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Decay Characteristics of Neutron Excess Germanium Nuclei

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Abstract
The properties of neutron excess germanium nuclei are predicted utilizing a single particle model. The single particle model calculations include alpha, beta, positron, electron capture, and spontaneous fission decay modes. Neutron emission decay modes that have short half-lives are not readily determined by the model. However, estimates of the neutron decay mode were evaluated using the methodology of Chowdhury *et al*. Using that model, spontaneous neutron emission is predicted to occur in the range of A = 101 – 103. The Japanese Nuclear Data Compilation terminate their calculations at A = 95. Given these results, single-particle model calculations are extended to encompass these values, and were extended to A = 102 with closure of the $2d_{3/2}$ neutron shell.
Single particle model calculations predict that A = 87 – 102 neutron excess germanium systems form bound systems that have limiting beta decay half-lives in the range of 0.853 – 32.0 ms. Model half-life results for the A = 87 – 95 germanium nuclei are within a factor of about two of the predictions of the Japanese Nuclear Data Compilation calculations.

Keywords: Nucleosynthesis, Neutron Excess Germanium Nuclei, Beta Decay, Nuclear Structure, Spontaneous Neutron Emission

1. Introduction

Interest in neutron excess nuclei [1-33] has stimulated both experimental and theoretical physics interest. Several physical processes generate neutron excess nuclei, but the r-process usually provides the most significant contribution. Production of neutron excess nuclei in mergers of astrophysical objects (e.g., black holes and neutron stars) is an active area of research in nuclear physics and astrophysics [1, 2].
This paper continues the investigation of neutron excess nuclei by focusing on the Z = 32 germanium systems. Neutron excess systems having Z = 9 - 31 were discussed in previous work [8-25, 29-33]. Studies of these systems provide additional insight into nuclear systematics involving the various nucleosynthesis mechanisms and decay modes, and their associated variation with atomic and mass numbers.

2. Calculational Methodology

Methods for investigating neutron excess nuclei are provided in Refs. 8-33. This paper follows the single particle methodology of Lukasiak and Sobiczewski [27] and Petrovich *et al*. [28]. Single particle energies of neutron excess nuclear systems are obtained by incorporating the numerical methods of Refs. 34 and 35.
The radial Schrödinger equation is utilized to determine binding energy of a nucleon interacting with a nuclear core [8-25, 29-33]:

$$[(\hbar^2/2\mu)\,(d^2/dr^2 - L(L+1)/r^2) - E_{NLSJ} - V_{LSJ}(r)]\,U_{NLSJ}(r) = 0 \qquad (1)$$

In Eq. 1, E_{NLSJ} is the nucleon binding energy, r is the radial coordinate, $V_{LSJ}(r)$ is the nuclear interaction, and $U_{NLSJ}(r)$ is the radial wave function. L, S, and J represent the orbital, spin, and total angular momentum quantum numbers, respectively. The model definition is completed by defining the radial quantum number (N) and reduced mass (μ).

3. Nuclear Interaction

The Rost interaction [36] is selected for the nuclear interaction. This interaction has a central strength:

$$V_0 = 51.6\ [1 \pm 0.73\ (N - Z)/A]\ \text{MeV} \quad (2)$$

In Eq. 2, the positive (negative) sign is assigned to protons (neutrons). The spin-orbit interaction strength (V_{so}) is defined in terms of the central interaction strength and the multiplier γ [36]:

$$V_{so} = \gamma\ V_0 / 180 \quad (3)$$

Inclusion of the pairing correction interaction of Blomqvist and Wahlborn [37] completes the definition of the model interaction.

The difficulties in defining an appropriate nuclear interaction are outlined in Refs. 38 and 39. Ray and Hodgson [38] and Schwierz, Wiedenhöver, and Volya [39] note that modifications, unique to each nuclear system, are required to ensure an accurate representation of the experimental energy levels and decay characteristics. In view of the conclusions of Refs. 38 and 39 and the results of previous excess neutron system calculations [8-25, 29-33], the Rost central interaction strength (V_A) is modified in the following manner:

$$V_A = V_0\ \lambda\ [1 \pm a(A)]\ \text{MeV} \quad (4)$$

Individual nuclear system characteristics are defined by incorporating a potential strength multiplier (λ) and a factor [a(A)] to adjust the potential strength as a function of A. For germanium systems, the multiplier λ is selected to have the value of 1.5. This multiplier value is consistent with previous excess neutron nuclei calculations [8-25, 29-33] that provided model results in agreement with available data [40-42].

4. Model Limitations

Previous calculations [8-25, 29-33, 43] provided a representative description of the various nuclear decay modes (e.g., alpha, beta, positron, electron capture, and spontaneous fission) that could be encountered in neutron excess nuclei. Neutron excess systems can also decay by neutron emission modes that are not well-described by single-particle models. Since these neutron emission modes have very short half-lives, single-particle models will likely overestimate the lifetimes of neutron excess nuclei.

