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Fractal Project: From Students' Alternative Ideas to Understanding Mathematical and Physical Fractals Through Elements of Fractal Geometry

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Abstract
This research investigates high school students' understanding of mathematical, physical, and biological systems exhibiting Fractal structures, with the aim of enhancing their comprehension of nature's intrinsic complexity. The research explores the pedagogical integration of Fractal Geometry, emphasizing key concepts such as scaling, self-similarity, and Fractal dimension, and examines their applications in both mathematical and natural systems.
The research employs an instructional intervention designed to evaluate students' conceptual understanding, the characterization of Fractal sets and systems, and the impact on their motivation for learning Natural Sciences. Both qualitative and quantitative methods were used, including pre- and post-test questionnaires, worksheets, and statistical analyses. Nonparametric tests (Wilcoxon Signed-Rank and Mann–Whitney U) were used due to nonnormal data distributions, and the reliability and validity of the instruments were verified.
The results reveal that Students' initial alternative conceptions shifted toward scientifically accepted ideas, and their motivation for the Natural Sciences increased substantially. Field-based activities involving authentic Fractal structures proved particularly effective in engaging learners, fostering active participation, retention, and meaningful knowledge construction.
The research demonstrates that the pedagogical use of Fractal Geometry provides an innovative and effective approach to teaching complex scientific phenomena. It bridges theoretical concepts and practical experience, enhances critical and creative thinking, and contributes to scientific literacy, offering new tools for interdisciplinary education and for teaching the natural world.

Keywords: Students' Conceptual Understanding, Alternative Conceptions, Mathematical, Physical and Biological Fractals

1. Introduction

Fractal geometry, as established by Benoit Mandelbrot, often referred to as the "geometry of nature," is a non-Euclidean geometric approach that allows for the description, modeling, and simulating/visualizing natural and biological objects with irregular/rough surfaces such as clouds, coastlines, trees, the blood vessel system, the branching of the bronchi in the lungs, nerve cells in the human brain, corals, sea sponges, to mention a few. Its basic concepts are scaling, scale invariance, and fractal dimension. Scale invariance can be exact, as in mathematical fractals, or approximate (statistical), as in physical fractals. Fractal dimension is the basic characteristic that measures the degree of roughness (irregularity, complexity) of a fractal set (Mandelbrot, 1982) [46]. Unlike Euclidean dimensions, which take integer values, fractal dimension usually takes non-integer values, serving as a measure of roughness, irregularity, and fragmentation of natural systems, closely linked to the study of natural systems that exhibit structure within structure and self-similarity.

This research focuses on the study of understanding systems (mathematical, physical, and biological) that exhibit fractal structure by students in the first years of high school, with the aim of introducing them to observation, study, and ultimately understand the complexity of nature. The study of the complexity of nature requires the use of non-linear models for its description and simulation, as traditional analytical techniques prove to be insufficient. The development of Computers and

Complexity Science, with its main areas being Dynamic Systems and Fractal Geometry, has provided the necessary tools for studying these specific systems (Mandelbrot, 1982, Bountis, 2004) [46, 5].

The introduction of Fractal concepts and models into high school physics courses can help students understand the composition, functioning, and evolution of both physical and biological systems, geological formations, as well as the organization of vital tissues and organs, not to mention other complex physical, chemical, biological, and environmental processes, and lastly determine their dimensions and other spatial parameters.

The novelty of this study lies in its systematic comparison of field-based and laboratory-based approaches to teaching fractal geometry in secondary education, combined with an explicit examination of students' understanding of fractal dimension as a measure of complexity in mathematical and natural systems. By linking conceptual learning outcomes with motivation across Physics, Chemistry, and Biology, the study provides evidence that interdisciplinary and real-world engagement with fractals supports conceptual change and enhances students' motivation for learning natural sciences.

The structure of the paper is the following: In Section 2, we present the theoretical framework of our research, in Section 3, the Research Methodology, while Section 4 contains the research results, and Section 5 contains the conclusions.

2. Theoretical Framework

2.1 The concept of Fractals

According to Mandelbrot (1982) [46], who first coined the term,

> "*Fractals are geometric shapes that do not exhibit any regularity. They are irregular and maintain this irregularity to the same degree at all scales of examination. A fractal object looks the same when viewed from a distance or up close- it is self-similar. When we approach it, we find that small parts of the whole, which appear from a distance to be unstructured masses, become well-defined objects whose configuration is approximately the same as that of the previously examined whole*."

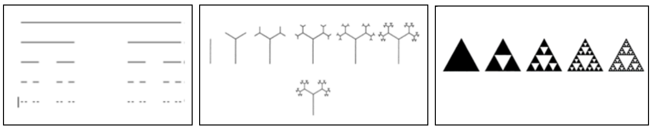
Falconer (1990) [28] believes that there can be no analytical definition for fractals because "they are like life. You can describe their basic properties and the fundamental elements that constitute them, but you cannot encapsulate them in a definition." Working within the same framework, Bountis (2004, p. 64) [5] gives the following definition:

"A set of points can be called a fractal if it has the following properties:

- *It is described by an infinite sequence of scales, exhibiting a 'structure within a structure', i.e. new details at each scale of examination.*
- *Parts of one scale are similar to other parts of a different scale, a situation characterized as self-similarity under scale change.*
- *This self-similarity can be exact, approximate, or a statistical property of the whole.*
- *It is often created through a repetitive process in which the same mathematical transformations are followed/applied at each step."*

Mandelbrot found that for objects in this category, a new non-Euclidean geometry should be established, Fractal Geometry, which describes them while providing a framework for describing complex objects in nature. According to Mandelbrot (p.122, 1992) [47], "*Fractal geometry... can also describe the geometry of mountains, clouds, and galaxies.*"

Since examples of fractal structures can be found in nature, starting with the branches of trees, ferns, broccoli, sponges, the pulmonary system or coastlines, it seems that the rules governing the developmental process of natural and biological structures require that characteristics relating to small scales be translated into characteristics of large scales.



