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Statistical Mechanics of Light in Free Space

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Abstract

The consequences of Bose's statistical theory with regard to the possible existence of zero-energy photons are discussed. It is pointed out that Einstein's mass dilation formula leaves open the distinct possibility that photons with zero energy can move with speed $v<c$. On this basis it is argued that the spontaneous radiative emission observed for atoms and molecules can be explained in a straightforward manner on the basis of the principle of conservation of energy, without invoking the assumption of quantum electrodynamics (QED) that vacuum fluctuations of the electromagnetic field are responsible. It is also shown that the quantization of electronic states can be explained by insisting that transitions resulting from photon interactions must obey the principle of conservation of momentum. Consideration of the spontaneous decay of positronium also opens up the possibility that the product is a zero-energy photon, thereby circumventing the requirement that the constituent electron and positron are destroyed in the transition and providing a possible description of the photon as a strongly bound (diatomic) state of an electron and positron. The ubiquitous use of the assumption that matter can be destroyed or annihilated in physical processes is therefore questioned on this basis.

Keywords: Bose-Einstein Statistics, Zero-Energy Photons, Spontaneous Radiative Emission, Quantization of Electronic States, Positronium Decay, Creation-Annihilation Assumption

1. Introduction

The discussion in earlier work [1] demonstrates first and foremost that there is no compelling proof that particles pass to and from existence in the decay of positronium. It is impossible to distinguish between objects which have gone out of existence from those which are simply lost from view for a period of time. The alternative assumption to the creation and annihilation of matter is thus that particles can exist in great abundance in a massless (zero-energy) state without being directly observable. Put more descriptively, this amounts to saying: *"We live in a sea of photons."* The question to be explored in the present work is how these concepts can be used to explain other fundamental observations in modern physics.

2. Statistical Mechanics of Massless Particles

In order to obtain a satisfactory explanation for spontaneous radiative emission which avoids the creation-annihilation hypothesis, it is not only necessary to assume that photons can exist with zero energy and momentum, but also that they exist in great numbers everywhere in the universe. One can approach this aspect of the problem on two levels. The first simply relies on the arguments of the last section and takes them a step further, namely if it is not possible to observe a single photon in this state, then it is also not possible to contradict the view that there are great numbers of such systems. However, it is also possible to find more positive indications regarding this point by considering the phenomenon of blackbody radiation. Quantum mechanics originated with Planck's discovery [2] that the observed intensity distribution in a perfect absorber can be quantitatively described within the framework of Maxwell-Boltzmann statistics, provided one assumes that only certain energy values are available for radiation of a given frequency. Specifically, Einstein showed that the mean value of the energy is obtained as:

$$\langle E\rangle_\nu = \sum_{n=0} nh\nu \; \exp(-nh\nu/kT) / \sum_{n=0} \exp(-nh\nu/kT) \quad (1)$$

rather than as a ratio of integrals in which n is treated as a continuous variable with non-integer values. The key point of interest in the present context is that *the n = 0 term in the above sums must be retained* to provide for an accurate representation of the observed spectral intensity distribution. This term does not alter the sum in the numerator, but it makes a decisive contribution to that in the denominator (partition function).

According to the theory of statistical mechanics, each term in the above sums corresponds to an allowed state for the system, in this case a collection of oscillators or photons with energy $E_n = nh\nu$. The zero-energy (n=0) photon is thus an ingredient in Planck's long-accepted solution to the black-body problem. Moreover, as the lowest-energy state available to a photon associated with a given frequency ν, it is also the most frequently populated according to the Boltzmann exponential law, and this at any temperature T. In order to obtain the total intensity, it is necessary to integrate over all frequencies from null upwards. It is important to note, however, that zero-energy photon states are present in the distribution for *each value of ν*. This situation is illustrated in Fig. 2 of Ref. 3, in which the various frequencies are represented by the spokes of a wheel. The allowed states for a given *ν* can be thought of as being plotted as points along the corresponding spoke at a distance from the center of the wheel which is proportional to their energy. Especially if the Boltzmann populations are taken into account, it is found that by far the largest concentration of photons is at the center of the wheel, i.e. with exactly zero energy and momentum.

