International Journal of Advanced Multidisciplinary Research and Studies

Nuclear Level Density in the X(1266, 354) System

JJ Bevelacqua

Bevelacqua Resources, 7531 Flint Crossing Circle SE, Owens Cross Roads, AL 35763, USA

Corresponding Author: **JJ Bevelacqua**

Abstract

Single particle levels for the X(1266, 354) system as a function of energy are modeled using various functional forms. A power law form $N(E) = \mathbf{a}\ E^b$ for the total number of energy levels provides a better fit than the constant temperature and equidistant models. The number of energy levels/MeV is not well-fit by the commonly used constant temperature model. Model results are compared to existing systems as well as postulated theoretical superheavy nuclei. The comparison suggests that the level density characteristics of light systems and theoretical superheavy systems are different.

1. Introduction

Level density data provide important information regarding the systematics of nuclear structure. These calculations have been performed over a range of mass values, and most are performed for nuclei with A ≥ 20 [1]. These calculations have been extended to superheavy nuclei including the Z=204 A=610 system X(610, 204) [2, 3], X(636, 204) [4, 5], X(692, 214) [4, 6], X(730, 226) [7, 8], X(766, 242) [7, 9], X(888, 274) [10, 11], X(926, 282) [12, 13], X(1062, 312) [14, 15], and X(1122, 330] [16, 17].

This paper extends those calculations to the X(1226, 354) system [18]. This system, and previous calculations in the ^{4}He system [19], represents endpoints for nuclear level density calculations. As such, they provide additional information regarding the level density systematics in nuclear systems.

2. Calculational Methodology

Nuclear **s**ingle particle energies are calculated from the Schrödinger Equation. For a spherically symmetric potential, single particle energies for a nuclear core plus nucleon, can be solved using the methodology of Lukasiak and Sobiczewski [20] and Petrovich *et al.* [21].

For a nucleon in the field of a nuclear core, the single particle binding energy E_{NLSJ} is derived from the solution of the radial Schrödinger Equation:

$$[(\hbar^2/2\mu)\ (d^2/dr^2 - L(L + 1)/r^2) - E_{NLSJ} - V_{LSJ}(r)]\ U_{NLSJ}(r) = 0 \tag{1}$$

In Eq. 1, the radial coordinate r defines the relative motion of the nuclear core and the particle, $V_{LSJ}(r)$ is the model interaction [22, 23] and $U_{NLSJ}(r)$ is the radial wave function. L, S, and J are the orbital, spin, and total angular momentum quantum numbers, respectively, and N is the radial quantum number. Finally, μ is the reduced mass.

Specific details of the model and its solution are provided in Refs. 2, 4, 7, 10, 12, 14, 16, and 18. The numerical methods of Refs. 24 and 25 are utilized to obtain a converged solution.

3. Single Particle Level Calculations

Fig. 1 summarizes the results of the X(1226, 354) level calculations [18]. The X(1226, 354) system has an alpha decay half-life of 4.8×10^{12} yr, and this system is stable with respect to beta decay [18]. These results are relative to other calculations performed with the same model interaction, and should not be taken as a definitive value. Extrapolations of the model interaction introduce uncertainties that preclude an absolute half-life determination [18].
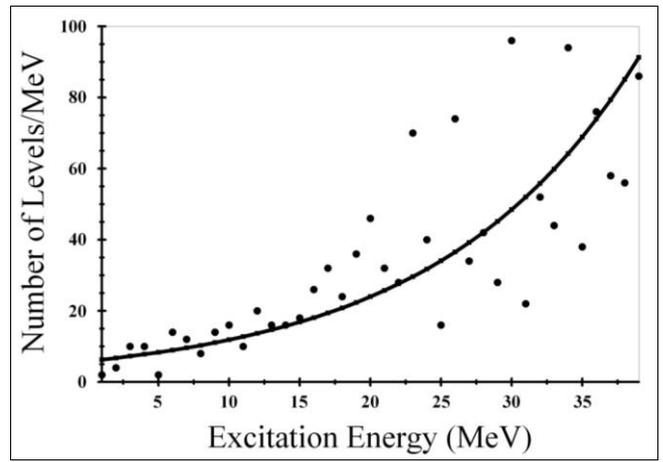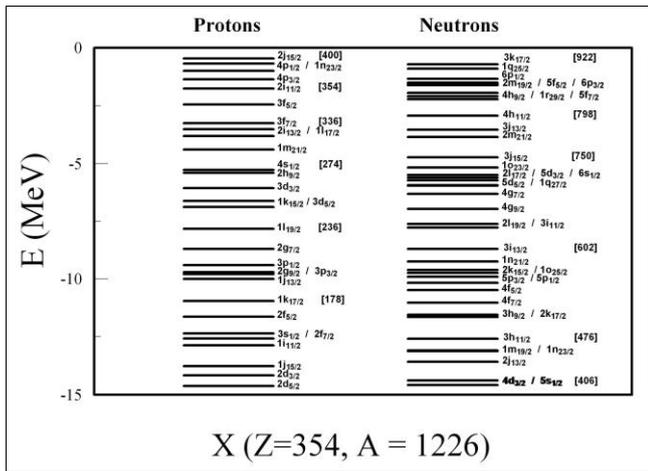
Fig 1: Calculated single particle energy levels for nucleus X(1266, 354). The notation A/B/C is used to indicate adjacent energy levels with level A more tightly bound than level B, which is more tightly bound than level C

