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Ampère's Equation and Maxwell's Alteration

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Abstract
James Clerk Maxwell took André-Marie Ampère's equation, which was for static charge, and added to it to include motion. Here in this paper the author will simplify the derivations to present a clear rendition of the mathematics. Both involve some calculation. Ampère's equation is: (1)

$$\oint B \cdot d\,l = \mu_0 I.$$

Maxwell added a term for motion, which resulted in this formula: (1)

$$\oint B \cdot d\,l = \mu_0 \epsilon_0 \int \partial \mathbf{E}/\partial t \cdot d\mathbf{S} + \mu_0 I.$$

Where μ_0 = permeability constant and ϵ_0 = permittivity constant. Now c, the speed of light, = $1 / (\mu_0 \epsilon_0)^{1/2}$. [2] Here the author will outline the calculations for Ampère's and Maxwell's equations. (2), (3), (4).

Introduction

It was noticed that when two parallel wires carrying current would attact or repel depending on whether they had the current going in the same direction or opposite. Ampère derived the formula by calculating the line integral of the magnetic field **B** from the long straight wire carrying current I around a concentric circular path with radius r. The magnetic field **B** is parallel to the path d l and is.

$$\mathbf{B} = (\mu_0 I) / (2 \pi r) \tag{1}$$

Ampère's law can also be derived from the Biot-Savart law. (3) Going further, we see that the line integral becomes this equation.

$$\oint \mathbf{B} \cdot d\,l = \oint \mathbf{B} \cdot r\,d\theta \tag{2}$$

Where r d θ is the projection of d l onto the circle passing through d l taken as a vector. Plugging in **B** from [1] we have.

$$\oint \mathbf{B} \cdot d\,l = \oint ((\mu_0 I) / (2 \pi r)) \cdot r\,d\theta \tag{3}$$

Since r = constant (radius of circular path) and I is constant, then $\oint d\theta = 2\pi$ for the circular path around the wire with current. So,

$$\oint \mathbf{B} \cdot d\,l = \oint ((\mu_0 I) / (2 \pi r)) \cdot r\,d\theta = ((\mu_0 I) / (2 \pi)) (2 \pi) = \mu_0 I \tag{4}$$

Eq. [4] is Ampère's law for static current.

Results

When Maxwell was studying the equations that became the four equations for electromagnetism he realized that Ampère's law had a deficiency in that it was for static charge only. Therefore there had to be included a term for the displacement current. In

1861, Maxwell added the displacement current, $\mathbf{J}_d$, to Ampere's conduction current, J. [2] So, we follow a derivation from Google [4].

$$\nabla \times \mathbf{B} = \mu_0 (J + \mathbf{J}_d) \qquad (5)$$

We need to evaluate $\mathbf{J}_d$. Maxwell then took the divergence of both sides and $\nabla \cdot (\nabla \times \mathbf{B}) = 0$. So,

$$0 = \nabla \cdot \mathbf{J} + \nabla \cdot \mathbf{J}_d \qquad (6)$$

For the conservation of charge we have.

$$\nabla \cdot \mathbf{J} = - \partial \rho / \partial t \qquad (7)$$

Substituting the conservation of charge into [6] gives.

$$\nabla \cdot \mathbf{J}_d = - \nabla \cdot \mathbf{J} = \partial \rho / \partial t \qquad (8)$$

Then he used Gauss's law, which is.

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0 \qquad (9)$$

Thus

$$\nabla \cdot \mathbf{J}_d = \partial / \partial t (\epsilon_0 \nabla \cdot \mathbf{E}) = \nabla \cdot (\epsilon_0 (\partial / \partial t\ \mathbf{E})) \qquad (10)$$

Maxwell saw that the displacement current density was.

$$\mathbf{J}_d = \epsilon_0 (\partial / \partial t\ \mathbf{E}) \qquad (11)$$

Then he substituted $\mathbf{J}_d$ back into $\nabla \times \mathbf{B}$ to get.

$$\nabla \times \mathbf{B} = \mu_0 (\mathbf{J} + \epsilon_0 (\partial / \partial t\ \mathbf{E})) = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 (\partial / \partial t\ \mathbf{E}) \qquad (12)$$

This (12) laid the foundation for electromagnetism along with two equations from Carl Friedrich Gauss and one from Michael Faraday.

Discussion
The permittivity has to do with the force between point charges in a vacuum and the symbol for it is ϵ_0. It is similar to the gravitation constant between masses. For a current the case is the same and the permeability of free space occurs in the force equation with the symbol μ_0 [1].

Conclusion
Ampère was a devout Catholic who prayed the Rosary and Maxwell was an evangelical Presbyterian. There are other Christians who contributed to mathematics and science such as Augustin-Louis Cauchy (mathematics) and Gerhard Ertl (Chemistry). The one who introduced me to scientific research was Stanley Middleman of UCSD and I was the first author of one paper where he was Principal Investigator [5].

Acknowledgments
James Clerk Maxwell was a devout Christian [6] and singlehandedly put together the four laws of electromagnetism. He invented the color photograph and solved the problem of the rings of Saturn, later confirmed by spectroscopic studies. [7] In understanding that light is electromagnetic, Maxwell started numerous advances that appear in our modern world. Maxwell is truly one of the greatest scientists of all time.

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