is α. C is the distance in z-coordinate from source to WF surface, L is half-length of WF for y coordinate. In general Fluence modification is $ΦU = Φ0 / (1 + z/F)^2$ according to [55, 56], and $ΦU = Φ0 / (1 + z/F)^2$ according to [28, 29, 30, 33, 34, 36], Equations 1-3. Then source fluence is Φ0 modified primarily for the dose-delivery depth z and secondly by the WF parameters. Parameters μw (WF material parameter), α (WF angle) and φ (photon-beam 2D divergence angle, here φ1, Figures 1-2) are defined at [1-20, 28, 29, 30, 33, 34, 36, 73]. Development of Equations 1-3 are extensively presented in [1-20, 28, 29, 30, 33, 34, 36, 73]. The angles Φ1 and Φ2 are the geometrical angles for 3D Omega Factor computation defined [1-20]. The foundation of this 2D-AAA solution for WF dose delivery, was firstly developed [30]. Simulations Experimental Dataset Table 1 shows the numerical parameters for 3D Isodoses simulations slightly improved/modified from [1-20, 73]. In brief, all these calculations are based on previous publications series [1-20, 73].

4. 3D Isodoses Dataset and Software-Programming Method

The computational method is based on previous software works [1-20, 73]. The integral equation complete analytical solution, Equations 1-3, is correctly simulated in dosimetry-matrices from 100 x 100 dimensions to 300 x 300 dimensions in the following results section and compared with simulations of equations of classical AEF [1-20] in AAA model foundations. The structure of the program comprises the summatory of every part of Equation 3 Erf function. Firstly, these parts of Erf functions are set independently one by one. Secondly all of them are summed. Finally, the resulting numerical values are set in the imaging subroutine. In this study, they are implemented in Matlab. Imaging processing tools and subroutines/options are applied [1-20, 73]. A number of previous publications develop WF AAA photon dose 3D graphical processed images for GNU-Octave and Freemat also [1-20]. But in these cases the programs involve rather significant modifications [1-20].

4.1 Dataset for Software Engineering

Table 1 shows the photon-beam parameter magnitudes and wedge filter dimensions. LINAC SSD parameter and field size is also specified, [1-20, 28, 29, 30, 33, 34, 36, 73].

Table 1: Numerical dataset to be implemented in software for a WF of 15 degrees and maximum φ1 and φ2 angles 30 degrees. Source-surface-distance, (SSD), is set at standard magnitude of 100 cm, and simulations are carried out for z-depth values of 5, 15 cm. Namely, where the dose deposition tends to be maximum. Fluence magnitudes are set mainly from [35, 37]

NUMERICAL DOSIMETRY DATASET			
RADIATION PARAMETER	EXPERIMENTAL REFERENCES AND SIMULATION SELECTED DATA	APPROXIMATED MAGNITUDE	AAA MODEL DATA
SSD [cm]	Simulation dataset [1-20,73,74]	Exactly 100 cm	[1-20,73,74]
Fluence Rate [N x cm^{-2} x s^{-1}] N:number of photons	$Φ_0$ ϵ Experimental Data with Foilers [74]	$Φ_0$ ϵ [4.16 x 10^9 , 2.82 x 10^{10}] At program, 10^9 was generally implemented	[1-20,73,74]
Field Size [cm^2] WF Angle	WF Angle=15° 12 x 12 [cm] approximated field size	WF Angle=15° 12 x 12 [cm]	[1-20,73,74]
Omega Factor Angles φ1=30° φ2=30° [approximately] WF angle α	From those articles coming from Au foils experimental dosimetry measurements [1,74]	φ1=30° φ2=30° WF Angle=15°	[1-20,73,74]
Dose Rate [Gy/s]	According to [74]	1 Gy dose delivery time At [Au foils] 1 Gy delivery time ϵ [14,19] seconds	[1-20,73,74]

4.2 Software Engineering Programming

The structure of the program comprises the summatory of every part of Equation 3 Erf function, but 3D modified. Firstly, these parts of Erf functions are set independently one by one. Secondly all of them are summed. Finally, the resulting numerical values are set in the imaging subroutine. In this study, they are implemented in Matlab. Imaging processing tools and subroutines/options are applied [1-20, 73]. Just to remark that Erf functions seem be rather complicated but for software programming are easy with modern systems, such as Matlab or GNU-Octave.

5 Results 3D Isodoses for TPO

The structure of the program comprises the summatory of every part of Equation 3 Erf function. Firstly, these parts of Erf functions are set independently one by one. Secondly all of them are summed. Finally, the resulting numerical values are set in the imaging subroutine. In this study, they are implemented in Matlab. Imaging processing tools and subroutines/options are applied [1-20, 73].

Figures 3-4 show Type I 3D Isodoses WF graphics for [z=5 cm, z=15 cm] depth doses with AAA model. The innovation of the study involves practical use for TPO, in contrast with classical 2D Isodoses plots. The visualization 3D advantages related to 2D Isodoses are evident. The number of 3D WF dose delivery images in Figures 5-6 is two, but it is not limited to any amount superposed 3D charts in the same image. Figure 7 is a WF dose-zone clarification for better reading.

5.1 Type I 3D Isodoses [Vertical]

Figures 2-3 show Type I 3D Isodoses WF graphics for [z=5 cm, z=15 cm] depth doses with AAA model. The innovation of the study involves practical use for TPO, in contrast with classical 2D Isodoses plots. The visualization 3D advantages related to 2D Isodoses are evident. The number of 3D WF dose delivery images in Figures 3-4 is two, but it is not limited to any amount superposed 3D charts in the same image.