The level systematics summarized in Sections 4.0, 5.0, and 6.0 are based on Fig. 1. These results are compared to the constant temperature model [26] in Section 4.0, power law in Section 5.0, and equidistant model [27-30] in Section 6.0. In addition to illustrating nuclear level systematics, level density calculations have also been applied to superheavy nuclei [31], nuclear astrophysical reaction rates [32], and machine learning [33].

4. Constant Temperature Model

The constant temperature model [26] is commonly utilized to evaluate nuclear level data and theoretical models, and it has the form:

$$\rho(E) = a \exp(E/T) \tag{2}$$

In Eq. 2, $\rho(E)$ is the number of energy levels per MeV. The terms **a** and T are constants derived from a numerical fit to the X(1226, 354) single particle levels summarized in Fig. 1. Simple level density approximations may not reproduce the nuclear density that exhibits a unique single particle level structure. Accordingly, the systematics of level density parameters illuminates the unique differences between nuclei.

The level density $\rho(E)$ functional form is expected to be a simple exponential form based on data from A = 36-66 even-even nuclei [26]. When the level scheme of Fig. 1 is fit to Eq. (2), the values **a** = 5.86487696 and T = 1/0.070386191= 14.207 are obtained. These values are similar to the fits to the X(610, 204) [3], X(636, 204) [5], X(692, 214) [6], X(730, 226) [8], X(730, 242) [9], X(888, 274) [11], X(929, 282) [13], X(1062, 312) [15], and X(1122, 330) [17]. Fig. 2 illustrates the fit of the constant temperature model to the model results for the X(1226, 354) system.

Fig 2: Energy level density for X(1266, 354) using the constant temperature model. The "•" symbol represents the state density for a 1 MeV energy bin. The solid curve is a fit to the constant temperature functional form $\rho(E) = a \exp(E/T)$

In view of the limited number of energy levels, there is considerable variation between the single particle model and constant temperature values of $\rho(E)$. This variation is minimized by considering the total number of levels N(E). N(E) is also assumed to have the constant temperature model form:

$$N(E) = c \exp(E/d) \tag{3}$$

The curve shown in Fig. 3 results when the X(1226, 354) levels N(E) of Fig. 1 are fit to the constant temperature model of Eq. (3). Fitting parameters c = 17.24197554 and d = 1/0.127510314 = 7.843 are utilized in Fig. 3. The values of these parameters are similar to the X(610, 204) [3], X(636, 204) [5], X(692, 214) [6], X(730, 226) [8], X(730, 242) [9], X(888, 274) [11], X(926, 282) [13], X(1062, 312) [15], and X(1122, 330) [17] results.