Since a blackbody of a given temperature displays the same intensity distribution regardless of its location, it must be assumed that this state of affairs exists everywhere. The relatively high density of zero-energy photons is a theoretical assumption apparently needed to explain observed phenomena. This circumstance does not constitute a proof of the hypothesis in the mathematical sense, but at least it can be said that the idea does not lead to a contradiction either. Put the other way around, it would be a very damaging piece of evidence to the massless photon concept if states of zero energy had to be *excluded* from the partition function in order to achieve a satisfactory representation of the experimental observations. Quite to the contrary, Einstein made the opposite assumption of a high density of zero-energy photons. Seemingly the most natural interpretation for this theoretical approach is to conclude that the populations of *all* the various photon states are given correctly by the Boltzmann exponential factors in eq. (1), not only for those corresponding to non-zero quantum numbers. More details on this general subject may be found elsewhere [4].

A different, and theoretically superior approach, was introduced by Bose,[5] however. He concluded that Maxwell-Boltzmann statistics do not apply for photons. Specifically, the population index for photons needs to be changed from $\exp(-E/kT)$ to $[\exp(E/kT)-1]^{-1}$. This change has the effect of *greatly increasing the population of low-energy photons relative to the prediction of Maxwell-Boltzmann statistics.* In the present context, the key conclusion is that photons of *zero energy must be ascribed infinite population.* This result is thus the same as is reached using Einstein's quantization assumption discussed above, but the manner in which it is reached, namely on the basis of Bose-Einstein statistics, is completely in line with applications for atoms and molecules with non-zero rest mass.

3. Spontaneous Radiative Transitions

In view of the above discussion, it is interesting to consider the well-known phenomena of fluoresence and phosphorescense, as well as other forms of *luminescence.* They are all the result of the spontaneous emission in atoms and molecules. This is the process in which a quantum mechanical system (such as a molecule, an atom or a subatomic particle) transitions from an excited energy state to a lower energy state (e.g., its ground state) and emits a quantized amount of energy in the form of a photon. Lasers start via spontaeous emission, then during continuous operation work by stimulated emission.

It is important to note that spontaneous emission cannot be explained by classical electromagnetic theory and is fundamentally a quantum process. Einstein first predicted the phenomenon of spontaneous emission in a series of papers starting in 1916, culminating in what is now called the Einstein A Coefficient [6, 7]. Einstein's quantum theory of radiation anticipated ideas later expressed in quantum electrodynamics and quantum optics by several decades [8].

Later, after the formal discovery of quantum mechanics in 1926, the rate of spontaneous emission was accurately described from first principles by Dirac in his quantum theory of radiation, [9] the precursor to the theory which he later called quantum electrodynamics [10].

In order to explain this effect, physicists generally invoke the zero-point energy of the electromagnetic field [11, 12]. In doing so, they disregard the evidence of statistical mechanics discussed in Sect. II., specifically the results of the Bose theory [5] which clearly indicate that the population of zero-energy photons approaches infinity. In other words, photons with zero energy exist in large quantities at all places in the universe. There is accordingly no need to distinguish between a zero-point energy of a field *and the energy expected for real photons each of which has zero energy*.

In quantum electrodynamics (or QED) justification, the electromagnetic field has a ground state, which is the QD vacuum, which can mix with the excited stationary states of the atom [10].

As a result of this interaction, the *stationary state* of the atom is no longer an eigenstate of the combined system of the atom plus electromagnetic field. The transition from the electronic excited state to the electronic ground state mixes with the transition of the electromagnetic field from the ground state to an excited state, a field state with one photon in it. Spontaneous emission in free space depends upon *vacuum fluctuations* to get started [13, 14]. Although there is only one electronic transition from the excited state to ground state, there are many ways in which the electromagnetic field may go from the ground state to a one-photon state. That is, the electromagnetic field has infinitely more degrees of freedom, corresponding to the different directions in which the photon can be emitted. Equivalently, one might say that the phase space offered by the electromagnetic field is infinitely larger than that offered by the atom. This infinite degree of freedom for the emission of the photon results in the apparent irreversible decay, i.e., spontaneous emission. In the presence of electromagnetic vacuum modes, the combined atom-vacuum system is explained by the superposition of the wavefunctions of the excited state atom with no photon and the ground state atom with a single emitted photon.