The onset of spontaneous neutron emission was estimated using the methodology of Chowdhury *et al.* [44]. Using the approach of Ref. 44, the onset of spontaneous neutron emission was estimated to occur in the range of A = 101 - 103 for germanium systems. Although independent of the calculated neutron single particle levels presented herein, the approach of Chowdhury *et al.* yields a logical value to terminate calculations for excess neutron germanium nuclei. The Japanese Nuclear Data Compilation terminate their calculations at A = 95. These results guide the A value selected to terminate the single-particle model calculations for germanium.

5. Results and Discussion

Table 1 summarizes the complete set of 102 ≥ A ≥ 79 germanium isotopes considered in this paper. The 102 ≥ A ≥ 79 germanium nuclei occupy a variety of neutron shells that are noted in Table 1. The heaviest observed germanium system is ^{86}Ge [42]. In view of the paucity of experimental data, extrapolations of nuclear characteristics beyond A > 86 become more uncertain.

5.1 86 ≥ A ≥ 79 Germanium Isotopes with Experimental Half-Life Data

The limiting decay mode (i.e., the transition that has the shortest decay half-life) for 86 ≥ A ≥ 79 germanium isotopes observed experimentally is summarized in Table 1. For example, the ^{79}Ge calculations predict five beta decay transitions (i.e., allowed $1f_{5/2}$(n) to $1f_{5/2}$(p) [22.4 s], allowed $2p_{3/2}$(n) to $2p_{1/2}$(p) [5.46 min], allowed $2p_{1/2}$(n) to $2p_{1/2}$(p) [21.1 s], allowed $1g_{9/2}$(n) to $1g_{9/2}$(p) [19.0 s], and first forbidden $1g_{9/2}$(n) to $1f_{5/2}$(p) [14.0 min]). For ^{79}Ge, the allowed $1g_{9/2}$(n) to $1g_{9/2}$(p) [19.0 s] transition is the limiting beta decay mode.

Table 1: Calculated Single-Particle and Experimental Decay Properties of Germanium Nuclei with 79 ≤ A ≤ 102

Nuclide (neutron shell)	a(A)	Half-Life (Decay Mode)	
		Experiment a,b,c/Theory d	This Work
^{79}Ge K ($1g_{9/2}$)	-0.0202 (+0.11%)	18.98 s c	19.0 s (β^-) e
^{80}Ge H ($1g_{9/2}$)	-0.0323 (-0.68%)	29.5 s c	29.3 s (β^-) e
^{81}Ge F ($1g_{9/2}$)	-0.0286 (+0.26%)	7.6 s c	7.62 s (β^-) e
^{82}Ge F ($1g_{9/2}$)	-0.0309 (-0.22%)	4.56 s c	4.55 s (β^-) e
^{83}Ge F ($2d_{5/2}$)	-0.0269 (0.00%)	1.85 s c	1.85 s (β^-) e
^{84}Ge G ($2d_{5/2}$)	-0.0239 (+0.21%)	949 ms c	951 ms (β^-) e
^{85}Ge E ($2d_{5/2}$)	-0.0192 (+0.20%)	503ms c	504 ms (β^-) e
^{86}Ge I ($2d_{5/2}$)	-0.0082 (0.00%)	226 ms c	226 ms (β^-) e
^{87}Ge C ($2d_{5/2}$)	0.0028	62.3 ms d	32.0 ms (β^-) f
^{88}Ge C ($2d_{5/2}$)	0.0138	43.0 ms d	21.6 ms (β^-) f
^{89}Ge C ($1g_{7/2}$)	0.0248	32.7 ms d	15.2 ms (β^-) f
^{90}Ge C ($1g_{7/2}$)	0.0358	21.4 ms d	11.0 ms (β^-) f
^{91}Ge C ($1g_{7/2}$)	0.0468	16.0 ms d	8.14 ms (β^-) f
^{92}Ge C ($1g_{7/2}$)	0.0578	10.4 ms d	6.19 ms (β^-) f
^{93}Ge C ($1g_{7/2}$)	0.0688	8.47 ms d	4.79 ms (β^-) f
^{94}Ge C ($1g_{7/2}$)	0.0798	6.11 ms d	3.79 ms (β^-) f
^{95}Ge C ($1g_{7/2}$)	0.0908	5.26 ms d	3.02 ms (β^-) f
^{96}Ge C ($1g_{7/2}$)	0.1018	f	2.45 ms (β^-) f
^{97}Ge C ($3s_{1/2}$)	0.1128	f	2.01 ms (β) f
^{98}Ge C ($3s_{1/2}$)	0.1238	f	1.66 ms (β) f
^{99}Ge C ($2d_{3/2}$)	0.1348	f	1.39 ms (β) f
^{100}Ge C ($2d_{3/2}$)	0.1458	f	1.17 ms (β) f
^{101}Ge C ($2d_{3/2}$)	0.1568	f	0.998 ms (β) f
^{102}Ge C ($2d_{3/2}$)	0.1678	f	0.853 ms (β) f

a Ref. 40. b Ref. 41. c Ref. 42.
d Japanese data Compilation calculation.
e Allowed $1g_{9/2}$(n) to $1g_{9/2}$(p) beta decay transition.
f Allowed $1g_{7/2}$(n) to $1g_{9/2}$(p) beta decay transition.
g No data provided in Ref. 40 - 42.