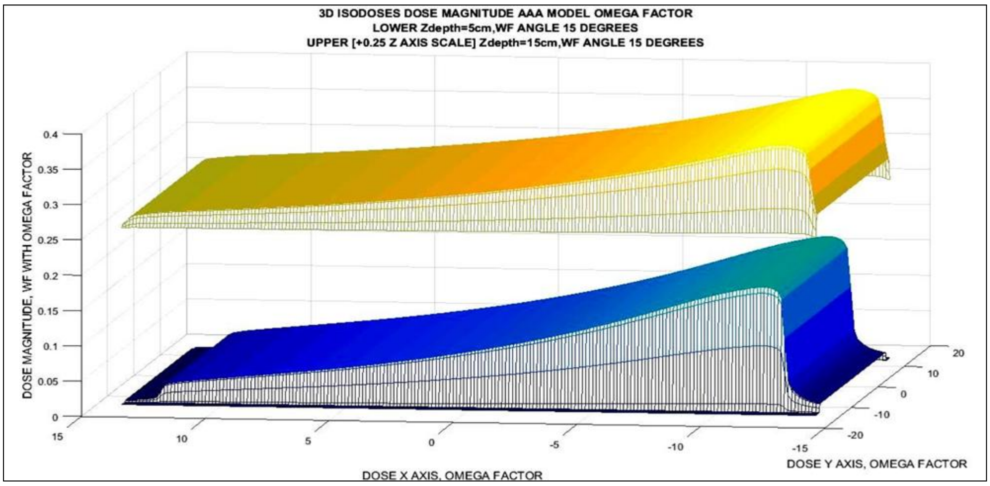

Fig 3: Type I lateral imaging perspective of 3D Isodoses for z=15 cm [upper, scaled +0.25], and z=5 cm [lower]. It is clear the dose difference related to depth absorbed dose deposition. Dosimetry calculations, TPO, and photon-dose approximations can be carried out with these 3D Isodoses charts

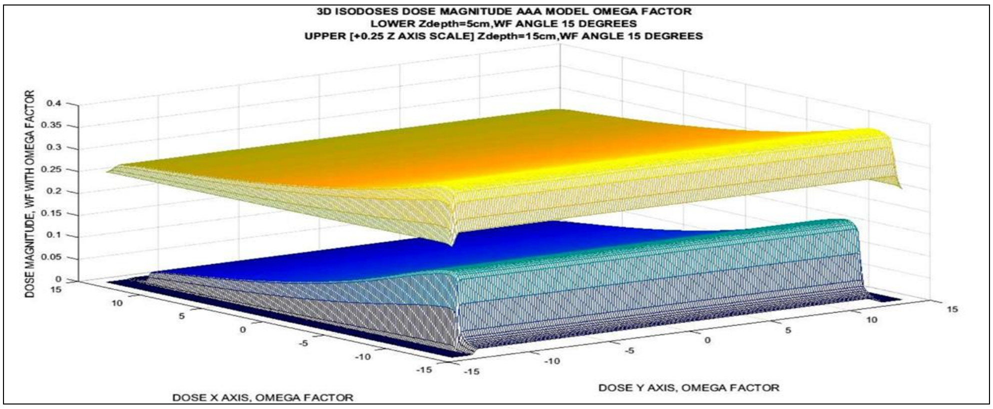

Fig 4: Type I oblique-lateral imaging perspective of 3D Isodoses for z=15 cm [upper, scaled +0.25], and z=5 cm [lower]. It is clear the dose difference related to depth absorbed dose deposition

5.2 Type II 3D Isodoses [Horizontal]

Figures 5-6 show Type II 3D Isodoses WF graphics for [z=5 cm, z=15 cm] depth doses with AAA model. The innovation of the study involves practical use for TPO, in contrast with classical 2D Isodoses plots. Figure 7, from [1] is a sharp reminder of the four dimensions that can be numerically determined in a 4D WF graph.

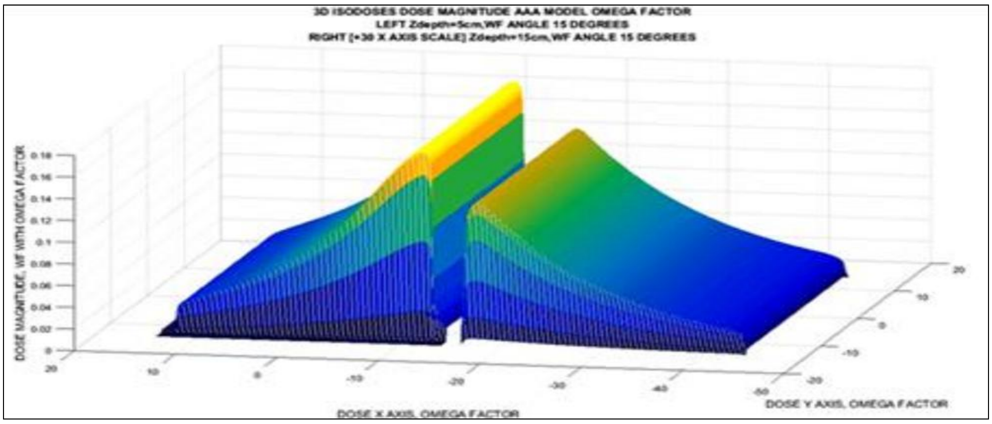

Fig 5: Type II lateral imaging perspective of 3D Isodoses for z=5 cm [left], and z=15 cm [right, scaled +30]. It is clear the height dose difference related to depth absorbed dose deposition

