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Evaluation of the B_c^+ and Excited B_c^{+*} States in Terms of a First-Order Meson Model

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Abstract
The B_c^+ and its excited states are evaluated in terms of a first-order meson model assuming interacting charm and antibottom quarks. This model yields possible $J^\pi = 0^-$ and 1^- values that include the ground B_c^+ state (0^-) and excited B_c^{+*} state (1^-) experimental values. The predicted ground state mass of 6313 MeV/c^2 is within 1% of the experimental value, and the excited state mass is within 3% of LHCb measured values.

Keywords: B_c^+ and B_c^{+*} Mesons, First-Order Mass Formula, Quark Model, Cluster Model

1. Introduction

The LHCb Collaboration recently reported the observation of structures in the $B_c^+\gamma$ mass spectrum. These structures were observed in the analysis of proton-proton collision data collected by the LHCb detector at center-of-mass energies of 7, 8, and 13 TeV. Ref. 1 reported the discovery of these orbitally excited beauty-charm states. The LHCb Collaboration evaluated these peak locations using a minimal two-peak model that effectively describes the observed structures. These peaks occur at 6704.8 and 6752.4 MeV/c^2. The results expand the known hadron spectroscopy. In addition, these results are important in advancing the understanding of the dynamics of hadrons and the nonperturbative characteristics of QCD.

The B_c^+ and its excited states are evaluated in terms of a first-order meson model assuming interacting charm and antibottom quarks. This model is based on the semiempirical mass formula of Zel'dovich and Sakharov [2, 3]. The first-order meson model provides the B_c^+ ground and its excited states mass as well their J^π values. Since the first-order model is limited in scope, it only permits a primitive angular momentum coupling structure. Meson pairs comprising tetraquark systems [4-24] were reasonably described by the first-order model.

2. Model and Formulation

The proposed first-order meson model is based on the semiempirical mass formula of Zel'dovich and Sakharov [2, 3]. This model assumes that two interacting quarks form the meson, with zero angular momentum between the particles.

The mesons (m) mass (M) is defined to have the form [2, 3]:

$$M_m = \delta_m + m_1 + m_2 + b_m \, [m_o^2 / (m_1 \, m_2)] \, \sigma_1 \cdot \sigma_2 \qquad (1)$$

In Eq. 1, δ_m is defined to have the value 40 MeV/c^2 [3], and m_i is the mass of the quark comprising the meson cluster (i = 1 and 2). The average mass of a first generation quark (u and d) is m_o [25, 26]. The scalar product of the quark spin vectors ($\boldsymbol{\sigma_1 \cdot \sigma_2}$) is -3/4 and +1/4 for pseudoscalar and vector mesons, respectively [3].

Effective quark masses provided by Griffiths [25] are used to determine the meson cluster mass. The d, u, s, c, b, and t quarks have effective mass values of 340, 336, 486, 1550, 4730, and 177000 MeV/c^2, respectively [25]. Following the convention of the Standard Model, these quarks are grouped into three generations: [d(-1/3), u(+2/3)], [s(-1/3), c(+2/3)], and [b(-1/3), t(+2/3)] [25, 26]. The specific quark charges, in units of the unit charge e, are given within parentheses.

3. First-Order Mass Formula for the B_c^+ Meson Ground State

The first-order mass formula only provides a limited angular momentum coupling structure, and the spin of a tetraquark is derived from the angular momentum coupling of the two quarks:

$$J^{\pi} = J^{\pi}(1) \times L \times J^{\pi}(2) \quad (2)$$

The first-order model summarized in Eq. 2 only provides a primitive angular momentum coupling structure for the J^{π} assignment, and the angular momentum between the quarks is zero (L = 0). These are limiting conditions of the model. Detailed quark structural information is not included in the model formulation.

These aforementioned simplifications minimize model complexity, and permit the meson mass formula to have the form of Eq. 1. The B_c^+ configuration is evaluated assuming interacting c and $\bar{b}$ quarks. Following the first-order model, the B_c^+ meson is modeled as a $1/2^+$ c quark coupled to a $1/2^-$ ($\bar{b}$) quark [26]. This meson has a model $J^{\pi} = 0^-$ and 1^- assignment ($1/2^+ \times 0 \times 1/2^-$ using Eq. 2). A 0^- ground state spin assignment was noted in Ref. 26 that includes the values predicted by the first-order model.

Using the first-order mass formula of Eq. 1, a mass of 6313 MeV/c^2 is predicted for the B_c^+. This value is about 1% larger than the value of Ref. 26. The model spin and parity values include the 0^- experimental value [26]. The model results for the B_c^+ and its excited states noted in Ref. 1 are summarized in Table 1.

Table 1: Comparison of First Order Model Results and Analysis of Refs. 1 and 26

Configuration	Model J^{π}	Experiment J^{π}	Model Mass (MeV/c^2)	Experimental Mass (MeV/c^2)
B_c^+	0^- and 1^-	0^{-a}	6313	6274.47^a
B_c^{+*}	0^- and 1^-	1^{-b}	6910	6704.8^b
B_c^{+*}	0^- and 1^-	1^{-b}	6910	6752.4^b
^{a} Ref. 26. ^{b} Ref. 1.				

4. First-Order Mass Formula for the B_c^{+*} Meson Excited States

The LHCb Collaboration noted two B_c^{+*} excited states that occur at 6704.8 and 6752.4 MeV/c^2. These states are modeled by modifying Eq. 1 to account for the difference in mass between the ground state and excited state [26]:

$$M(B_c^{+*}) = M_m + \Delta(B_c^+(2S) - B_c^+) \quad (3)$$

Where $\Delta(B_c^+(2S) - B_c^+)$ is 596.73 MeV/c^2 [26]. Using Eq. 3, the first-order model predicts a mass of 6910 MeV/c^2. This result is within about 3% of the candidate values noted in Ref. 1. The J^{π} values predicted by the first order model are 0^- and 1^-. The 1^- value is predicted in Ref. 1.

5. First-Order Tetraquark Model Uncertainties and Limitations

There are a number of uncertainties and limitations that affect the model results. The limited angular momentum coupling structure restricts the available J^{π} values that can be evaluated. In addition, assuming zero angular momentum between the quarks also limits the complexity of possible states.

The values for the effective quark masses [25] are not definitive. In spite of these uncertainties, the model continues to provide reasonably credible results [4-24].

6. Conclusions

The B_c^+ and its excited states are evaluated in terms of a first-order meson model. The B_c^+ and its excited states model yields $J^{\pi} = 0^-$ and 1^- values. The ground (excited) state experimental values are 0^- (1^-), and are consistent with the first-order model $J^{\pi} = 0^-$ and 1^- predictions. In addition, the predicted ground (excited) state mass is within 1% (3%) of the measured value.

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