The Cantor Set A Tree Sierpinski's Triangle

Fig 1: Examples of Fractal shapes

But how is a mathematical Fractal set created? Usually through an iterative process in which a certain geometric shape is repeatedly replaced by other shapes, creating the Fractal set. Three types of iterative processes are commonly used to create fractal sets:

- Generator iteration
- Construction using iterated function systems (IFS)
- Formula Iteration

Examples of Fractal sets created through a repetitive process using a generator are the Cantor set, the binary tree, the Sierpinski triangle and carpet, the Koch curve and snowflake (Peitgen, Jürgens, Saupe, 1992). Fig 1 presents some of them.

It is important to start with the notion of dimension. Let us begin by selecting certain mathematical/geometric objects from classical Euclidean geometry that are locally smooth except for their vertices and angles. In the first category, we place geometric objects that appear as individual points when enlarged enough which are local/point-like. We assign these objects a dimension of 0 (zero), which is the dimension of a point. In the next category, we include objects that appear locally as a line. We assign these objects a dimension of 1 (one), as in a line. Continuing with the same logic, we include in the third category objects that locally resemble a square and in the fourth category objects that locally resemble the interior of a cube to which we assign dimensions 2 (two) and 3 (three).

Geometric objects such as straight lines, squares, cubes, but also Cantor sets, Sierpinski triangles, etc. shared a common property. They can be repeatedly subdivided into a number of copies smaller than the original object in a scaling process. More specifically, the ratio of the "size of the copy" to the "size of the original segment" defines the scaling ratio. The original object and the segments into which it was subdivided are self-similar in this case. The way they are created can lead to the determination of a relationship between the scaling ratio r, the number of segments, N, resulting from the original in any subdivision, and their dimension, d, which in this case has the form:

$$Nr^d = 1 \tag{1}$$

In this case, we can choose the following relationship to define the dimension of these geometric objects, which is consistent with the equation previously formulated, as it results from it by taking the logarithms of nominator and denominator:

$$d = \frac{\log(N)}{\log(1/r)} \quad (2)$$

We define this dimension as fractional/fractal dimension or self-similarity dimension.

Applying the new relationship for the dimension we determined for the cases of the Cantor set and the Koch curve, we will have, respectively:

$$d_{Cantor\ set} = \frac{\log(2)}{\log(3)} = 0.63092 \ldots.$$

and

$$d_{Koch\ curve} = \frac{\log(4)}{\log(3)} = 1{,}26185 \ldots \ldots$$

One possible interpretation of fractional dimension is that it provides a general measure of a system's complexity. For example, speaking of the Koch curve, its non-integer dimension is d=1.26185... and can be interpreted as occupying more space than a straight line. The Sierpinski triangle and the Koch curve are shaped in such a way that they do not suggest the idea of a smooth and even curve that completely covers a surface. The consideration of non-integer dimensions suggests treating dimensions as a continuous range of values rather than discrete values, and, according to Mandelbrot (1982) [46], researchers should treat dimension as a variable that takes values from zero to infinity.

It should be noted that attempting to measure any geometric object with a measuring tool smaller than its own dimension yields an infinite value, whereas using a measuring tool larger than its own dimension yields a zero value. This also brings about the concept of the space occupied/immersion space within which the Fractal object develops and we know that its dimension can never be greater than the topological dimension of this space.

Fractals are also found in nature; however, without the infinite detail in magnification/scaling that is found in fractals derived from mathematical relationships. The range of magnification scales is small due to the practical limits of natural time and space. Examples of systems whose structure can be simulated by Fractals in nature include: certain trees, plant root systems (from the most primitive, such as lichens, mosses, and algae, to the more familiar cauliflower, ferns, etc.) and leaf vein branching, natural sponges, the surface of clouds, river beds, mountain surfaces, coastlines, snowflakes, foam, and galaxies. The branches of lightning and/or other electrical discharges, the turbulent flow of fluids, the phenomenon of "viscous fingering" and, more generally, the storage of liquids in the subsoil, the deposition of metal particles on electrodes during electrolysis, the process of crystal growth through diffusion-controlled aggregation and, more generally, dendrite forms, can also be simulated by Fractal structures.

Many biological systems have been studied and found to exhibit striking similarities to various forms of Fractals. These include proteins and enzymes, cell nuclei, single-celled organisms, neurons, the surface of the brain, lung bronchial trees, the nervous network of the heart, the bile ducts, the venous and arterial systems, the walls of the uterus, the urinary system, and the intestinal villi. As we will see below, they characterize the development and functioning of basic systems of living organisms (Mandelbrot, 1982) [46].

It should be noted, however, that in such instances self-similarity is of a statistical nature. A small part of a cloud resembles the whole, just as a small section of lightning or cauliflower are equally smaller copies that resemble the whole on average. This statistical, rather than exact, self-similarity is also maintained for several but not infinite scales, such as mathematical Fractals.

But how is the dimension of a natural object with a Fractal structure defined? A common method follows. We consider an image of the object and cover it with "boxes"[1] whose side length is s and whose number is N(s). We repeat the process with "boxes" of different side lengths s, counting the corresponding N(s). We identify the relationship between N(s) and s. Let us assume that this relationship has the form of a power law:

$$N(s) = k\cdot(1/s)^d \quad (3)$$

We observe that for Euclidean geometry shapes k=1 and for one-dimensional, two-dimensional, and three-dimensional shapes where we have N(s)= 1/s1, 1/s2, 1/s3, respectively, we get d=1, 2, and 3, which is to be expected. By taking the logarithms of both sides of the equation (3), we obtain:

$$\mathrm{Log}(N(s)) = d\cdot\mathrm{Log}(1/s) + \mathrm{Log}(k) \quad (4)$$

Solving for d and taking the limit as s → 0, we have,

$$d_b = \lim_{r\to 0} (\mathrm{Log}(N(s))/\mathrm{Log}(1/s)) \quad (5)$$

In the case the limit exists, it is defined as the box-counting dimension of the Fractal object (Garnet, 1997, Bountis, 2004) [25, 5]. In practice, relation (4) defines a linear equation with slope d. Therefore, by working experimentally and plotting Log(N(s)) against Log(1/s), points are obtained that approximate the straight line with slope d_b. This method is the log-log approach to finding the box-counting dimension.