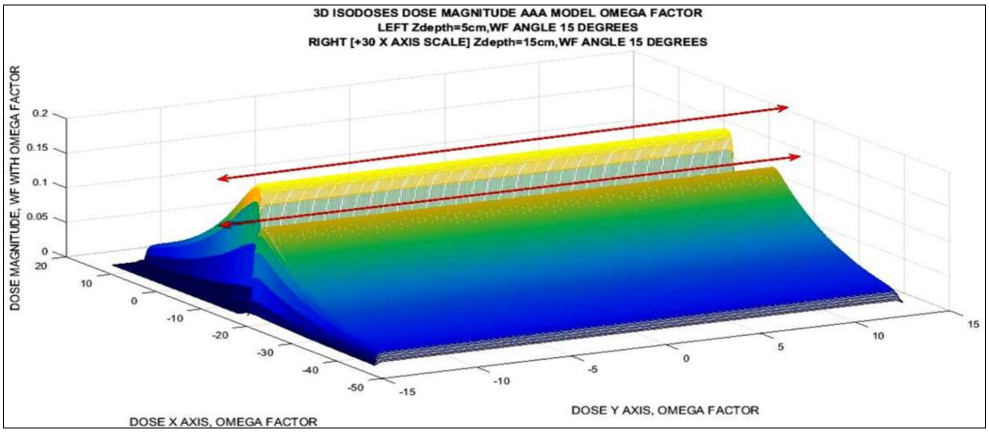

Fig 6: Type II oblique-lateral imaging perspective of 3D Isodoses for z=5 cm [left], and z=15 cm [right, scaled +30]. It is clear, red arrows inset, the height dose difference related to depth absorbed dose deposition

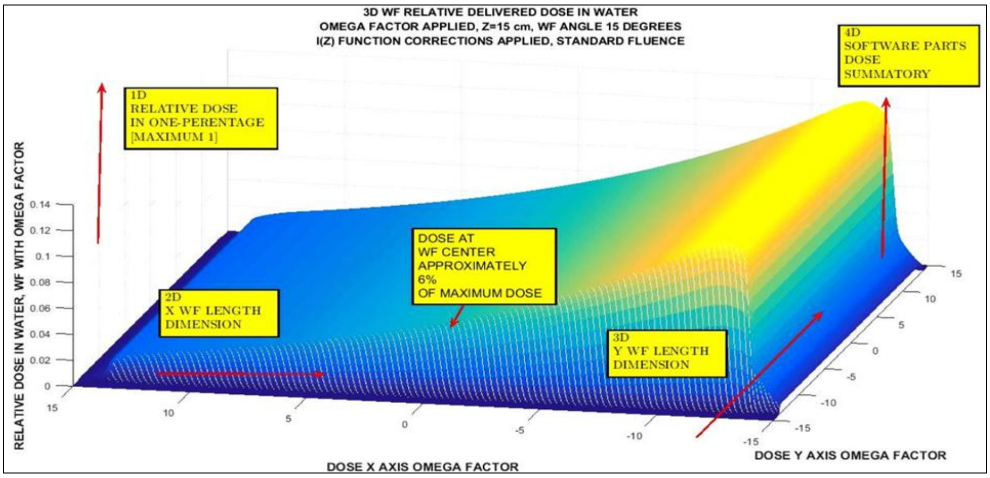

Fig 7: From [1], a Matlab simulation 3D image for 18Mev photon-beam at 15 cm depth-dose with Omega Factor and I(z) corrected. Numerical data that can be obtained in simulation comprises four dimensions. Namely, relative dose in one-percent, X coordinate at WF, Y coordinate at WF, and program part relative dose. Fluence magnitude according to Sections 2-4. Matrices for Image Processing have about 103 elements. The superposition of imaging-processing erf functions implementation can be observed

6. Medical Physics Breast Tumors TPO Applications

Table 2 shows a resume of breast tumor [9.1-9.4, 52, 70, 73, 75] medical physics principal applications for 3D Isodoses radiotherapy TPO. 3D Isodoses Those prospective applications are also useful for further radiotherapy research/applications on breast tumors and other types of cancer.

Table 2: Briefing of medical physics applications of the study, both for TPO and I+D

3D ISODOSES CLINICAL MEDICAL PHYSICS TPO BREAST TUMOR APPLICATIONS				
TYPE	CLINICAL	RESEARCH	MIXED	COMMENTS
TREATMENT PLANNING OPTIMIZATION	TPO precise for breast tumors with AAA and Biomodels	TPO Modelling developments	Clinical improvements with BMs and NTCP models after research	Inverse planning system set up on Photon-Dose Model, BMs, and NTCP
BREAST CANCER RADIOPROTECTION	OARs determination with CT	Optimization of minimum OARs irradiation	Clinical improvements with BMs and NTCP models after research	NTCP models are important in breast cancer radioprotection
LINAC MANUFACTURING OPTIMIZATION	Optimization of photon-dose for BMs	LINACs BMs , NTCP, Usage for IMRT, IMPT	Exploration of new possibilities	Manufacturing adaptation of LINACs for BMs and NTCP ones
THEORETICAL NEW MODELS	Dosimetry improvements in accuracy according to radiobiology experimental	From tumor survival clinical statistics advances in BMs and NTCP	New BMs, Photon-Dose, and NTCP models research, both theory and clinical experimental trials	BMs got experimental evidences to be set on TPO

