Int. j. adv. multidisc. res. stud. 2025; 5(6):972-974

Received: 08-10-2025
Accepted: 18-11-2025

International Journal of Advanced Multidisciplinary Research and Studies

ISSN: 2583-049X

Possible $B\overline{B}$ Tetraquark for the $\chi_b(2P)$ at 10.236 GeV/c^2

JJ Bevelacqua
Bevelacqua Resources, 7531 Flint Crossing Circle SE, Owens Cross Roads, AL 35763, USA

DOI: https://doi.org/10.62225/2583049X.2025.5.6.5294
Corresponding Author: **JJ Bevelacqua**

Abstract
The $\chi_b(2P)$ is investigated using a first-order tetraquark model assuming weakly interacting $B\bar{B}$ meson clusters. The $\chi_b(2P)$ model yields a $J^\pi = 0^+$ value in agreement with other analysis that also suggests 1^+ and 2^+ possible assignments. A mass of 10.154 GeV/c^2 is predicted by the model. This value is about 1.1% larger than the value suggested by the Brambilla *et al.* and the Belle Collaboration.

Keywords: $\chi_b(2P)$ Tetraquark, First-Order Mass Formula, Quark Model, Cluster Model

1. Introduction
Two decades ago the $\chi_{c1}(3872)$ state was observed in the spectrum with two heavy quarks [1]. This discovery fostered additional experimental research that led to the observation of additional exotic states, and theoretical investigations into new forms of matter (e.g., quark-gluon hybrids, tetraquarks, and pentaquarks). The tetraquarks included the $\chi_{c1}(3872)$ and $T_{cc}^{+}(3875)$.

As noted in Ref. 1, the discovery of the $\chi_{c1}(3872)$ in the charmonium spectrum was a significant event since it added a tetraquark to the collection of particle states that were previously comprised of mesons and baryons. The $\chi_{c1}(3872)$ had the form of a $q\bar{q}Q\bar{Q}$ where q is a light quark and Q is a heavy quark. Using the basis of this resonance, Brambilla *et al.* [1], predicted a new resonance in the bottomonium sector based on a $B\bar{B}$ bound state of a b meson and its antiparticle. Additionally, Ref. 1 predicted three bound states with masses 9.883, 10.236, and 10.519 GeV/c^2, respectively. These states were identified with the spin-averaged $\chi_b(1P)$, $\chi_b(2P)$, and $\chi_b(3P)$.

The $\chi_b(2P)$ is investigated in terms of a first-order tetraquark model. This model is based on the semiempirical mass formula of Zel'dovich and Sakharov [2, 3]. The first-order tetraquark model provides the $\chi_b(2P)$ mass as well its J^π value. Since the first-order model is limited in scope, it only permits a primitive angular momentum coupling structure. Other tetraquark systems [4-24] were reasonably described by the first-order model. In particular, the $\chi_b(2P)$ was also investigates as ω + ′Υ(1S) configuration [24].

2. Model and Formulation
The proposed first-order model is based on the semiempirical mass formula of Zel'dovich and Sakharov [2, 3]. This model assumes that two weakly bound meson clusters form the tetraquark, with zero angular momentum between the clusters.

The mesons (m) mass (M) is defined to have the form [2, 3]:

$$\mathbf{M_m = \delta_m + m_1 + m_2 + b_m\, [m_o^2 / (m_1\, m_2)]\, \sigma_1 \cdot \sigma_2} \qquad (1)$$

In Eq. 1, δ_m is defined to have the value 40 MeV/c^2 [3], and m_i is the mass of the quark comprising the meson cluster (i = 1 and 2). The average mass of a first generation quark (u and d) is m_o [25, 26]. The scalar product of the quark spin vectors ($\boldsymbol{\sigma_1 \cdot \sigma_2}$) is -3/4 and +1/4 for pseudoscalar and vector mesons, respectively [3].

Effective quark masses provided by Griffiths [25] are used to determine the meson cluster mass. The d, u, s, c, b, and t quarks have effective mass values of 340, 336, 486, 1550, 4730, and 177000 MeV/c^2, respectively [25]. Following the convention of the Standard Model, these quarks are grouped into three generations: [d(-1/3), u(+2/3)], [s(-1/3), c(+2/3)], and [b(-1/3), t(+2/3)] [25, 26]. The specific quark charges, in units of the unit charge e, are given within parentheses.