Fractal models are generally created with the use of appropriate software. For example, trees, ferns, cells of the nervous system (Falconer, 2003) [21], blood and lung vessels, electrochemical patterns, circadian rhythms (Strogatz, 1994) [66], and other similar patterns in nature can be modeled on a computer using recursive algorithms and L-system [2] techniques (Prusinkiewicz & Hanan 1989) [59]. One limitation of Fractal modeling is that the similarity of the model to the natural phenomenon does not prove that the natural phenomenon being modeled is formed by a process similar to that described by the modeling algorithms.

In conclusion, Fractal geometry allows us to adequately describe and represent complex physical and biological

[1] Suitable dimensions d=1, d=2, d=3, where these can be lines, parallelograms, cubes, etc.

[2] An L-System or Lindenmayer System is a method for constructing an algorithm consisting of a set of commands, usually applying a simple form. It can be used to create mathematical curves that obey the principle of self-similarity. The resulting shapes are widely used in the field of computer graphics.

structures, taking steps towards understanding their structure. In addition, it allows us to create Fractal structures for a variety of applications, including technological applications, but also for purely aesthetic enjoyment.

2.2 Introduction of Fractal Geometry in Secondary Education. Literature Review

Considering Fractal geometry as one of the modern mathematical tools with which physical processes involving irregularity and roughness in terms of structure can be described and analyzed, as well as the fact that fractal structures appear in mathematics, sciences as well as in music, literature, and the fine arts in general, a broad field emerges that allows for interdisciplinary connection and teaching of science and the arts (Frame & Mandelbrot 2002) [30].

We now have significant research experience from the relevant literature (Stavrou & Assimopoulos, 2011) [67] on the elements/concepts of fractal geometry (scaling, repetition, recursion, self-similarity, fractional-fractal dimension, fractal creation) which can be incorporated into school teaching, mainly in mathematics and, to a smaller extent, in natural sciences and computer science at various levels of education, as well as teaching suggestions on how this incorporation can be beneficial and effective (Gounari, 2017a, Narimbetov, 2020 [52], Tzanaki *et al*., 2024a,b) [26, 52, 72, 73].

This introduction leads to an increase in students' interest in mathematics, while the creation of fractal forms by students increases their creativity and understanding while at the same time creates positive emotions and aesthetic enjoyment. In addition, the introduction allows for the interconnection of both scientific disciplines and their teaching in a way that enhances and improves learning outcomes for students (Sorgo 2010, Drakopoulos & Sioulas, 2020) [63, 18] and university students (Chirkova & Testova, 2019) [12] in related disciplines.

The introduction of fractal concepts into the study of everyday materials, or rather the natural universe of students, is considered important because it offers a privileged field of interconnection among mathematics, physics, and education with direct reference to reality (Esbenshade, 1991, Zembrowska & Kuzma, 2002, Zanoni, 2002, Knutson & Dahlberg, 2003) [20, 76, 75, 38].

Teaching suggestions for introducing fractal concepts and studying their properties include (indicatively):

- *In middle school* (Junior High school): Buldyrev *et al*., 1994 [7], Naylor, 1999 [53], Yildiz *et al*., 2011 [74], Gounari, 2017b [27], Kovács, 2019 [41], Tzanaki *et al*., 2024a [72],
- *In high school*: Lewis & Kaye, 1990 [44], Peitgen *et al*., 1991 [56], Camp, 1995 [9], Langille, 1996 [42], Komorek *et al*., 2001 [40], Stavrou, Duit & Komorek, 2008 [68], Reda, 2014 [60], Narimbetov, 2020 [52], Tzanaki *et al*., 2024 b [73],
- *At university*: Talanquer & Irazoque, 1993 [69], Karakus & Kösaa, 2010 [35], Monferrer-Sales, Lorenzo-Valentín & Mas, 2014 [51],
- *For teachers* (training): Cuéllar & Salazar, 2017 [14].

The following teaching suggestions are interdisciplinary / multidisciplinary in nature:

- Esbenshade, 1991 [20], Frantz & Lazarnick, 1991 [22], Naylor, 1999 [53], Komorek *et al*., 2001 [40], Pereira *et al*., 2010 [58], Kirwan & Tobias, 2014 [37], Monferrer-Sales, Lorenzo-Valentín & Mas, 2014 [51], Chirkova & Testova, 2019 [12], Drakopoulos & Sioulas, 2020 [18], Roanes-Lozano & Solano-Macías, 2021 [61], Betancourt *et al*., 2025 [4].

The activities proposed for the study of fractals refer to:

- The creation of mathematical fractal sets in the calculation of their dimension (indicatively: Monferrer-Sales, Lorenzo-Valentín & Mas, 2014, Karakus, 2015a, Cuéllar & Salazar, 2017, Gounari, 2017b, Kovács, 2019, Tzanaki *et al*., 2024a) [51, 34, 14, 27, 41, 72].
- The observation, identification, and study of natural fractal sets and the calculation of their dimension (indicatively: Lewis & Kaye, 1990, Esbenshade, 1991, Zembrowska & Kuzma, 2002, Zanoni, 2002, Knutson & Dahlberg, 2003, McCartney *et al*, 2008, Karakus & Kösaa, 2010, Cañibano, Vazquez & Gandini, 2011, Tzanaki *et al*., 2024 b [73]) [44, 20, 76, 75, 38, 50, 35, 10, 73].

The main source of inspiration for activities at the teaching level is the work of Peitgen *et al*. (1991 [56], 1992) and the classic writings of Mandelbrot & Frame (2002) [48] and Devaney (2014) [16]. They use: Fractal trees, Koch snowflakes, Cantor dust, Sierpinski triangle and carpet, variations of structures, Mandelbrot and Julia sets, and participants usually create their own fractals by working with hands-on activities as well as software and computer programs (XaoS, GeoGebra, Sketchpad). Students are exposed to iterative processes, the identification of generalized relationships, the creation and execution of algorithms and, finally, the calculation of perimeters and areas of increasingly complex shapes.