7. Discussion and Conclusions

The first objective of the research was to demonstrate and prove mathematically the feasibility and practical TPO usage of new 3D Isodoses for WF photon-dose delivery with 3D Omega Factor. Omega Factor convolution integral equation solution is demonstrated. Specially, elaborated/illustrative sketchs, Figures 1-2, to prove the analytic geometry algorithm were done. The second objective was to implement and develop software engineering for 3D imaging processing, provided to previous algorithms implementation. The model for calculations and images was, as usual in research series, the AAA one extensively used/improved in current photon-dose LINACs. Breast cancer TPO, based on [1-20, 74, 75] criteria, was the practical usage of 3D Isodoses proven. Images obtained were got sharp and contrasted, Figures 2-7. The precision of the analytic determination of the Omega Factor was demonstrated properly. The exact convolution of the corrected 3D AEF with Omega Factor was detailed. The programming system used was Matlab. However both Freemat and GNU-Octave have demonstrated efficacy for these studies [1-20, 73].

For the second objective, the software required several combined WF calculations implemented in an unique program. However, the increase running time was less than expected. The reason was the optimal set of patterns, loops, image subroutine selected, and avoiding redundant calculations. These techniques were rather arduous in the programming task. Method dataset is shown in Table 1. Water AAA model approaches to breast tissue adipose density with acceptable approximations.

The improvement of Omega Factor within the integral equation of AAA model shows a higher precision, detailed by a curved dose-distribution at image-processing. The disadvantage is that it requires further calculations and implementation in software. The objective to get more exact algorithms related to [Ulmer, 29,30] initial AAA 2D model was reached.

Advantages of 3D isodoses for TPO are the imaging perspectives, better calculations, fast calculus of dose or approximate dose delivery, and others, Figures 1-5. Inconvenients could be the rather more difficult programming and numerical refinements to process 3D optimal Isodoses images, compared to classical 2D Isodoses graphs.

In brief, 3D Isodoses WF photon-dose imaging processing graphs were simulated-presented for Omega Factor AAA model in water without tissue heterogeneities inset at programming. Omega Factor Equations were proven. The specific focus was breast cancer [9.1-9.4, 38, 39, 73, 74] applications as done in [1-20] for TPO WF dose delivery applications.

8. Scientific Ethics Standards

Omega Factor was created by Author at Philadelphia city in spring 2015. All the software was done by Author, based on radiotherapy and hip wear previous articles and materials engineering/bioengineering software. This type of software is developed from Author's series of previous publications in hip prostheses wear and other computational contributions. The article contains few reviews of the previous publications essential for complete understanding. The 3D multidigital simulations are original from the Author, software, design, patterns and 3D image processing. The Graphical Optimization software-programs were developed by Author, based on literature and Radiotherapy publications, and previous experience. Article has very short review from literature to make all more sharp/perceptive. No artificial intelligence (AI) tools or systems were used for programming anyway. No artificial intelligence (AI) information from browsers was included, and if it would be so, it is always declared. Radiotherapy references are included as they are the programming base. This study was carried out, and their contents are done according to the International Scientific Community and European Union Technology and Science Ethics, [43-45]. References: 'European Textbook on Ethics in Research'. European Commission, Directorate-General for Research. Unit L3. Governance and Ethics. European Research Area. Science and Society. EUR 24452 EN. And based on 'The European Code of Conduct for Research Integrity'. Revised Edition. ALLEA. 2017, [43-45]. When a mathematical statement, algorithm, proposition or theorem is presented, demonstration is always included. When a formula is presented, all parameters are detailed or referred. If any results inconsistency is found after publication, it is clarified in subsequent. When a citation such as [Casesnoves, 'year'] is set, it is exclusively to clarify intellectual property at current times, without intention to brag. The article is exclusively scientific, without any commercial, institutional, academic, deliberate obfuscation, religious or religious influences, religious-similar, non-scientific theories, personal opinions, political ideas, political or economical influences. When anything is taken from a source, it is adequately recognized.

9. Some Specific References for Breast Tumor Radiotherapy

9.1 Pinheiro M, *et al*. Towards a breast phantom: An evaluation of total mass attenuation coefficients for simulating breast tissues and pathologies. Academia Engineering, 2025. Doi: https://doi.org/10.20935/AcadEng7940

9.2 Ko E, *et al*. Analysis of the effect of breast magnetic resonance imaging on the outcome in women undergoing breast conservation surgery with radiation therapy. J Surg Oncol. 2013; 107:815-821.