It is much easier to explain what happens in spontaneous emission when it is assumed that there is an infinite population of zero-energy photons in the neighbourhood of the atomic system, as Bose's theory [5] predicts. Accordingly, all the energy lost in the transition occurring in the atom is transferred to a single zero-energy photon. All that is involved is the conservation-of-energy principle.

The theory of the spontaneous emission under the QED framework was first calculated by Weisskopf and Wigner in 1930 in a landmark paper [15-17]. The Weisskopf-Wigner calculation remains the standard approach to spontaneous radiation emission in atomic and molecular physics [18]. Dirac had also developed the same calculation a couple of years prior to the paper by Wigner and Weisskopf [19]. None of this is affected by a change in interpretation in terms of actual zero-energy photons. This includes the formula for the rate of spontaneous emission in the dipole approximation, which is shown to be proportional to the third power of the transition frequency.

But why not actual particles? A typical argument is drawn from Einstein's version of relativity theory, asserting that once a particle with zero rest mass (as one assumes for a photon *in vacuo)* does not move with the speed of light c, it ceases to exist. The justification comes from the law of mass dilation [20]: $m=m_o (1- v^2/c^2)^{-1/2} = m_o\gamma$, where m_o is the rest mass and m is the relativistic mass of the particle moving relative to the observer with speed v. Accordingly, if $m_o= 0$ and $v < c$, then γ is finite and $m = 0$. On this basis it is generally concluded that the corresponding particle cannot exist. Nevertheless, all one learns with certainty from the above formula is that $0 < v < c$ is a condition under which the relativistic mass of such a particle vanishes.

On this basis, it is reasonable to expect that there are photons of vanishing mass which *exist in the rest frame of the atom* undergoing spontaneous emission of energy. It is then straightforward to assume that the energy lost in the atomic transition is taken up by such a zero-energy photon. This is more likely to occur than that the energy is passed on to a photon in the area which is moving with high relative speed.

4. Processes Involving Light Absorption; Quantzation Explained

The quantum jumps associated with photon interactions provided an important clue regarding the particle nature of light. In his explanation of the photoelectric effect, [21] Einstein reversed a trend away from the Newtonian view [22] of light as "corpuscles". He showed that surface ionization of metals could be most consistently explained by assuming that a single quantum of light gives up all its energy to a single electron. He used the word "heuristic" in describing his ideas because the (exclusively) wave theory of electromagnetic radiation was widely accepted by the physics community at that time. While there can be general agreement that the photoelectric effect is inconsistent with a totally wave-like nature for light, it still must be regarded as extraordinary that any particle would transmit *all* its translational energy to a single electron in given interaction. Such a property of photons is consistent with the concept of annihilation, because it is reasonable to assume that a particle which has gone out of existence does so by leaving behind all its energy and momentum. However, if it is assumed instead that the photon retains its existence after photoionization has occurred and simply assumes a massless state which defies direct experimental observation, it is necessary to look more closely at the dynamics of this process to better understand the nature of the quantization phenomenon.

To this end it is instructive to apply the laws of energy and momentum conservation to the absorption process, as depicted in Fig. 1. If the photon γ were to give off an arbitrary amount ΔE of its energy to an atom A with mass M_A, its momentum would decrease by $\Delta p_\gamma = \Delta E/c$. If the atom were to remain in the same internal state, this amount would appear in the form of translational energy, which means that the momentum of the atom would change by $\Delta p_A = (2M_A\Delta E)^{1/2}$. Conservation of momentum requires that Δp_A and Δp_γ be equal. For small ΔE this can never be the case, however, in view of the large mass of A. Setting Δp_A equal to Δp_γ shows that ΔE would have to be equal to twice the rest energy of A or $2M_Ac^2$, which corresponds to the GeV range [23]. There is a solution to this dilemma, however, namely to have a part of the photon's energy be added to the internal energy of the atom, i.e. that another electronic state of the more massive system be reached. If the excited electronic state differs by $h\nu$ in energy from that of the initial state, conservation of momentum requires that:

$$\Delta p_\gamma = \Delta E/c = \Delta p_A = (2M_A)^{1/2} (\Delta E - h\nu)^{1/2} \quad (2)$$

Which is possible provided $h\nu$ is only slightly smaller than ΔE, again by virtue of the relatively large mass of A as well as the magnitude of c.