Working with fractals in the classroom engages students in exploring a wide range of mathematical topics such as similarity, ratios and proportions, measurements and fractions, symmetries, sequences, progressions, and functions (exponential and logarithmic) and their composition, affine transformations, number systems beyond the decimal system, and complex numbers.

The aforementioned suggest that there appears to be room for research in the 15-17 age group, which covers grades A and B of high school in Greece, in the relevant cognitive area. What is needed is not a traditional teaching intervention but rather a project that includes both the study of mathematical and physical fractals and their properties and the study of materials and phenomena in the everyday lives of students, which are presented or simulated by fractal structures, as it offers a privileged field of interconnection between mathematics, physical sciences, information technology, and fine arts. A project that can be carried out using mathematics and computer science teaching hours, as well as physics laboratory hours and visits to the natural environment to identify fractal structures.

By studying the Curriculum for Mathematics, it is necessary to identify the prior knowledge and skills that students theoretically possess and check the initial ideas of students regarding their actual knowledge, skills, and difficulties. An important and not widely discussed aspect of the research effort should be the identification of the initial motivations that lead them to voluntarily engage in such an educational activity along with the possible identification of the change that this may bring about in their motivations. Another useful dimension is the link between cognitive outcomes and motivation, which is a new research finding.

3. Research Methodology

The purpose of this research effort is to investigate the possibility of introducing fractal concepts and their geometry into the mathematics and natural sciences curriculum of the first and second years of high school, to study their effect on

- the understanding of concepts in mathematics and natural sciences
- the development of critical and creative thinking skills, as well as skills for visualizing concepts and phenomena
- student motivation and interest and finally
- assessing the overall pedagogical value of this introduction

More specifically, the aim of the study was to examine whether the initial/alternative ideas of the students regarding the concepts of fractals and the determination of their properties changed towards those scientifically accepted under the experimental teaching intervention, which included hands-on activities and the use of appropriate software, as there is a limited number of relevant studies in the field of physics teaching.

The research questions examined were as follows:

1. Which mathematical concepts are acquired and/or reinforced through the proposed teaching intervention for the introduction of fractals?
2. To what extent do students understand the basic properties of fractals (mathematical and physical) before and after teaching?
3. How does the use of computers and relevant software for creating fractal structures and calculating their properties affect students' understanding and participation?
4. How does fieldwork (school grounds, outdoor areas, residence, etc.) affect students' understanding and participation?
5. What motivates students who voluntarily participate in educational activities? Do their motivations remain stable or change after the activity is completed, and in what direction?
6. Is there a connection between motivation and learning outcomes?

The main tool of the investigation was an exploratory/constructive experimental teaching intervention consisting of three phases, during which the following were implemented:

- **First phase:** study of fractal mathematics and their properties.
- **Second phase:** study of selected natural fractals and their properties in the laboratory, as well as searching for and studying fractals in the natural environment.
- **Third phase:** study of fractals and their creation by students using software in the school computer lab.

The second phase was divided into two parts. Initially, students worked with images of natural fractals and discussed their properties, while in the second part they had the opportunity to continue their study, freely choosing whether they will continue studying natural fractals in the school laboratory or in the field (outside the school, at home, in the natural environment, etc.). The students worked together on the first phase, the first part of the second phase, and the third phase, while their choice in the second part of the second phase divided them into an experimental group (field work) and a control group (laboratory work). The thesis compares the learning outcomes of the students in the experimental group with those of the students in the control group.

The teaching intervention was carried out by the first author of this paper, with the support of the teachers who teach physics and mathematics. During the comparison, both the experimental and control groups participating in the study were treated identically in terms of objectives, cognitive content, and use of laboratories, while the experimental group focused its work beyond the laboratory, in the field.

The sample of the main study consisted of 122 students from two schools in the region of Greece, the 1st Vocational High School of Oinoi-Schimatari, which is located in an area of intense industrial activity, and the High School of Pyli, a relatively small school located in an agricultural and livestock farming area. Both schools belong to the Secondary Education Directorate of the Prefecture of Boeotia. Out of the total number of students who participated in the study, 94 (77%) were boys and 28 (33%) were girls. The ages of the students ranged from 15 to 17 years, with an average age of 16 years.

Prior to the implementation of the main study, a pilot study was conducted, the results of which led to the selection of the final form of the teaching intervention, the worksheets, and the teaching intervention evaluation questionnaire.

The effectiveness of the experimental teaching intervention was evaluated by:

- Examining the responses to the questionnaire developed for the purposes of the study in order to identify the students' ideas, before and after the completion of the teaching intervention, regarding their understanding of concepts related to mathematics, natural and biological fractals.
- Examining the data obtained from the analysis of the students' responses on the activity worksheets.
- Examining the researcher's interviews with participants in the experimental and control groups through focus groups.

The questionnaire used in this study, which was designed to systematically measure the understanding and teaching of fractal structures by students in the first and second years of high school, covers multiple dimensions such as: recognition of fractals, their definition and characteristics, production through rules, dimension of fractals, applications in nature.

In its development, previous studies and tools were considered, which despite the fact that they refer to different samples (Karakuş 2013, 2015 [32, 33], Chen, 2004), provide useful ideas for selecting questions. Finally, the combination of multiple-choice and open-ended questions allows for both quantitative and qualitative assessment, while the tables matching question dimensions and learning objectives ensure the most complete coverage possible of the learning objects and conceptual areas it aims to measure. The tool is therefore a comprehensive means of recording students' knowledge and understanding of Fractal structures.

The validity of the tool was ensured through the involvement of experts in the field of fractals and their educational application, while its reliability was assessed in terms of item analysis of the questionnaire based on the Corrected Item-Total Correlation index and in terms of its internal consistency with Cronbach's alpha index (a) for all questions and for both participating groups, experimental and control, with index values greater than 0.7, which also indicates its reliability (Nunnally & Bernstein, 1994, Anastasi & Urbina, 1997, Crocker & Algina, 2008, DeVellis, 2017, Field, 2018) [54, 2, 13, 17, 29]. The analysis of the results also showed that the reliability of the questionnaire in terms of its internal consistency and structural validity is confirmed by Cronbach's alpha (a) coefficient, which was found to be 0.958 in the pre-test procedure and equal to 0.993 in the post-test procedure.