9.3 Bellon J, *et al*. Radiation Therapy Techniques and Treatment Planning for Breast Cancer. Springer, 2016.

9.4 Haydaroglu A, Ozyigit G. Principles and Practice of Modern Radiotherapy Techniques in Breast Cancer. Springer, 2013. Doi: 10.1007/978-1-4614-5116-7

10. General References with Further Reading

1. Casesnoves F. Radiotherapy Wedge Filter AAA Model 18 Mev- Dose Delivery 3D Simulations with Several Software Systems for Medical Physics Applications. Applications. Biomed J Sci & Tech Res. 2022; 40(5). Doi: 10.26717/BJSTR.2022.46.007337
2. Casesnoves F. Mathematical Exact 3D Integral Equation Determination for Radiotherapy Wedge Filter Convolution Factor with Algorithms and Numerical Simulations. Journal of Numerical Analysis and Applied Mathematics. 2016; 1(2):39-59. ISSN Online: 2381-7704

3. Casesnoves F. Radiotherapy Conformal Wedge Computational Simulations, Optimization Algorithms, and Exact Limit Angle Approach. International Journal of Scientific Research in Science, Engineering and Technology (IJSRSET). 2015; 1(2):353-362. Print ISSN: 2395-1990. Online ISSN: 2394-4099
4. Casesnoves F. Improvements in Simulations for Radiotherapy Wedge Filter dose and AAA-Convolution Factor Algorithms. International Journal of Scientific Research in Science, Engineering and Technology (IJSRSET). 2019; 6(4):194-219. Print ISSN: 2395-1990. Online ISSN: 2394-4099
5. Casesnoves F. Exact/Approximated Geometrical Determinations of IMRT Photon Pencil-Beam Path Through Alloy Static Wedges in Radiotherapy Using Anisothropic Analytic Algorithm (AAA). Peer-reviewed ASME Conference Paper. ASME 2011 International Mechanical Eng Congress. Denver. USA. IMECE2011-65435, 2011.
6. Casesnoves F. Geometrical Determinations of Limit angle (LA) related to maximum Pencil-Beam Divergence Angle in Radiotherapy Wedges. Peer-reviewed ASME Conference Paper. ASME 2012 International Mechanical Eng Congress. Houston. USA. IMECE2012-86638, 2012.
7. Casesnoves F. A Conformal Radiotherapy Wedge Filter Design. Computational and Mathematical Model/Simulation'. Peer-Reviewed Poster IEEE (Institute for Electrical and Electronics Engineers), Northeast Bioengineering Conference. Syracuse New York, USA. April 6th, 2013. Peer-Reviewed Poster Session on 6th April 2013. Sessions 1 and 3 with Poster Number 35. Page 15 of Conference Booklet Printed, 2013.
8. Casesnoves F. Mathematical and Geometrical Formulation/Analysis for Beam Limit Divergence Angle in Radiotherapy Wedges. Peer-Reviewed International Engineering Article. International Journal of Engineering and Innovative Technology (IJEIT). 2014; 3(7). ISSN: 2277-3754. ISO 9001:2008 Certified
9. Casesnoves F. Geometrical determinations of IMRT photon pencil-beam path in radiotherapy wedges and limit divergence angle with the Anisotropic Analytic Algorithm (AAA) Casesnoves, F. Peer- Reviewed scientific paper, both Print and online. International Journal of Cancer Therapy and Oncology. 2014; 2(3):2031. Doi: 10.14319/IJCTO.0203.1. Corpus ID: 460308
10. Casesnoves F. Radiotherapy Conformal Wedge Computational Simulations and Nonlinear Optimization Algorithms. Peer-reviewed Article, Special Double-Blind Peer-reviewed paper by International Scientific Board with contributed talk. Official Proceedings of Bio- and Medical Informatics and Cybernetics: BMIC 2014 in the context of the 18th Multi-conference on Systemics, Cybernetics and Informatics: WMSCI 2014 July 15 - 18, 2014, Orlando, Florida, USA, 2014. ISBN: 978-1-941763-03-2 (Collection). ISBN: 978-1-941763-10-0 (Volume II)