It is important to distinguish between two aspects of the absorption process in the foregoing discussion. First, the quantized nature of the atomic spectrum is seen to be directly connected with the large disparity between the respective masses of the atom and the photon. When one considers the translational motion of the atom, it is recognized that the energy levels available to it are actually continuous. It is the requirement of momentum conservation which restricts the possible transitions between different states of the same atom and thereby produces the quantization phenomenon. On the other hand, on the basis of these arguments by themselves there is no restriction put on the magnitude ΔE of the energy lost by a photon in the absorption process, save that it be less than its total energy, $E = m_\gamma c^2$. Indeed, the analogous excitation brought about by electron impact is well known, [24]. One is thus still left with the conclusion that there is something special about a zero-energy, zero-momentum state of the photon, even though many aspects of the absorption phenomenon can be understood by just assuming that the photon is a particle of relatively small mass compared to the system with which it interacts.

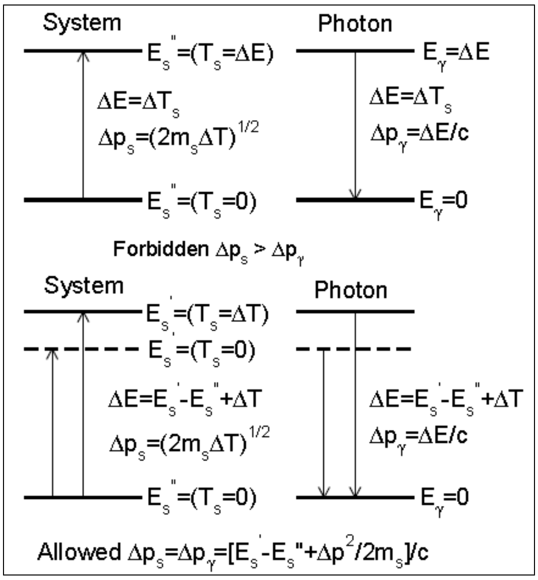

FIG. 1. Energy level diagram detailing the role of conservation laws in determining whether a given radiative absorption process is allowed or not. At the top of the diagram, the system is to retain the same internal energy E_s" in the transition, i.e. the two levels shown differ only in translational energy $\Delta T_s = \Delta p_s^2/2m_s$ (non-relativistic theory), where m_s is the mass of the system and Δp_s is the corresponding change in the momentum of its center of mass. Such a radiative process is always forbidden by the law of conservation of linear momentum because the rest mass of the photon is so much smaller than that of the system ($\Delta p_s > \Delta p_\gamma$). The only way for radiative absorption to occur is if the system changes its internal (from E_s" to E_s') as well as its translational energy, as depicted in the lower part of the diagram. Under these circumstances the momentum conservation law can be satisfied for a particular value of Δp_s, namely one that is equal to (E_s' - E_s" + Δp_s^2/2m_s)/c, where c is the speed of light. This condition rules out the occurrence of a radiative absorption process in which the system's translational energy does not change at all, also as indicated. Thus the "quantized" nature of radiative transitions is seen to be intimately connected with the photon's vanishing rest mass.