The questionnaire contains 38 questions, which are divided into eight groups, the subjects of which are shown in the following table.

Table 1: The questionnaire questions and the topics they examine

Questions	Examine (concepts, properties, descriptions, prerequisites)
1-6	Scale / Scaling
7, 9-11	Similarity / Self-similarity
12, 13, 27	Dimension / Fractal dimension
15-23	Description of the properties of physical fractals
24-28	Discovery of laws governing the creation of natural fractals and their definition
30, 31, 33	Fractal creation, software & computers
8, 14, 36, 37, 38	Assessment of prerequisite knowledge in mathematics
29,32, 34, 35	Assessment of prerequisite knowledge in informatics

The data was analyzed using the Statistical Package for Social Sciences (SPSS) V.26.0, which was accessed via the institutional license held by the University of Thessaly. Our data was checked for normality using the Kolmogorov-Smirnov (K-S) criterion. According to the results a) the control group C.G. did not form a normal distribution in either the pre-test $p < 0.001$ or the post-test $p < 0.001$, b) the experimental group E.G did not form a normal distribution in the pre-test $p < 0.001$ and in the post-test $p < 0.001$. The following criteria were selected as appropriate for the analysis:

- for the intra-group test (E.G before-after & C.G before-after) where we have a case of two dependent samples, the non-parametric statistical criterion Wilcoxon (T).
- for the between-group test (E.G - C.G before & E.G – C.G after) where we have two independent samples, the non-parametric Mann-Whitney (U) statistical criterion

The students' answers to the questionnaire initially give us an initial idea of their conceptual background before the implementation of our teaching intervention, while their answers after the intervention help us determine the degree of conceptual change that may have resulted from the implementation of our teaching intervention.

In order to study motivation and changes in motivation, we used the Students' Motivation Towards Science Learning (SMTSL) questionnaire, which was developed by Tuan *et al*. (2005a) [71] and adapted to Greek context by Dermitzaki *et al*. (2013) [15]. The reliability and validity of this questionnaire have been successfully tested in Greek (Dermitzaki Stavroussi, Vavougios, Kotsis, 2013; Andressa, Mavrikaki, Dermitzaki, 2015) [15, 3], and a factor analysis was performed which confirmed the specific factors. The data and relevant results from the motivation survey have been included in the paper by Mantzavinos, Dermitzaki & Vavougios (2024) [48] and will not be further analysed.

4. Results

Through the teaching intervention, the students initially understood the concepts of scale and scaling, with which although they come into contact in the 3rd Grade of Junior High School curriculum, it appears that a large percentage of them has not understood them, and this applied to both the experimental and the control group. Subsequently, after the completion of the teaching intervention, understanding improved, being much greater for the experimental group than for the control group, with the difference being statistically significant. This conclusion is consistent with the work of Reda (2014) [60].

4.1 Question Group 1: Understanding the concepts of scale and scaling

Table 2: Understanding the concepts of scale and scaling

n	Purpose	Questions	(CG vs EG) before	Difference in accepted answers before ($MeanRank_{EG}$-$MeanRank_{CG}$)	(CG vs EG) after	Difference of accepted answers after ($MeanRank_{EG}$-$MeanRank_{CG}$)
1	Understanding of concepts ▪ scale και ▪ scaling	1	Not Sig.	+1,98	Sig	+39,05
		2	Not Sig.	+7,50	Sig	+38,42
		3	Not Sig.	+3,25	Sig	+34,28
		4	Not Sig.	+7,60	Sig	+33,36
		5	Not Sig.	+5,35	Sig	+32,47
		6a	Not Sig.	+10,87	Sig	+32,01
		6b	Not Sig.	+10,87	Sig	+32,61

An analysis of the activity worksheets shows that by carrying out the activities, they understood how the iterative process works by implementing the successive patterns produced during its application and observing in practice the "structure within a structure" created by it. After completing the relevant activities, they appeared to be familiar with iterative processes, recognizing their basic characteristics and able to find general patterns after n iterations, at least for the activities they were given, while in discussion with the researcher they seemed to understand the data of a problem based on repetitions of specific actions and suggest ways it might be solved. Details are presented in the Table 2.

4.2 Question Group 2: Understanding the concepts of similarity and self-similarity

The concept of similarity is generally considered to be a concept that has been taught and is therefore acquired in the A and B grade of high school, but we observe that the reality is different. A small percentage of students in both groups initially appear to understand and be able to handle the relevant concepts, while after the intervention, the experimental group shows a significant improvement that is statistically significant compared to the control group, for which there is no statistically significant difference in the responses of its members. The same conclusions apply to the understanding of the concept of self-similarity and the importance of self-similarity under scale change. Details are presented in the Table 3.

Table 3: Understanding the concepts of similarity and self-similarity

n	Purpose	Questions	(CG vs EG) before	Difference in accepted answers before ($MeanRank_{EG}$-$MeanRank_{CG}$)	(CG vs EG) aafter	Difference in accepted answers after ($MeanRank_{EG}$-$MeanRank_{CG}$)
2	Understanding of concepts ▪ similarity ▪ self-similarity	7	Not Sig.	+11,19	Sig.	+32,96
		9	Not Sig.	+1,50	Sig.	+32,3
		10	Not Sig.	-6,85	Sig.	+34,22
		11	Not Sig.	+1,21	Sig.	+29,13

4.3 Question Group 3: Understanding the concepts of Euclidean dimension and fractal dimension of geometric and physical objects

With regard to the concept of Euclidean dimension, which was introduced in the third year of secondary school, students initially do not seem able to answer what a dimension is and in turn identify the dimension of Euclidean shapes. However, due to the teaching intervention, the situation improves, with the experimental group performing better and its improvement being statistically significant compared to the control group. Analysis of the students' answers on the worksheets for the relevant activities leads to the same result.