The model predicts the correct decay mode for the known 86 ≥ A ≥ 79 germanium systems [40-42]. As noted in Table 1, the model half-lives are also consistent with data [40-42].

^{79}Ge – ^{82}Ge nuclei occupy the $1g_{9/2}$ neutron shell. These systems decay through allowed $1g_{9/2}$(n) to $1g_{9/2}$(p) beta transitions. Model predictions for the beta decay half-lives of ^{79}Ge – ^{82}Ge are within about 0.7% of the experimental values [42]. In addition beta decay is the predicted decay mode in agreement with Ref. 42.

The ^{83}Ge – ^{86}Ge nuclei partially fill the $2d_{5/2}$ neutron shell. These systems decay through allowed $1g_{9/2}(n)$ to $1g_{9/2}(p)$ beta transitions. The half-life values of the ^{83}Ge -^{86}Ge systems are within 0.3% of the data [42]. Model calculations also predict the correct decay mode for these $2d_{5/2}$ germanium nuclei.

5.2 102 ≥ A ≥ 87 Germanium Isotopes without Experimental Half-Life Data

The a(A) values for 102 ≥ A ≥ 87 germanium isotopes were obtained from a linear fit based on the half-lives of ^{85}Ge and ^{86}Ge. The resulting a(A) values are listed in Table 1.

^{87}Ge and ^{88}Ge complete the $2d_{5/2}$ neutron shell, and decay through an allowed $1g_{7/2}(n)$ to $1g_{9/2}(p)$ beta decay transition. These germanium systems have beta decay half-life of 32.0 and 21.6 ms, respectively. Both values are is within a factor of 2 of the Japanese Data Compilation calculations [42].

^{89}Ge – ^{96}Ge nuclei fill the $1g_{7/2}$ neutron shell, and have calculated beta decay half-lives in the range of 2.45 to 15.2 ms. These systems decay through allowed $1g_{7/2}(n)$ to $1g_{9/2}(p)$ beta decay transitions. The model results for ^{89}Ge – ^{95}Ge are within about a factor of 2 of the Japanese Data Compilation calculations [42]. ^{95}Ga is the last germanium system included in the Japanese Data Compilation calculations [42].

^{97}Ge and ^{98}Ge fill the $3s_{1/2}$ neutron shell, and these systems decay through allowed $1g_{7/2}(n)$ to $1g_{9/2}(p)$ beta decay transitions. The half-lives of ^{97}Ge and ^{98}Ge are 2.01 and 1.66 ms, respectively.

^{99}Ge - ^{102}Ge fill the $2d_{3/2}$ neutron shell, and these systems decay through allowed $1g_{7/2}(n)$ to $1g_{9/2}(p)$ beta decay transitions. The half-lives of ^{99}Ge and ^{102}Ge are in the range of 0.853 – 1.39 ms.

The germanium calculations are terminated at A = 102 to account for the spontaneous neutron emission as the mass increases. Using the methodology of Chowdhury *et al.* [44], spontaneous emission is expected to occur at A = 101 – 103. The Japanese Data Compilation calculations [42] suggest that A = 95 terminates the germanium systems. Model calculations were extended to A = 102 to account for uncertainties in estimates of Refs. 42 and 44, and were terminated with filling the $2d_{3/2}$ neutron shell.

6. Conclusions

Single-particle model calculations readily incorporate alpha, beta, positron, electron capture, and spontaneous fission decay modes. Neutron emission decay modes have short half-lives that are not readily determined by a single-particle model. However, estimates of the neutron decay mode were evaluated using the methodology of Chowdhury *et al*. Using that model, spontaneous neutron emission in germanium nuclei is predicted to occur in the range of A = 101 – 103. The Japanese Nuclear Data Compilation calculations terminate their germanium calculations at A = 95. Given these results, single-particle model calculations are extended to encompass these values, and were terminated at A = 102 with filling of the $2d_{3/2}$ neutron shell.

Single particle model calculations predict that A = 87 – 102 neutron excess germanium systems form bound systems that have limiting beta decay half-lives in the range of 0.853 – 32.0 ms. Model half-life results for the A = 87 – 95 germanium nuclei are within about a factor of about 2 of the predictions of the Japanese Nuclear Data Compilation calculations.

These neutron excess germanium systems decay through allowed $1g_{9/2}(n)$ to $1g_{9/2}(p)$ and $1g_{7/2}(n)$ to $1g_{9/2}(p)$ beta decay transitions. The model likely overestimates the actual half-life values, because it does not explicitly include the short-lived neutron emission decay modes.

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