The fact that the energy transferred in the above process is exactly equal to $E_\gamma = m_\gamma c^2$ is thus seen to be a separate issue from the photo-ionization phenomenon itself. In other words, why doesn't the photon give off only part of its energy in inducing a transition in another system? Dirac used time-dependent perturbation theory [25] to answer this question, arguing that the incident radiation introduces a frequency-dependent term in the Hamiltonian of the atomic system. A resonance condition results according to which the energy of the most probable atomic transition, $h\nu = E_i - E_f$, must be the same as the energy of the incident photon, $E_\gamma = m_\gamma c^2$. The prospect of a massless photon being formed as a result of this energy exchange (rather than that the original photon is annihilated in the process) suggests a somewhat different interpretation for this phenomenon, however, one which does not rely on the *ad hoc* assumption of wavelike properties for the incident radiation. If one simply looks upon the process as a collision between an atom and a photon moving with speed c, it seems plausible to demand that the observed energy exchange take place over a relatively small but finite period of time. As a consequence, the temporal requirements of the interaction are more readily fulfilled by an outgoing system whose velocity has been considerably reduced below the speed of light in free space. As long as the departing photon possesses a non-zero amount of energy, this condition can never be fulfilled, but as has been pointed out in Sect. III, a *massless* photon is free of any such restriction, and thus can move at any speed less than c, including zero. In this view, the only practical means available to a photon to reduce its energy by virtue of an atomic collision is to assume a massless state, so that its relative speed compared to the system with which it interacts can be made as close to zero as possible. Accordingly, this interaction mode represents the only inelastic collision process available to a system of zero rest mass, since it is otherwise forced to move with the speed of light as long as it possesses any non-zero amount of translational energy.

By combining this result with the conservation of energy and momentum arguments first discussed, it is seen that the quantum characteristic associated with radiative absorption (and emission [26]) can be deduced exclusively on the basis of the rest mass values of the photon and the interacting system, respectively. There is no need to postulate any wave characteristics for the field inducing the transition. Rather, one is led to conclude from knowledge of the internal energy states of the interacting system and the magnitude of its rest mass exactly which photon energy is required to induce maximum transition probability. The magnitude of this transition probability itself cannot be determined quantitatively on the basis of the above information alone, and thus for this purpose one does have to introduce some additional information about the nature of the perturbing Hamiltonian, which itself is ultimately based on other experimental observations. This state of affairs does not affect the main conclusion in the present discussion, however, namely that the properties expected on the basis of relativity theory for a massless but nonetheless existent system are sufficient in themselves to allow for a suitable explanation of the observed tendency of photons to give up all their translational energy upon interacting with other particles.

5. Positronium Decay; e^+e^- Composition of the Photon

The above observations are also relevant to the positronium decay process [27]. In order for the de-excitation process to occur from the positronium 1s state to the proposed tightly-bound e^+e^- photon state (as depicted in Fig. 1 on p. 8 of Ref. 27), it again seems highly desirable that there be a minimum of relative motion between its initial and final systems. This condition cannot be said to be satisfactorily fulfilled when the product photon carries translational energy, because it must then move away from the original point of interaction with the speed of light. That would be something akin to a business transaction carried out between two people, one of which is riding on a speeding train while the other is standing on the station platform. In its massless state the photon can move with exactly the same velocity as the initial positronium system, thereby greatly improving the chances for such a transition. In this way, momentum can be conserved in the process, but the energy lost by the positronium complex still has to be carried away.

As shown in Fig. 2, the simplest way to accomplish this objective is to have the released energy divided up equally between two other photons which are in the neighborhood of the interaction locale, which again means they must initially possess zero translational energy. The conservation laws can then be satisfied by dividing the emitted energy equally among the two departing photons and having them move with exactly opposed momenta [4]. There are also angular momentum conditions to be satisfied, which is why the number of emitted photons is different depending on the multiplicity of the positronium state prior to its decay.