11. Casesnoves F. Large-Scale Matlab Optimization Toolbox (MOT) Computing Methods in Radiotherapy Inverse Treatment Planning. High Performance Computing Meeting. Nottingham University. Conference Poster, 2007.
12. Casesnoves F. A Computational Radiotherapy Optimization Method for Inverse Planning with Static Wedges. High Performance Computing Conference. Nottingham University. Conference Poster, 2008.
13. Casesnoves F. Radiotherapy Conformal Wedge Computational Simulations, Optimization Algorithms, and Exact Limit Angle Approach. International Journal of Scientific Research in Science, Engineering and Technology. 2015; 1(2). Print ISSN: 2395-1990, Online ISSN: 2394-4099
14. Casesnoves F. Radiotherapy Standard/Conformal Wedge IMRT-Beamlet Divergence Angle Limit Exact Method, Mathematical Formulation, and Bioengineering Applications. International Article-Poster. Published in Proceedings of Conference. 41st Annual Northeast Bioengineering Conference. Rensselaer Polytechnic Institute. Troy, New York USA, April, 2015, 17-19. Doi: 10.1109/NEBEC.2015.7117152. Corpus ID: 30285689
15. Casesnoves F. Radiotherapy Standard/Conformal Wedge IMRT-Beamlet Divergence Angle Limit Exact Method, Mathematical Formulation, and Bioengineering Applications. IEEE (Institute for Electrical and Electronics Engineers), International Article-Poster, 2015. http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7117152
16. Casesnoves F. Abstract-Journal. Radiotherapy Standard/ Conformal Wedge IMRT-Beamlet Divergence Angle Limit Exact Method, Mathematical Formulation. International Conference on Significant Advances in Biomedical Engineering. 252nd OMICS International Conference 5(1). Francisco Casesnoves, J Bioengineer & Biomedical Sci. 2015; 5(1). Doi: http://dx.doi.org/10.4172/2155-9538.S1.003
17. Casesnoves F. Determination of absorbed doses in common radio diagnostic explorations. 5th National Meeting of Medical Physics. Madrid, Spain. September 1985. Treatment Planning, 2001.
18. Casesnoves F. Master Thesis in Medical Physics. Eastern Finland University. Radiotherapy Department of Kuopio University Hospital and Radiotherapy Physics Grouversity-Kuopio. Defense approved in 2001. Library of Eastern finland University. Finland, 2001.
19. Casesnoves F. A Conformal Radiotherapy Wedge Filter Design. Computational and Mathematical Model/Simulation. Peer-Reviewed Poster IEEE (Institute for Electrical and Electronics Engineers), Northeast Bioengineering Conference. Syracuse New York, USA. Presented in the Peer-Reviewed Poster Session on 6th April 2013. Sessions 1 and 3 with Poster Number 35. Page 15 of Conference Booklet. April 6th, 2013, 2013.
20. Casesnoves F. Radiotherapy Biological Tumor Control Probability Integral Equation Model with Analytic Determination. International Journal of Mathematics and Computer Research. 2022; 10(8):2840-2846. Doi: https://doi.org/10.47191/ijmcr/v10i10.01
21. Casesnoves F. Radiotherapy Wedge Filter AAA Model 3D Simulations For 18 MEV 5 cm-Depth Dose with Medical Physics Applications. International Journal of Scientific Research in Computer Science, Engineering and Information Technology (IJSRCSEIT). 2022; 8(1):261-274. ISSN: 2456-3307 (www.ijsrcseit.com). Doi: https://doi.org/10.32628/CSEIT228141
22. Walsh S. Radiobiological modelling in Radiation Oncology. PhD Thesis. School of Physics. National University of Galway, 2011. http://hdl.handle.net/10379/3027