3. First-Order Mass Formula for the $B + \bar{B}$ Tetraquark Description of the $\chi_b(2P)$ Resonance

The first-order mass formula only provides a limited angular momentum coupling structure, and the spin of a tetraquark is derived from the angular momentum coupling of the two meson clusters:

$$\mathbf{J^\pi = J^\pi(1) \times L \times J^\pi(2)} \quad (2)$$

The first-order model summarized in Eq. 2 only provides a primitive angular momentum coupling structure for the J^π assignment, and the angular momentum between the clusters is zero (L = 0). These are limiting conditions of the model. Detailed meson cluster structural information, and strong coupling between the clusters are not included in the model formulation.

These aforementioned simplifications minimize model complexity, and permit the tetraquark mass formula to have the form [5-24]:

$$\mathbf{M = M_m(1) + M_m(2) + \Phi} \quad (3)$$

In Eq. 3, the two meson clusters are denoted by the numbers 1 and 2, and the individual meson cluster mass is given by Eq. 1. The final term in Eq. 3 (**Φ**) is the interaction between the meson clusters that is assumed to be negligible relative to the magnitude of the meson masses. Accordingly, Eq. 3 suggests a quasimolecular four quark system that is characteristic of a weakly bound meson-meson system.

The $\chi_b(2P)$ configuration is evaluated assuming weakly interacting B + $\bar{B}$ meson clusters. The $\chi_b(2P)$ is modeled as a 0^- $(b\bar{d})$ B meson cluster coupled to a 0^- $(\bar{b}d)$ $\bar{B}$ meson cluster. This tetraquark has a model assignment of $J^\pi = 0^+$ assignment (0^- x 0 x 0^- using Eq. 2). A 0^+ spin assignment was noted in Ref. 1 that is agreement with other analysis that also suggests 1^+ and 2^+ possible assignments [1].

The predicted first-order mass is based on Eq. 3:

$$\mathbf{M(B + \bar{B}) = M(B) + M(\bar{B}) + \Phi} \quad (4)$$

Using Eq. 4 and the first-order mass formula of Eq. 1, a mass of 10.154 GeV/c^2 is predicted. This value is about 1.3% larger than the value of Ref. 1. Ref. 24 also evaluated this state in terms of an ω + ϒ(1S) tetraquark and predicted a 10.371 GeV/c^2 mass. The first-order model is not sufficiently accurate to determine if the $B + \bar{B}$ or ω + ϒ(1S) configuration provides the best description of the $\chi_b(2P)$.

Table 1: Comparison of First Order Model Results and Analysis of Ref. 1

Configuration	Mass (GeV/c^2)	% Error (Relative to Ref. 1)
ω + ϒ(1S) (Ref. 24)	10.371	+1.3%
$B\bar{B}$ (This work)	10.154	-0.8%
$B\bar{B}$ (Ref. 1)	10.236	---

4. First-Order Tetraquark Model Uncertainties and Limitations

There are a number of uncertainties and limitations that affect the model results. The limited angular momentum coupling structure restricts the available J^π values that can be evaluated. In addition, assuming zero angular momentum between the clusters also limits the evaluation of possible states.

The values for the effective quark masses [25] are not definitive. Although the weak coupling assumption appears to be reasonable, the exact magnitude for the interaction strength between the clusters is unknown [4-24]. The coupling strength will likely depend on the physical properties of the interacting systems. In spite of these uncertainties, the model continues to provide reasonably credible results [4-24] for candidate tetraquark systems.

5. Conclusions

The $\chi_b(2P)$ mass and J^π value are investigated using a first-order tetraquark model. A B + $\bar{B}$ configuration is evaluated assuming weakly interacting meson clusters. The $\chi_b(2P)$ model yields a $J^\pi = 0^+$ value that is agreement with experiment, and Ref. 1 also included possible 1^+ and 2^+ values. A mass of 10.154 GeV/c^2 is predicted by the model. This value is about 0.8% smaller than the value suggested by Brambilla *et al.*