The fit to the functional form of Eq. 3 to the X(1226, 354) system underestimates the number of energy levels below about 28 MeV, and overestimates N(E) above about 28 MeV. Table 1 provides a comparison of the X(1226, 354) system d value to lighter systems using the constant temperature model.
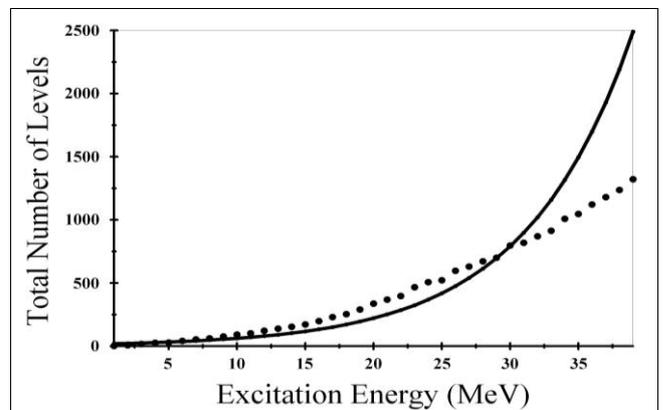

Fig 3: Total number of energy levels N(E) for X(1266, 354) as a function of energy. The "•" symbol represents the total number of energy levels up to energy E. The solid curve is a fit to the constant temperature functional form N(E) = c exp(E/d).

Table 1: Constant Temperature Model Parameters for Nuclear Densities[a-k]

Nucleus	d (MeV)
^{4}He	2.79[a]
^{36}Ar	1.87
^{38}Ar	1.47
^{40}Ca	1.73
^{50}Cr	1.29
^{52}Cr	1.43
^{54}Cr	1.22
^{54}Fe	1.40
^{56}Fe	1.40
^{58}Fe	1.31
^{68}Zn	0.90
X(610, 204)	7.31[b]
X(636, 204)	7.15[c]
X(692, 214)	6.90[d]
X(730, 226)	7.26[e]
X(766, 242)	8.25[f]
X(888, 274)	7.65[g]
X(926, 282)	7.41[h]
X(1062, 312)	7.51[i]
X(1122, 330)	8.01[j]
X(1226 354)	7.84[k]

[a]Ref. 6. [b]Ref. 3. [c]Ref. 5. [d]Ref. 6. [e]Ref. 8. [f]Ref. 9. [g]Ref. 11, [h] Ref, 13, and [i]Ref. 15, and [j]Ref. 17. [k]This work. All others Ref. 26.

Table 1 summarizes the systematics of the constant temperature model parameter d. This parameter behaves differently in the A < 70 region compared to A > 600 systems. The A > 600 d values are about a factor of four larger than those in the A < 70 region.

5. Power Law Model
The total number of levels N(E) is also fit to the power law functional form:

$$N(E) = a E^b \qquad (4)$$

Where **a** = 1.71892297 and b = 1.77600325. Fig. 4 summarizes the use of these parameters in Eq. 4 to fit the X(1226, 354) energy levels of Fig, 1. The power law (Eq. 4) yields an improved fit compared to the constant temperature model (Eq. 3) for the total number of energy levels. The X(1226, 354) parameters are similar to the values derived from X(610, 204) [3], X(636, 204) [5], X(692, 214) [6], X(730, 226) [8], X(730, 242) [9], X(788, 274) [11], X(926, 282) [13], X(1062, 312) [15], and X(1122, 330) [17].
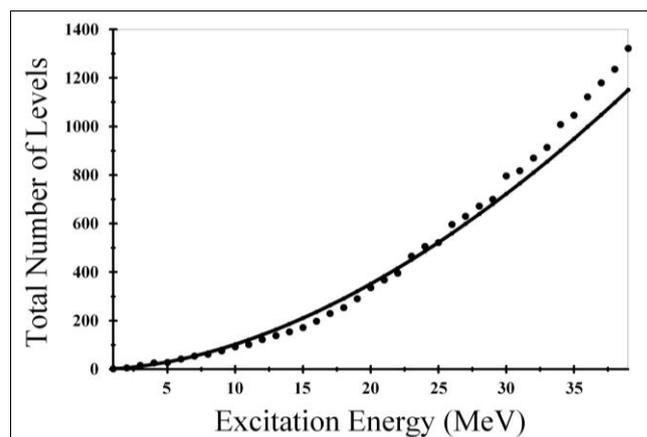