23. Chapman D, Nahum A. Radiotherapy Treatment Planning, Linear- Quadratic Radiobiology. CRC Press, 2015. ISBN: 9780367866433
24. Mayles W, Nahum A. Rosenwald, J. Editors. Handbook of Radiotherapy Physics. Second Edition. CRC Press, 2015. ISBN: 9780367192075. International Standard Book Number-13: 978-1-4987-2146-2
25. Nahum A, Webb S. A model for calculating tumour control probability in radiotherapy including the effects of inhomogeneous distributions of dose and clonogenic cell density. Physics in Medicine and Biology; v. 1993; 38(6):653-666. ISSN: 0031-9155
26. Haydaroglu A, Ozyigit G. Principles and Practice of Modern Radiotherapy Techniques in Breast Cancer. Springer, 2013. Doi: 10.1007/978-1-4614-5116-7
27. Casesnoves F. Die numerische Reuleaux-Methode Rechnerische und dynamische Grundlagen mit Anwendungen (Erster Teil), 2019-20. ISBN-13 : 978-620-0-89560-8, ISBN-10: 6200895600. Publishing House: Sciencia Scripts. 2019-20.
28. Ulmer W, Harder D. Corrected Tables of the Area Integral I(z) for the Triple Gaussian Pencil Beam Model. Z Med Phys. 1997; 7:192-193. Doi: https://doi.org/10.1016/S0939-3889(15)70255-2
29. Ulmer W, Harder D. A triple Gaussian pencil beam model for photon beam treatment planning. Med. Phys. 1995; 5:25-30. Doi: 10.1016/S0939-3889(15)70758-0
30. Ulmer W, Harder D. Applications of a triple Gaussian pencil beam model for photon beam treatment planning. Med Phys. 1996; 6:68-74. Doi: https://doi.org/10.1016/S0939-3889(15)70784-1
31. Ma C, Lomax T. Proton and Carbon Ion Therapy. CRC Press, 2013. Doi: https://doi.org/10.1201/b13070
32. Censor Y, Zenios S. Parallel Optimization: Theory, Algorithms and Applications. UOP, 1997. Doi: 10.12694/SCPE.V3I4.207. Corpus ID: 19584334
33. Ulmer W, Pyyry J, Kaissl W. A 3D photon superposition/ convolution algorithm and its foundation on results of Monte Carlo calculations. Phys Med Biol, 2005, 50. Doi: 10.1088/0031-9155/50/8/010
34. Ulmer W, Harder D. Applications of the triple Gaussian Photon Pencil Beam Model to irregular Fields, dynamical Collimators and circular Fields. Phys Med Biol, 1997. Doi: https://doi.org/10.1023/B:JORA.0000015192.56164.a5
35. Haddad K, Anjak O, Yousef B. Neutron and high energy photon fluence estimation in CLINAC using gold activation foils. Reports of practical oncology and radiotherapy. 2019; 24:41-46. Doi: 10.1016/j.rpor.2018.08.009
36. Sievinen J, Waldemar U, Kaissl W. AAA Photon Dose Calculation Model in Eclipse™. Varian Medical Systems Report. Rad #7170A, Kaipio corrupt.
37. Vagena E, Stoulos S, Manolopoulou M. GEANT4 Simulations on Medical LINAC operation at 18MV: Experimental validation based on activation foils. Radiation Physics and Chemistry, 2016. Doi: 10.1016/j.radphyschem.2015.11.030
38. Ethics for Researchers. EU Commission. Directorate-General for Research and Innovation. Science in society/Capacities FP7, 2013. https://data.europa.eu/doi/10.2777/7491
39. Casesnoves F. Surgical Pathology I course class notes and clinical practice of Surgical Pathology Madrid Clinical Hospital [Professor Surgeon Dr Santiago Tamames Escobar]. 4th academic year course for graduation in Medicine and Surgery. Lessons and practice Breast Cancer Surgical and Medical Treatment. 1980-1981. Madrid Complutense University, 1981.
40. Tamames Escobar S. Cirugia/ Surgery: Aparato Digestivo. Aparato Circulatorio. Aparato Respiratorio/ Digestive System. Circulatory System. Respiratory System (Spanish Edition), 2000. ISBN: 10: 8479034955. ISBN: 13: 9788479034955
41. Formenti S, Demaria S. Combining Radiotherapy and Cancer Immunotherapy: A Paradigm Shift Silvia C. Formenti, Sandra Demaria. J Natl Cancer Inst. 2013; 105:256-265. Doi: 10.1093/jnci/djs629
42. Numrich R. The computational energy spectrum of a program as it executes. Journal of Supercomputing. 2010; 52. Doi: 10.1007/s11227-009-0273-x
43. European Commission, Directorate-General for Research. Unit L3. Governance and Ethics. European Research Area. Science and Society, 2021.
44. Allea. The European Code of Conduct for Research Integrity, Revised Edn.; ALLEA: Berlin Barndenburg Academy of Sciences, 2017.
45. Good Research Practice. Swedish Research Council, 2017. ISBN 978-91- 7307-354-7
46. Ulmer W, Schaffner B. Foundation of an analytical proton beamlet model for inclusion in a general proton dose calculation system. Radiation Physics and Chemistry. 2011; 80:378-389. Doi: 10.1016/j.radphyschem.2010.10.006
47. Sharma S. Beam Modification Devices in Radiotherapy. Lecture at Radiotherapy Department, PGIMER. India, 2008.
48. Barrett A, Colls. Practical Radiotherapy Planning. Fourth Edition. Hodder Arnold, 2009. ISBN: 9780340927731
49. Ahnesjö A, Saxner M, Trepp A. A pencil beam model for photon dose calculations. Med Phys, 1992, 263-273. Doi: 10.1118/1.596856
50. Brahime A. Development of Radiation Therapy Optimization. Acta Oncologica. 2000; 39(5). Doi: 10.1080/028418600750013267
51. Bortfeld T, Hong T, Craft D, Carlsson F. Multicriteria Optimization in Intensity-Modulated Radiation Therapy Treatment Planning for Locally Advanced Cancer of the Pancreatic Head. International Journal of Radiation Oncology and Biology Physics. 2008; 72(4). Doi: 10.1016/j.ijrobp.2008.07.015
52. Brown B, Cols. Clinician-led improvement in cancer care (CLICC) - testing a multifaceted implementation strategy to increase evidence-based prostate cancer care: Phased randomised controlled trial - study protocol. Implementation Science. 2014; 9:64. Doi: https://doi.org/10.1186/1748-5908-9-64
53. Bortifield T. IMRT: A review and preview. Phys Med Biol. 2006; 51:R363-R379. Doi: 10.1088/0031-9155/51/13/R21
54. Censor Y. Mathematical Optimization for the Inverse problem of Intensity-Modulated Radiation Therapy. Laboratory Report, Department of Mathematics, University of Haifa, Israel, 1996.