With reference to the calculation of the fractal dimension (self-similarity), the students initially calculated it in approximation using paper, pencil, and a calculator, and then repeated the same calculation using GEOGEBRA6 (Version 6.0.x) and the use of an exponential function resulting from the iterative method of generating generations (connection of generation segments, scale, and dimension), a method taught to them by the researcher, and with its help, they determined the (exact) dimension of the fractal shapes given to them (Cantor dust, Sierpinski triangle and carpet, etc.). Two main difficulties were identified at this level. The first refers to the fact that the fractal dimension is in many cases an irrational number. Although irrational numbers are introduced in the mathematics curriculum of the second year of junior high school and are also taught later in high school, it seems that this knowledge is not solidly grounded in mathematics and needs to be repeated and clarified. The use of GEOGEBRA6 provides a practical solution, though a deeper understanding of the mechanism requires an understanding of the exponential function, which is introduced / taught in the first year of high school and completed in higher grades. Addressing these difficulties leads to a meaningful understanding, while the use of logarithms is one of the most effective teaching solutions, but they are taught in the second year of high school and their earlier use requires special introduction.

Simultaneously, students were introduced to a simple and functional definition of Box dimension so that they could characterize the complexity of physical and biological objects with a Fractal structure. The answers of the students in the experimental group compared to the control group to the relevant question in the questionnaire (regarding the characterization of complexity through dimension for natural objects) show a significant improvement which, however, is not statistically significant compared to the control group. It seems that ultimately the difficulty of characterizing the complexity of natural and biological Fractals through dimension is challenging for both groups and requires a more focused teaching process.

Table 4: Understanding the concepts of Euclidean dimension and fractal dimension of geometric and physical objects

n	Purpose	Questions	(CG vs EG) before	Difference in accepted answers Before ($MeanRank_{EG}$-$MeanRank_{CG}$)	(CG vs EG) after	Difference in accepted answers After ($MeanRank_{EG}$-$MeanRank_{CG}$)
3	Understanding of concepts: ▪ Euclidian dimension ▪ Geometric Fractal dimension and Fractal dimension of natural objects	12a	Not Sig.	+0,25	Sig.	+33,68
		12b	Not Sig.	+0,95	Sig.	+32,03
		13a	Not Sig.	-2,07	Sig.	+33,36
		13b	Not Sig.	-1,73	Sig.	+33,65
		13c	Not Sig.	-4,02	Sig.	+33,8
		13c	Not Sig.	-3,91	Sig.	+33,8
		27a	Not Sig.	+3,71	Sig.	+34,72
		27b	Not Sig.	-1,21	Not Sig	+18,52

Considering the two groups, experimental and control, the students, either by observing images of fractal natural and biological objects (control group) or by working in the field (experimental group), sought ways to characterize the dimension of biological objects. Analysis of the worksheets for the relevant activities and the presentations of the objects chosen by the students showed that the experimental group that worked in the field gave the objects they identified as having a fractal structure dimensions between 2 and 3, while the control group gave dimensions between 1 and 2. A discussion with them showed that they consider dimensions according to the way they are presented or the environment "immersion," i.e. three dimensions for nature and physical and biological objects, or two dimensions determined by the image/photograph they hold in their hands and process, which is extremely important. When the researcher asked them to calculate the fractal dimension in mathematical and real structures and shapes, it turned out that they could do so in mathematical fractal structures, while for structures in the natural world they only had suggestions on how it could be done. The control group suggestions mainly involved calculation using Box dimension with two-dimensional "boxes" while the experimental group suggested three-dimensional "boxes". Details are presented in the Table 4.

4.4 Question Group 4: Description of natural Fractals and understanding their properties (scaling, self-similarity, dimension)

We will now move on to presenting our conclusions regarding how students perceive the scaling, self-similarity, and dimension presented in real Fractal structures and shapes that are presented. From the analysis of the worksheets for the relevant activities and the questionnaire, it appears that while initially we have either an inability to describe or descriptions that do not reveal the fractal characteristics, the situation then improves and we have descriptions that present scaling for at least some scales, as well as the similarity of parts of the natural object at different scales, which shows an understanding of the imprecise, statistical nature of self-similarity.

Table 5: Description of natural Fractals and understanding their properties (scaling, self-similarity, dimension)

n	Purpose	Questions	(CG vs EG) before	Difference of accepted answers before ($MeanRank_{EG}$-$MeanRank_{CG}$)	(CG vs EG) after	Difference of accepted answers after ($MeanRank_{EG}$-$MeanRank_{CG}$)
4	▪ Description of natural Fractals ▪ Understanding of their properties (scaling, self-similarity, dimension)	15	Not Sig.	-7,33	Sig.	+39,78
		16	Not Sig.	-13,83	Sig.	+35,89
		17	Not Sig.	-0,77	Sig.	+35,89
		18	Not Sig.	-13,35	Sig.	+35,03
		19	Not Sig.	-1,047	Sig.	+31,84
		20	Not Sig.	-1,079	Sig.	+33,13
		21	Not Sig.	-11,39	Sig.	+29,99
		22	Not Sig.	-10,33	Sig.	+33,13
		23	Not Sig.	+3,45	Sig.	+32,49

The students in both groups improved their answers, with the experimental group showing much greater improvement. Both the control group and the experimental group, while initially not responding, subsequently and after the intervention gave correct descriptions of physical and biological systems, highlighting their Fractal nature, with the improvement in the answers of the experimental group being higher than that of the control group and statistically significant. Details are presented in the Table 5.

4.5 Question Group 5: Discovery of mathematical laws governing the creation of natural fractals and their definition

We will further explore whether students can discover the mathematical laws that govern the creation of natural and biological Fractals, arrive at a functional definition of them, and, having already discussed attempts to calculate their dimension and, through this, characterize their complexity. They discovered the common elements of the natural Fractals they studied and discovered the rules governing their creation. With difficulty but successfully, they pondered and concluded that there is a common rule, with appropriate changes each time, can create the corresponding fractals, and they assigned them a dimension, without fully combining the characterization of their complexity through dimension. Finally, they provided a simple rule for what a natural fractal is. Garbin's research (2007) reached similar conclusions for a sample of students.