Fig 4: Total number of energy levels N(E) for X(1266, 354) as a function of energy. The "•" symbol represents the total number of energy levels up to energy E. The solid curve is a fit to the power law functional form N(E) = aE[b]

6. Equidistant Model
Single particle energy levels are assumed to be equidistant and nondegenerate in the equidistant model [27-30]. The total number of energy levels for a system of neutrons and protons is given by:

$$N(E) = (\pi)^{1/2} \exp (2 [a E]^{1/2}) / (12 E^{5/4} a^{1/4}) \qquad (5)$$

Where **a** is a level density parameter. For known nuclei, the parameter **a** has the value of A/8 [30].

The X(1122, 330) energy levels are fit to the functional form of Eq. (5), and are illustrated in Fig. 5. An **a** = 1.278 value is utilized in Eq. 5. This **a** value is similar to the X(610, 204) [3], X(636, 204) [5], X(692, 214) [6], X(730, 226) [8], X(730, 242) [9], X(788, 274) [11], X(926, 282) [13], X(1062, 312) [15], and X(1122, 330) [17] parameters.

The reader should note that Eq. 5 is an asymptotic expression that should become more accurate as the atomic mass tends to infinity. This suggests that Eq. 5 should provide an improved description of the level density as the nucleus mass increases. However, the equidistant model of Eq. 5 does not reproduce the systematics of the level spectrum of Fig. 1 as well as the power law model.
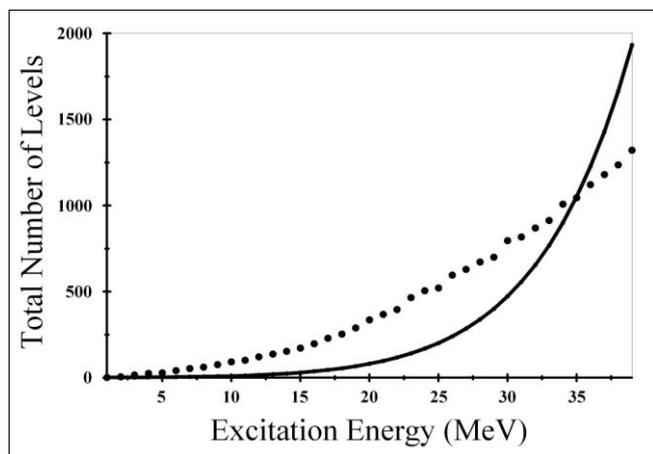
Fig 5: Total number of energy levels N(E) for X(1266, 354) as a function of energy using the equidistant model. The "•" symbol represents the total number of energy levels up to energy E

7. Conclusions

The single particle levels for the X(1226, 354) system as a function of energy can be modeled using various functional forms. A power law form N(E) = **a** E^b for the total number of energy levels provides a better fit than the than the constant temperature and equidistant models. The number of energy levels/MeV is not well-fit by the commonly used constant temperature model.