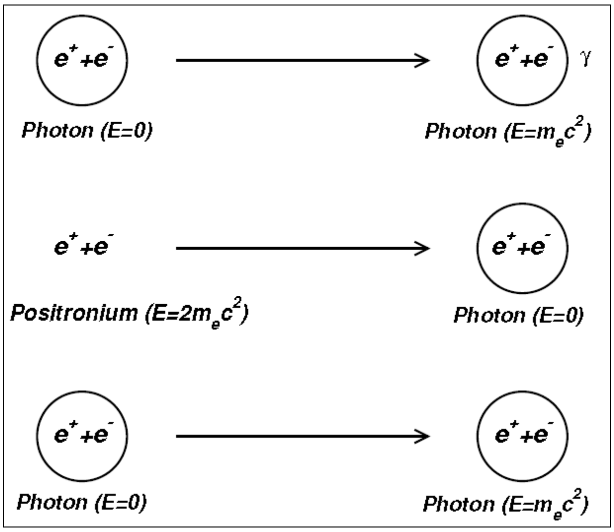

FIG. 2. Schematic diagram for the two-photon decay of (singlet) positronium. By assuming that the photon also has an e^+e^- composition, it is possible to describe this transition without assuming that particles are either created or annihilated in the process. In this model three e^+e^- binaries are involved, two of which are massless photons at the start of the process. They share the energy released by the positronium decay, and are observed as γ photons of equal energy at its conclusion. The final state of the original positronium system is another massless photon which, as its two counterparts at the start of the transition, escapes detection by virtue of its lack of energy.

To summarize, it is possible in this way to look upon the most commonly occurring positronium decay process (Fig. 2) as involving three distinct photons, each of which exists in its massless state at some point in the interaction. One of them is formed as a result of the de-excitation of the (singlet) positronium 1s state (Fig. 2 and Fig. 1 of Ref. 27), thus eluding detection by virtue of its null frequency. The other two are already present at the start of the reaction and are also unobservable as a consequence of their masslessness. Upon taking up their share of the energy released in the decay process they are detected, however, giving rise to the "two-photon" classification commonly ascribed to this interaction.

6. Conclusion

The introduction of the Bose statistical distribution for bosons brought with it the clear indication that there is an inexhaustible supply of zero-energy photons throughout the universe; in other words, *we live in a sea of zero-energy photons.* Along with this comes the realization on the basis of the Einstein mass-dilation formula of his special theory of relativity that there is no restriction on the speed of photons with zero inertial mass. This opens up the possibility in processes of spontaneous emission from states of atoms and molecules that the energy lost can be taken up entirely by zero-energy photons which are at rest relative to the decaying systems. There is accordingly no need to assume that *vacuum fluctuations* are required, as per QED theory, in order to initiate spontaneous emission.

The cause of quantization of electronic states in radiative absorption can be explained on the basis of the conservation of energy and momentum principles, as shown in Fig. 1.

This rules out transitions between different translational states except for the one with the exactly required energy, ultimately as a consequence of the fact that the mass of the participating atom or molecule is greater than that of a photon. As a result, the only allowed transitions caused by interaction with photons must involve different electronic states of the atom or molecule, which fact is therefore perceived as quantization of same. In addition, there is a clear relationship between radiative absorption and the photelectric effect. As long as the energy available in the approaching light waves is less than the required amount, no increase in the intensity of the light waves is sufficient to cause transitions to occur, just as increasing the intensity of impacting waves of electrons by itself is not sufficient to induce electronic ionization in metals. In other words, accumulation of energy in the waves is not a decisive factor in either case.

The existence of zero-energy photons also allows insight into the composition of photons.

When an electron and positron come together, they produce a state referred to as positronium. It is closely related to the ground state of the hydrogen atom. Unlike the latter, however, positronium is only metastable, with a lifetime of 10^{-10} s. It decays with an energy of 1.022 Mev, which is equal to $2m_ec^2$, where m_e is the inertial mass of both the electron (e^-) and positron (e^+). The conventional view of physicists is that the electron and positron have been annihilated in this process, i.e., they have simply passed out of existence. An alternative view is presented in Fig. 2, however. Accordingly, the result of the interaction is a strongly bound state of e^+e^- composition which has null energy *and is still in existence*. The 1.022 Mev of released energy can then be divided equally in the most common process between two zero-energy photons which are initially at rest with respect to the centre-of-mass of the e^+e^- system. Momentum is conserved by having the two photons depart in opposite directions, each with speed v=c. The main conclusion to be drawn from Fig. 2 is that the zero-energy state resulting from the (spontaneous) positronium emission process is the photon itself.

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