Table 6: Discovery of mathematical laws governing the creation of natural fractals and their definition

n	Purpose	Questions	(CG vs EG)before	Difference in accepted answers before ($MeanRank_{EG}$-$MeanRank_{CG}$)	(CG vs EG) after	Difference of accepted answers after ($MeanRank_{EG}$-$MeanRank_{CG}$)
5	Finding/Exploring mathematical laws governing the creation of natural Fractals and their definition	24	Not Sig.	+1,61	Sig.	+35,51
		25	Not Sig.	-2,62	Sig.	+35,15
		26	Not Sig.	+1,46	Sig.	+25,34
		28a	Not Sig	+4,58	Sig.	+35,00
		28b	Not Sig	-0.60	Sig.	+32,26

In the general discussion, the researchers found that the vast majority of students distinguish between natural and mathematical fractals, adopting the view that "…they show similarity on limited scales…". Finally, while they attribute "… different complexities …" to them, they do not associate them with the dimension for which they believe "… the dimension definitely exists since it is a Fractal …". It was also found that although they were taught how to calculate the Fractal dimension of a natural object and stated that they understood it, most of them did not believe in the concept of Fractal dimension as a dimension like the Euclidean dimension. It seems that they hold the view, without explicitly stating it, that there is only one unique dimension, the Euclidean dimension. This conclusion agrees with the conclusions of Karakus & Kösaa (2010) [35] in their study of students on how they approached the concept of dimension by participating in the implementation of two related projects. At this point the problem that arises is to what extent the activities designed in relation to fractals and their dimensions are appropriate (Ko & Bean 1991) [39]. Although literature states that such activities excite high school students (Lewis & Kaye 1990) [44] and that students understood the significance of fractal dimension in relation to its connection with the roughness or complexity of the structure they experimented with in their attempt to calculate the fractal dimension of physical objects from actual measurement data, they encounter the same problems as those students encounter in physics experiments when plotting interdependent variables and calculating the value of a variable using its slope.

However, despite the difficulty of calculating the dimension, fieldwork and laboratory activities involving the study of specific physical/biological and generally real objects with a fractal structure proved to be exciting for them. Even the children who did not work in the field successfully completed the study of the coastline. In general, the conclusions drawn from linking the study of fractal structures to the natural world, which is innovative, increased the interest of the students and appears to be effective in different school environments. These conclusions are consistent with the conclusions of the research and proposals of Esbenshade (1991) [20], Amaku, da Silva & Pizzinga (1999) [1], Zanoni (2002) [75], Cañibano, Vazquez & Gandini (2011) [10], Souza, Alves & Balthazar (2018) [64]. Details are presented in the Table 6.

4.6 Question Group 6: Creating Fractals & Computers

The conclusions regarding the third phase of the teaching intervention that aimed to encourage students to exploit the capabilities of computers and appropriate software in the study of iterative processes that produce various fractal structures under changing initial conditions and use appropriate programs to create and/or simulate fractals will be presented. Teaching and practice at this stage of the teaching intervention were common to both the experimental and control groups. In this section, students were taught what an L-System is and used it to construct algorithms that "run" iterative processes.

Table 7: Creating Fractals & Computers

n	Purpose	Questions	(CG vs EG) before	Difference of accepted Answers before ($MeanRank_{EG}$-$MeanRank_{CG}$)	(CG vs EG) after	Difference of accepted answers after ($MeanRank_{EG}$-$MeanRank_{CG}$)
6	Creating Fractals& Computers	30a	Not Sig.	-2,42	Sig.	+34,11
		30b	Not Sig.	+0,60	Sig.	+25,05
		31	Not Sig.	-7,77	Sig.	+27,61
		33	Not Sig.	-7,60	Sig.	+33,51

The algorithms were created using SCRATCH programming, and then Koch's Snowflake, Sierpinski's Carpet, Sierpinski's Triangle, the Dragon Curve, Peano's Curve, and other mathematical Fractals were created. They were then asked to create their own Fractals, which they did. The vast majority experimented with the parameters of the existing fractals, while a small minority created new fractals and enjoyed the results. In this section, it emerged that the combination of L-Systems in the construction of algorithms that implement iterative processes and the use of SCRATCH programming, which the students are already familiar with, leads to the optimal use of programming structures in the composition of programs in the field of fractal design and implementation that students can handle functionally. Details are presented in the Table 7.

5. Discussion and Conclusions

Fractal geometry is a difficult and demanding subject that is not covered by the high school curriculum and, according to other researchers who have attempted to implement it (Langille 1996) [42], it requires careful gradual implementation in the curriculum and adequate infrastructure in terms of hardware, software, and educational resources in general. Despite this, however, the presentation of topics not included in school curricula, such as the elements of fractal geometry, make students amuse/enjoy themselves, become interested, and/or discover concepts that are not the usual ones, giving substance to the view that they are not just mathematics in the classroom but necessary tools for our real life (Cañibano, Vazquez & Gandini 2011) [10].

According to Mandelbrot & Frame (2002) [48], Fractal geometry is regarded as one of the modern mathematical tools that can now be used to describe physical processes, and in particular the first tool that focuses on roughness and irregularity and, more generally, complexity in nature, an attempt was made to approach them partially through the design, implementation, and evaluation of a teaching intervention suitable for application to first and second grade high school students.

In addition to the creation of the teaching intervention and its assessment tools, apart from the appropriately selected conceptual content, the difficulties that students have in understanding the relevant concepts, as we know from the literature, were also considered (Bowers, 1991) [6].

In order for the intervention to be implemented, a community of practice was created with the aim of studying the creation of Fractal structures (Buldyrev *et al*., 1994) [7] in which the teacher/researcher would play the role of the scientific advisor and the students would form the research community. Such a community offered opportunities to explore models and develop their intuition about fractals before their formal mathematical study at university, as they are not included in the school curriculum. The students collaborated effectively with each other and with the teacher, learning from each other, while the teacher's role was that of a mentor who highlighted the "big picture" and helping to complete the learning projects by helping students build their knowledge both individually and in the social interaction environment of the community.

During the teaching intervention, students were able to ask questions and freely express their opinions, discuss and exchange arguments and views with their classmates, formulate hypotheses, make predictions, and assess the progress of the learning projects they were implementing. During the implementation of the activities, their initial predictions/ hypotheses were confirmed or rejected, while through the summaries and discussions in which their opinions and projects were presented and discussed, they were able to compare their prior knowledge with the newly acquired knowledge, helping them to check how they had learned.