8. References

1. Smith BC, Choudhury FN, Das Gupta S. Nuclear level density with schematic forces, Phys. Rev. C. 1978; 17:318.
2. Bevelacqua JJ. 570 ≤ A ≤ 620 superheavy nuclei, Physics Essays. 2012; 25:475.
3. Bevelacqua JJ. Nuclear Level Density in the X(610, 204) System, Qeios **OVWPON**. 2022; 1. Doi: https://doi.org/10.32388/OVWPON
4. Bevelacqua JJ. Superheavy nuclei II: 620 <A< 700 systems, Physics Essays. 2013; 26:516.
5. Bevelacqua JJ. Nuclear Level Density in the X(636, 204) System, Qeios **CBI922**. 2022; 1. Doi: https://doi.org/10.32388/CBI922
6. Bevelacqua JJ. Nuclear Level Density in the X(692, 214) System, Qeios **8SBVPK**. 2022; 1. Doi: https://doi.org/10.32388/8SBVPK
7. Bevelacqua JJ. Superheavy Nuclei III: 700 ≤ A < 800 Systems, Physics Essays. 2014; 27:655.
8. Bevelacqua JJ. Nuclear Level Density in the X(730, 226) System, Qeios **9LZL9I**. 2023; 1. Doi: https://doi.org/10.32388/9LZL9I
9. Bevelacqua JJ. Nuclear Level Density in the X(766, 242) System, Qeios **C2V8T7**. 2023; 1. Doi: https://doi.org/10.32388/C2V8T7
10. Bevelacqua JJ. Superheavy Nuclei IV: 800 ≤ A < 900 Systems, Physics Essays. 2016; 29(4):490.
11. Bevelacqua JJ. Nuclear Level Density in the X(888, 274) System, Qeios **QEOPGL**. 2024; 1. Doi: https://doi.org/10.32388/QEOPGL
12. Bevelacqua JJ. Superheavy Nuclei V: 900 ≤ A < 1000 Systems, Physics Essays. 2017; 30(1):1.
13. Bevelacqua JJ. Nuclear Level Density in the X(926, 282) System, Qeios **05USEO**. 2024; 1. Doi: https://doi.org/10.32388/05USEO
14. Bevelacqua JJ. Superheavy Nuclei VI: 1000 ≤ A < 1100 Systems, Physics Essays. 2017; 30(4):392.
15. Bevelacqua JJ. Nuclear Level Density in the X(1062, 312) System, Qeios **HFIEM2**. 2025; 1. Doi: doi.org/10.32388/HFIEM2
16. Bevelacqua JJ. Superheavy Nuclei VII: 1100 ≤ A < 1200 Systems, Physics Essays. 2018; 31(3):235.
17. Bevelacqua JJ. Nuclear Level Density in the X(1122, 330) System. Journal of Physics & Optics Sciences SRC/JPOS-398. 2025; 7(7):1.
18. Bevelacqua JJ. Superheavy Nuclei VIII: 1200 ≤ A < 1300 Systems, Physics Essays. 2020; 33(3):276.
19. Bevelacqua JJ. Nuclear Level Density in the ^{4}He System, Phys. Rev. C. 1979; 19:2050.
20. Lukasiak A, Sobiczewski A. Estimations of half-lives of far-superheavy nuclei with Z approx. = 154 - 164, Acta Phys. Pol. **B6**. 1975; 147.
21. Petrovich F, Philpott RJ, Robson D, Bevelacqua JJ, Golin M, Stanley D. Comments on Primordial Superheavy Elements, Phys. Rev. Lett. 1976; 37:558.
22. Rost E. Proton shell-model potentials for lead and the stability of superheavy nuclei, Phys. Lett. 1968; 26B:184.
23. Blomqvist J, Wahlborn S. Shell Model Calculations in the Lead Region with a Diffuse Nuclear Potential, Ark. Fys. 1959; 16:545.
24. Brown GE, Gunn JH, Gould P. Effective mass in nuclei, Nucl. Phys. 1963; 46:598.
25. Fox L, Godwin ET. Some new methods for the numerical integration of ordinary differential equations, Proc. Cambridge Philos. Soc. 1949; 45:373.
26. Huizenga JR, Katsa AA. Distribution of spacings of nuclear energy levels of mixed spin and parity, Nucl. Phys. A. 1967; 98:614.
27. Bethe H. An Attempt to Calculate the Number of Energy Levels of a Heavy Nucleus, Phys. Rev. 1936; 50:332.
28. Bethe HA, Bacher RF. Stationary States of Nuclei, Rev. Mod. Phys. 1936; 8:82.
29. Bethe H. Nuclear Dynamics, Theoretical, Rev. Mod. Phys. 1937; 9:69.
30. Huizenga JR. In Statistical Properties of Nuclei, edited by J. B. Garg (Plenum, New York, 1972), p. 425.
31. Rahmatinejad A, *et al*. Energy dependent ratios of level-density parameters in superheavy nuclei, Phys. Rev. 2022; 105:044328.
32. Sangeeta T, *et al*. Astrophysical reaction rates with realistic nuclear level densities, Phys. Rev. C. 2022; 105:044320.
33. Wu XH, Ren ZX, Zhao PW. Nuclear energy density functionals from machine learning, Phys. Rev. C. 2022; 105:L031303.