55. Capizzello A, Tsekeris PG, Pakos EE, Papathanasopoulou V, Pitouli EJ. Adjuvant Chemo-Radiotherapy in Patients with Gastric Cancer. Indian Journal of Cancer. 2006; 43(4). ISSN: 019-509X
56. Tamer Dawod EM Abdelrazek Mostafa Elnaggar, Rehab Omar. Dose Validation of Physical Wedged symmetric Fields in Artiste Linear Accelerator. International Journal of Medical Physics, Clinical Engineering and Radiation Oncology. 2014; 3:201-209. Doi: 10.4236/ijmpcero.2014.34026
57. Do SY, David A, Bush Jerry D Slater. Comorbidity-Adjusted Survival in Early-Stage Lung Cancer Patients Treated with Hypofractioned Proton Therapy. Journal of Oncology, 2010. Doi: 10.1155/2010/251208
58. Ehrgott M, Burjony M. Radiation Therapy Planning by Multicriteria Optimization. Department of Engineering Science. University of Auckland. New Zealand. Conference Paper, 1999.
59. Ezzel G. Genetic and geometric optimization of three-dimensional radiation therapy treatment planning. Med Phys. 1996; 23:293-305. Doi: 10.1118/1.597660
60. Effective Health Care. Number 13. Comparative Efectiveness of Therapies for Clinically Localized Prostate cancer. Bookshelf ID: NBK554842, 2008.
61. Hansen P. Rank-deficient and discrete ill-posed problems: Numerical aspects of linear inversion. SIAM monographs on mathematical modelling and computation, 1998. ISBN-13: 978-0898714036
62. Hashemiparast S, Fallahgoul H. Modified Gauss quadrature for ill-posed integral transform. International Journal of Mathematics and Computation. 2011; 13(11). ISSN: 0974-570X
63. Isa N. Evidence based radiation oncology with existing technology. Reports of practical oncology and radiotherapy. 2014; 19:259-266. Doi: 10.1016/j.rpor.2013.09.002
64. Johansson KA, Mattsson S, Brahme A, Turesson I. Radiation Therapy Dose Delivery. Acta Oncologica. 2003; 42(2). Doi: 10.1080/02841860310004922
65. Khanna P, Blais N, Gaudreau PO, Corrales-Rodriguez L. Immunotherapy Comes of Age in Lung Cancer, Clinical Lung Cancer, 2016. Doi: 10.1016/j.cllc.2016.06.006
66. Kufer KH, Hamacher HW, Bortfeld T. A multicriteria optimisation approach for inverse radiotherapy planning. University of Kaiserslautern, Germany, 2000. Doi: 10.1007/978-3-642-59758-9_10
67. Kirsch A. An introduction to the Mathematical Theory of Inverse Problems. Springer Applied Mathematical Sciences, 1996. Series E-ISSN2196-968X
68. Luenberger D. Linear and Nonlinear Programming (2nd Edn). Addison-Wesley, 1989. ISBN-13: 978-3030854492
69. Moczko J, Roszak A. Application of Mathematical Modeling in Survival Time Prediction for Females with Advanced Cervical cancer treated Radio-chemotherapy. Computational Methods in Science and Technology. 2006; 12(2). Doi: 10.12921/cmst.2006.12.02.143-147
70. Ragaz J, Ivo A Olivotto, John J Spinelli, Norman Phillips, Stewart M Jackson, *et al*. Regional Radiation Therapy in Patients with High-risk Breast Cancer Receiving Adjuvant Chemotherapy: 20-Year Results of the Columbia Randomized Trial. Journal of National Cancer Institute. 2005; 97(2). Doi: 10.1093/jnci/djh297
71. Steuer R. Multiple Criteria Optimization: Theory, Computation and Application. Wiley, 1986. https://doi.org/10.1002/oca.4660100109
72. Spirou SV, Chui CS. A gradient inverse planning algorithm with dose-volume constraints. Med Phys. 1998; 25:321-323. Doi: 10.1118/1.598202
73. Casesnoves F. Radiotherapy Linear Quadratic Bio Model 3D Wedge Filter Dose Simulations for AAA Photon-Model [18 Mev, Z= 5,15 cm] with Mathematical Method System. Biomed J Sci & Tech Res. 2022; 46(2)-2022. BJSTR. MS.ID.007337. DOI: 10.26717/BJSTR.2022.46.007337
74. Casesnoves F. Practical Radiotherapy TPO course and practice with Cyberknife. Robotic simulations for breathing movements during radiotherapy treatment. Sigulda Radiotherapy Cyberknife Center. Latvia. Riga National Health Oncology Hospital Varian LINACs TPO practice/lessons several Varian LINACs. Riga Technical University Bioengineering Training-Course Nonlinear Life, August 2018.
75. Das I, Colls. Patterns of dose variability in radiation prescription of breast cancer. Radiotherapy and Oncology. 1997; 44:83-89. Doi: 10.1016/s0167-8140(97)00054-6
76. Casesnoves F. Radiotherapy Linear Quadratic Bio Model 3D Wedge Filter Dose Simulations for AAA Photon-Model [18 Mev, Z= 5,15 cm] with Mathematical Method System. Biomed J Sci & Tech Res. 2022; 46(2)-2022. BJSTR. MS.ID.007337. DOI: 10.26717/BJSTR.2022.46.007337
77. Casesnoves F. Master in Philosophy Thesis at Medical Physics Department. Protection of the Patient in Routinary Radiological Explorations. Experimental Low Energies RX Dosimetry. Medicine Faculty. Madrid Complutense University, 1985, 1984-1985.
78. Casesnoves F. Ionization Chamber Low Energies Experimental Measurements for M-640 General Electric RX Tube with Radcheck ionization camera, Radcheck Beam Kilovoltimeter and TLD dosimeters. Radiology Department practice and measurements. Madrid Central Defense Hospital. Medical Physics Department. Master in Philosophy Thesis. Medicine Faculty. Complutense University. Madrid, 1983-1985.
79. Casesnoves F. Determination of Absorbed Doses in Routinary Radiological Explorations. Medical Physics Conference organized by Medical Physics Society Proceedings Printed. San Lorenzo del Escorial. Madrid, September 1985.
80. Greening J. Fundamentals of Radiation Dosimetry. Taylor and Francis. Second Edition, 1985. Doi: https://doi.org/10.1201/9780203755198
81. International Commission of Radiation Protection. Bulletin 26th. The International Commission on Radiological Protection. Recommendations of the International Commission on Radiological Protection. Pergamon Press. Copyright © 1977 The International Commission on Radiological Protection, 1977.

82. Stanton P, Colls. Cell kinetics *in vivo* of human breast cancer. British Journal of Surgery. 1996; 83:98-102. Doi: https://doi.org/10.1002/bjs.1800830130
83. Hedman M, Bjork-Eriksson T, Brodin O, Toma-Dasu I. Predictive value of modelled tumour control probability based on individual measurements of *in vitro* radiosensitivity and potential doubling time. Br J Radiol. 2013; 86:20130015. Doi: 10.1259/bjr.20130015
84. Fowler J. 21 years of Biologically Effective Dose. The British Journal of Radiology. 2010; 83:554-568.
85. Marcu L, *et al*. Radiotherapy and Clinical Radiobiology of Head and Neck Cancer. Series in Medical Physics and Biomedical Engineering. CRC Press, 2018.
86. Casesnoves F. Radiotherapy 3D Isodose Simulations for Wedge Filter 18 Mev-Dose [z = 5,15 cm] with AAA Model with Breast Cancer Applications. International Journal on Research Methodologies in Physics and Chemistry (IJRPC). 2022; 9(2). ISSN: 2349-7963