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Comparison of Nucleation from Liquid to Gas and Nucleation of Rain Droplets in a Cloud

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Abstract

The nucleation rate for liquid to gas and rain droplets in a cloud is of the form for the nucleation rate $J = M \exp(N)$. We see that $|d\ln(M)/dN| \ll 1$ and in this paper is presented the expressions for N and the values for N. In Blander and Katz, 1975, page 837 they state this for liquid to gas and the same is true in Young, 1993, page 40 for rain droplets in a cloud.

Here are the expressions for J for each case: first Blander and Katz then Young.

$$J \approx 3.73 \ (10^{35}) \ [d^2 \ \sigma \ / \ M^3 \ B]^{1/2} \exp \ [- \ 1.182 \ (10^5) \ \sigma^3/(T \ (P_V - P_L)^2)]$$

$$J = 4\alpha_c/N_L(\sigma/2\pi m)^{1/2}(e/kT)^2 S^2 \exp[-16\pi\sigma^3/3(kT)^3(N_L \ \ln S)^2]$$

Here for Blander and Katz $J = A \exp(K)$ and for Young it is $J = C \exp(D)$. In Blander and Katz, page 837 it is noted that the presence of σ^3 in the numerator of K means that surface tension has the largest effect on temperature of nucleation. In Young, page 40 there is the comment that "it is the exponential term in the nucleation equation that is tested by these observations and not the pre-exponantial factor." The author will produce the values for K and D, the dominating expressions, and the fact that Jennings, 2020 eliminated the prefactor in Blander and Katz.

Keywords: Simplifying Classical Nucleation Theory, Rain Droplets in a Cloud, Theory of Corresponding States

Introduction
For the Blander and Katz expression, Jennings (2020) [3] eliminated the prefactor. In Jennings' paper we see that it is proved that:

$$\ln A = (1/12)\ln(K^2) + C \tag{1}$$

Where C was determined from the average of ten solvents: heptane, hexane, carbon tetrachloride, cyclohexane, pentane, cyclopentane, benzene, ethanol, methanol and water.

$$C = 75.379 \pm 1.073 \tag{2}$$

The details are in the Jennings, 2020 paper: get A by plugging in d, σ, M, B and get K by plugging in σ, T, Pe, P_L, δ in (1), solving for C, the integration constant, for each solvent at the limit of superheat. Including water in the list did not throw the calculation off. Further, we note.

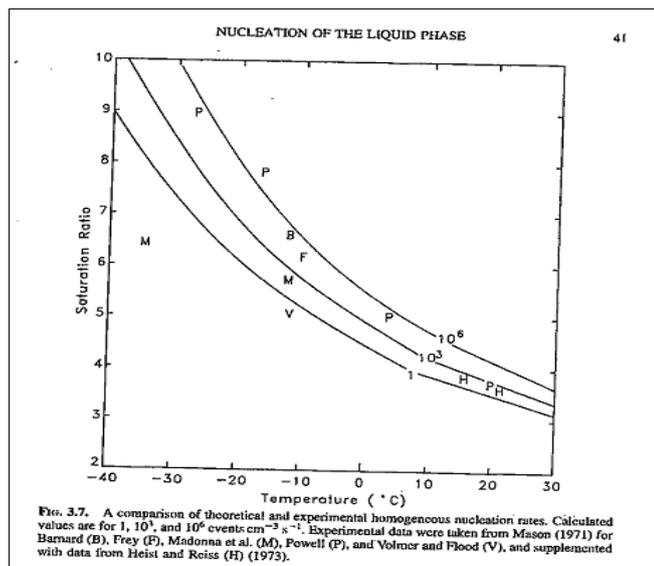$$\ln J = \ln A + K \tag{3}$$

So, we get.

$$\ln J = (1/12)\ln(K^2) + 75.379 + K \tag{4}$$

Blander and Katz say that J is approximately 10^4 to 10^6 on page 837 so we take the geometric mean 10^5 for J.

To get K, we have to guess K, plug it into (4) on a scientific calculator and cycle. K = - 64.5605.

Results

To arrive at the value for D, we have to look at Jennings (2023) [4]. There is a Table, reproduced here from Jennings (2023) [4]. Fig. 3.7 in Young, page 41 (Figure) has curves for J at 1, 10^3 and 10^6 events/cm -sec for experimental data of the raindrop nucleation rate, so the author took the geometric mean 10^3 curve. The Y axis is S and the X axis is T. Here is a partial table of what Young, page 41 did. This is only in the vicinity of zero degrees Centigrade. NOMENCLATURE is omitted.

Fig. 3.7. A comparison of theoretical and experimental homogeneous nucleation rates. Calculated values are for 1, 10^3, and 10^6 events $cm^{-3} s^{-1}$. Experimental data were taken from Mason (1971) for Barnard (B), Frey (F), Madonna et al. (M), Powell (P), and Volmer and Flood (V), and supplemented with data from Heist and Reiss (H) (1973).

T ºC	σ ergs/cm²	S(calculated)	S(from graph)	D(from S calculated)	D(S from graph)
- 8	77.0	5.1672	5.6	- 51.7	- 47.0
0	75.6	4.784	4.9	- 49.3	- 47.8
+10	74.22	4.4035	4.2	- 46.7	- 49.8

Only taking interest in calculated values of D(from S calculated) we get D(calculated) = - 49.2 ± 2.0. What can be concluded from getting K and D is 1) the exponential expression dominates and 2) D(S from graph) gives D(from data) = - 48.2 ± 1.2. Jennings eliminated the prefactor in Blander/Katz. Finally, we have the simplified version of Blander and Katz, Eq. (6).

$$1830.84 = T (Pe - P_L)^2 \delta^2 / \sigma^3 \qquad (6)$$

So, Jennings has simplified classical nucleation theory.

In a Max Planck Institute URL, REFERENCE 5), the critical cluster is supposed to be less than 100 molecules, similar to what is taken in Young, page 41, or 63 molecules of water in clouds. The author performed some research from 1980-83, Jennings/Middleman (1985) [6], on the limit of superheat in polymer solutions with the theory substantially wrapped up in Jennings (2020) [3].

It is like there is a corresponding states situation between the two nucleation equations, in that it is actually true that one can get the value for the exponent in J = M exp(N) for each one, Guggenheim (1945) [9].

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