6. References

1. Brambilla N, *et al*. Nature of $\chi_{c1}(3872)$ and $T_{cc}^{+}(3875)$, Phys. Rev. Lett. 2025; 135:131902.
2. Ya B Zel'dovich, Sakharov AD. Kvarkovaia struktura i massy sil'novzaimodeistvuyushchikh chastits, Yad. Fiz. 1966; 4:395.
3. Sakharov AD. Mass formula for mesons and baryons, Sov. Phys. JETP. 1980; 51:1059.
4. Bevelacqua JJ. First-Order Tetraquark Mass Formula, Physics Essays.2016; 29:198.
5. Bevelacqua JJ. Description of the X(5568) and Proposed 750 GeV/c^2 State in Terms of a First-Order Tetraquark Mass Formula, Physics Essays. 2016; 29:367.
6. Bevelacqua JJ. Fusion of Doubly Heavy Mesons into a Tetraquark, Physics Essays. 2018; 31:167.
7. Bevelacqua JJ. Possible Tetraquark Explanation for the Proposed X(3872), Physics Essays. 2019; 32:469.
8. Bevelacqua JJ. Description of the X(6900) as a Four Charmed Quark State in Terms of a First-Order Tetraquark Mass Formula, Qeios **KLXLKJ**. 2020; 1. Doi: https://doi.org/10.32388/KLXLKJ
9. Bevelacqua JJ. Description of the X(2900) as an Open Flavor Tetraquark in Terms of a First-Order Mass Formula, Qeios, **OVLMEB**. 2020; 1. Doi: https://doi.org/10.32388/OVLMEB
10. Bevelacqua JJ. Possible Tetraquark Explanation for the Proposed $Z_{cs}(3985)^-$, Qeios **GLTEU2**. 2021; 1. Doi: https://doi.org/10.32388/GLTEU2
11. Bevelacqua JJ. Possible Tetraquark Explanation for the X(6200), Qeios **J6AFYW**. 2021; 1. Doi: https://doi.org/10.32388/J6AFYW
12. Bevelacqua JJ. Possible Tetraquark Explanation for the T_{cc}^+, Qeios **OMDGAQ**. 2021; 1. Doi: https://doi.org/10.32388/OMDGAQ
13. Bevelacqua JJ. Possible Tetraquark Explanation for the Proposed $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$, Qeios **PPLMWV**. 2021; 1. Doi: https://doi.org/10.32388/PPLMWV
14. Bevelacqua JJ. Possible Tetraquark Explanation for the Proposed X(3960), Qeios, **O1L0YM**. 2022; 1. Doi: https://doi.org/10.32388/O1L0YM
15. Bevelacqua JJ. Possible Tetraquark Explanation for the Proposed $T(2900)^{++}$ and $T(2900)^0$ Structures, Qeios **V6WLTS**. 2022; 1. Doi: https://doi.org/10.32388/V6WLTS

16. Bevelacqua JJ. Possible K K bar Tetraquark Explanation for the $f_0(1370)$, Qeios **HBDQXV**. 2023; 1. Doi: https://doi.org/10.32388/HBDQXV
17. Bevelacqua JJ. Possible f Quark Model of Tetraquarks and Pentaquarks, Qeios, **8T3IVE**. 2023; 1. Doi: https://doi.org/10.32388/8T3IVE
18. Bevelacqua JJ. Possible Tetraquark Explanation for the Y(10753), Qeios **NZRGH3**. 2023; 1. Doi: https://doi.org/10.32388/NZRGH3
19. Bevelacqua JJ. Possible Tetraquark Explanation for the ψ(4230), ψ(4360), and ψ(4415), Qeios **D5HKO0**. 2024; 1. Doi: https://doi.org/10.32388/D5HKO0
20. Bevelacqua JJ. Possible Tetraquark Explanation for the $J/\psi K^0_S$ Structure Observed in Proton-Proton Collision Data at Center-of-Mass Energies of 7, 8, and 13 TeV, Qeios **YFT8L5**. 2024; 1. Doi: https://doi.org/10.32388/YFT8L5
21. Bevelacqua JJ. Possible Explanations for the Proposed J/Ψ + J/Ψ and J/Ψ + Ψ(2S) Tetraquark States, Qeios **W2A4LD**. 2024; 1. Doi: https://doi.org/10.32388/W2A4LD
22. Bevelacqua JJ. Possible Tetraquark Explanations for the B D-bar and B-star D-bar States Proposed by Lattice QCD Calculations, Qeios **8MWBEY**. 2024; 1. Doi: https://doi.org/10.32388/8MWBEY
23. Bevelacqua JJ. Possible Tetraquark Explanations for the R(3780), Qeios **WRVP7E**. 2025; 1. Doi: https://doi.org/10.32388/WRVP7E
24. Bevelacqua JJ. Possible ω ΄Y(1S) Tetraquark Explanation for the $\chi_{b0}(2P)$, Int. J. Adv. Multidisc. Res. Stud. 2025; 5(6):35.
25. Griffiths D. Introduction to Elementary Particles, 2nd ed., Wiley-VCH, Weinheim, 2008.
26. Particle Data Group, Review of Particle Physics, Phys. Rev. D. 2024; 110:30001.