What did the participants gain? The following conclusions can be drawn from the aforementioned for the students who participated in the teaching process:

1. They became familiar with geometric fractal objects and their creation through the use of iterative processes.
2. They understood the concept of scaling and the property of self-similarity that characterizes fractal objects.
3. They acquired the ability to characterize the geometric fractal objects they practiced with through their fractal dimension.
4. They realized that there are physical, chemical, biological, geological, meteorological, etc. systems that can be modeled using fractal structures.
5. They understood that natural fractal objects can be characterized by their fractal dimension, which is related to their geometric and physical characteristics.
6. They encountered difficulty in calculating the fractal dimension of natural and biological objects.

7. They used L-Systems to construct algorithms that implement iterative processes and programming in SCRATCH to design and implement the Fractals they had been taught and created themselves.

Their initial alternative ideas changed significantly towards scientifically accepted ones, with the percentages of scientifically accepted answers and, more generally, the performance of the experimental group, which worked in the field searching for and studying natural objects with a Fractal structure was higher than that of the control group, with the difference being statistically significant. It seems that working with real objects with fractal structures in the field (open spaces, school grounds, residences, etc.) activates students and engages them in ways that knowledge is acquired, retained, and can be retrieved from memory in an optimal way, influencing their understanding and participation in the various teaching processes of this type. In general, linking teaching activities to real life actively engages students in learning processes, and in this respect, the introduction of Fractals is a privileged field (Karakosta, 2016) [36]. The above points cover research questions 1-4 and show that:

- The teaching intervention increased the percentage of correct answers to comprehension questions about the concepts and properties related to fractals for both the PO and OE groups, with the percentage of correct answers being higher for the experimental group, a statistically significant result.
- The use of software to create Fractals and study their properties enhanced understanding of the relevant concepts and unleashed creative abilities.
- Fieldwork (courtyard, school grounds, residence, etc.) led to a greater improvement in the understanding of the students who participated in the PO compared to the students in the OE.

With regard to high school student motivation for natural sciences, the study within the framework of the dissertation aimed to examine whether it changes or remains relatively stable after the completion of the annual teaching intervention. The main hypothesis of the study was that after the teaching intervention the motivation of students in the experimental group in the three subjects examined would improve significantly compared to the motivation of students in the control group. This hypothesis was confirmed based on the findings of the present study. The central finding of the study was that after the teaching intervention, the overall motivation of the students in the experimental group improved significantly in all three subjects examined compared to the motivation of the students in the control group. Our results are partly consistent, though in a moderate sample, with those found for secondary school students by Caballero-Garcia & Fernandez (2019) and Spandana, Rani & Devi (2020) [65] for teaching science subjects, Eman Fathi Jalal Gad (2021) [19] for teaching biology subjects, and Lazarevic *et al*. (2024) [43] for teaching physics. Therefore, this finding is considered to be related to the nature of the teaching intervention, according to which the experimental group participated in field research and teaching activities with laboratory and digital characteristics, which are factors that influence student motivation, reinforcing it according to the literature (Gagné & Deci, 2005, Hofstein & Lunetta, 2003, Tseng *et al*., 2007, Sevinç, Özmen & Yiğit, 2011) [23, 31, 70, 62].

Furthermore, as students prefer not to undertake learning projects if they are optional, this resulted in the control group not engaging in the corresponding activities, which explains the lack of improvement in their motivation to some extent.

However, there were also individual findings for the six motivational factors examined. For example, the 'performance goals' factor was found not to differ as a result of the Time × Group interaction in both Physics and Chemistry although it did differ in Biology. This type of motivation reflects students' competitive motivations; for instance, when they want to get the best grade or be the ones who draw the teacher's attention. While 'achievement goals', which express students' willingness to improve and learn, did not differ as a result of the Time X Group interaction, this was only applied in Physics. It should be noted that with regard to performance goals, the statement that 'participation is voluntary and has no impact on grades' may have led to a reduction in competition between students and an increase in their cooperation, since their participation had already drawn the attention of their teacher.

Furthermore, comparisons between the two groups (PO, OE) before and after the intervention showed that in Biology, after the intervention, PO significantly outperformed OE in all motivational factors. In Chemistry, only the 'performance goals' did not differ between the two groups after the intervention, while in Physics, both the 'value of the course' and the 'achievement goals' did not differ between the two groups after the intervention. Therefore, although in general after the intervention there was a significant improvement in motivation in all three subjects for the PO compared to the OE, there were some exceptions.

As for the question of whether the motivation of PO students before and after the intervention changed in the same or different ways in the three different courses examined, the results of the analyses showed that the motivation of PO students for Natural Sciences changed in different ways in the three courses. The difference/change index of motivation before and after the intervention for each school subject showed that the greatest improvement in motivation was in Biology, followed by Chemistry, while the least extensive change in motivation was observed in Physics. Thus, the greatest difference in the change in motivation was observed in Physics and Biology, which outweighed Physics.

Overall, based on the nature of the intervention and the results of the analyses already presented, we conclude that the involvement of students in teaching interventions that concern objects and structures of the real/physical world with a Fractal structure, are interdisciplinary/cross-curricular and involve students in fieldwork, laboratory work, and experimentation using computers and appropriate software tools, can significantly improve student motivation for the natural sciences. Such interdisciplinary teaching interventions appear to have an extensive positive contribution to student motivation in all three subjects examined, namely biology, chemistry, and physics, and most of all in biology.

In conclusion, we believe that, although further study and larger samples of participants are needed, the positive change in the six motivational factors related to learning Natural Sciences, identified in this study, can contribute significantly to the teaching and learning of Natural Sciences, in agreement with recent studies. We also believe that the findings of this study can contribute to broader record the motivations of Greek students in the field of science during a school year with reference to three different high school courses: Physics, Chemistry, and Biology.

The previous conclusions cover research questions 5 and 6 and answer the research hypothesis regarding whether student motivations will change as a result of their involvement in the teaching intervention. They highlight that the motivations of the students in the experimental group show a significant improvement compared to those of the control group.

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