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Decay Characteristics of Neutron Excess Copper Nuclei

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Abstract
The properties of neutron excess copper nuclei are determined utilizing a single-particle model. The single particle model calculations include alpha, beta, positron, electron capture, and spontaneous fission decay modes. Neutron emission decay modes that have short half-lives are not readily determined by the model. However, estimates of the neutron decay mode were evaluated using the methodology of Chowdhury *et al*. Using that model, spontaneous neutron emission is predicted to occur in the range of A = 91 – 93. The Japanese Nuclear Data Compilation calculations terminate their calculations at A = 87. Given these results, single-particle model calculations are extended to encompass these values, and were extended to A = 95.
Single particle model calculations predict that A = 82 – 95 neutron excess copper systems form bound systems that have limiting beta decay half-lives in the range of 1.40 – 44.3 ms. Model half-life results for the A = 82 – 87 copper nuclei are within a factor of about two of the predictions of the Japanese Nuclear Data Compilation calculations.

Keywords: Nucleosynthesis, Neutron Excess Copper Nuclei, Beta Decay, Nuclear Structure, Spontaneous Neutron Emission

1. Introduction

Interest in neutron excess nuclei [1-31] has stimulated both experimental and theoretical physics interest. Several physical processes generate neutron excess nuclei, but the r-process usually provides the most significant contribution. Production of neutron excess nuclei in mergers of astrophysical objects (e.g., black holes and neutron stars) is an active area of research in nuclear physics and astrophysics [1,2].

This paper continues the investigation of neutron excess nuclei by focusing on the Z = 29 copper systems. Neutron excess systems having Z = 9 - 28, and 30 were discussed in previous work [8-25, 29-31]. Studies of these systems provide additional insight into nuclear systematics involving the various nucleosynthesis mechanisms and decay modes, and their associated variation with atomic and mass numbers.

2. Calculational Methodology

Methods for investigating neutron excess nuclei are provided in Refs. 8-31. This paper follows the single particle methodology of Lukasiak and Sobiczewski [27] and Petrovich *et al*. [28]. Single particle energies of neutron excess nuclear systems are obtained by incorporating the numerical methods of Refs. 32 and 33.

The radial Schrödinger equation is utilized to determine binding energy of a nucleon interacting with a nuclear core [8-25, 29-31]

$$[(\hbar^2/2\mu)\,(d^2/dr^2 - L(L+1)/r^2) - E_{NLSJ} - V_{LSJ}(r)]\,U_{NLSJ}(r) = 0 \quad (1)$$

In Eq. 1, E_{NLSJ} is the nucleon binding energy, r is the radial coordinate, $V_{LSJ}(r)$ is the nuclear interaction, and $U_{NLSJ}(r)$ is the radial wave function. L, S, and J represent the orbital, spin, and total angular momentum quantum numbers, respectively. The model definition is completed by defining the radial quantum number (N) and reduced mass (μ).

3. Nuclear Interaction

The Rost interaction [34] is selected for the nuclear interaction. This interaction has a central strength:

$$V_0 = 51.6\,[1 \pm 0.73\,(N - Z)/A]\ \text{MeV} \quad (2)$$

In Eq. 2, the positive (negative) sign is assigned to protons (neutrons). The spin-orbit interaction strength (V_{so}) is defined in terms of the central interaction strength and the multiplier γ [34]:

$$V_{so} = \gamma \, V_0 / 180 \quad (3)$$

Inclusion of the pairing correction interaction of Blomqvist and Wahlborn [35] completes the definition of the model interaction.

The difficulties in defining an appropriate nuclear interaction are outlined in Refs. 36 and 37. Ray and Hodgson [36] and Schwierz, Wiedenhöver, and Volya [37] note that modifications, unique to each nuclear system, are required to ensure an accurate representation of the experimental energy levels and decay characteristics. In view of the conclusions of Refs. 36 and 37 and the results of previous excess neutron system calculations [8-25, 29-31], the Rost central interaction strength (V_A) is modified in the following manner.

$$V_A = V_0 \, \lambda \, [1 \pm a(A)] \text{ MeV} \quad (4)$$

Individual nuclear system characteristics are defined by incorporating a potential strength multiplier (λ) and a factor [a(A)] to adjust the potential strength as a function of A. For copper systems, the multiplier λ is selected to have the value of 1.5. This multiplier value is consistent with previous excess neutron nuclei calculations [8-25, 29-31] that provided model results in agreement with available data [38-40].

4. Model Limitations

Previous calculations [8-25, 29-31, 41] provided a representative description of the various nuclear decay modes (e.g., alpha, beta, positron, electron capture, and spontaneous fission) that could be encountered in neutron excess nuclei. Neutron excess systems can also decay by neutron emission modes that are not well-described by single-particle models. Since these neutron emission modes have very short half-lives, single-particle models will likely overestimate the lifetimes of neutron excess nuclei.

The onset of spontaneous neutron emission was estimated using the methodology of Chowdhury *et al.* [42]. Using the approach of Ref. 42, the onset of spontaneous neutron emission was estimated to occur in the range of A = 91 - 93 for copper systems, Although independent of the calculated neutron single particle levels presented herein, the approach of Chowdhury *et al.* is a logical value to terminate calculations for excess neutron copper nuclei. The Japanese Nuclear Data Compilation terminate their calculations at A = 87. These results guide the A value selected to terminate the single-particle model calculations for copper.

5. Results and Discussion

Table 1 summarizes the complete set of 95 ≥ A ≥ 70 copper isotopes considered in this paper. The 95 ≥ A ≥ 70 copper nuclei occupy a variety of neutron shells that are noted in Table 1. The heaviest observed copper system is ^{81}Cu [40]. In view of the paucity of experimental data, extrapolations of nuclear characteristics beyond A > 81 become more uncertain.

5.1 81 ≥ A ≥ 70 Copper Isotopes with Experimental Half-Life Data

The limiting decay mode (i.e., the transition that has the shortest decay half-life) for 70 ≥ A ≥ 81 copper isotopes observed experimentally is summarized in Table 1. For example, the ^{71}Cu calculations predict five beta decay transitions (i.e., allowed $1f_{5/2}(n)$ to $1f_{5/2}(p)$ [8.24 min], allowed $2p_{1/2}(n)$ to $2p_{13/2}(p)$ [19.5 s], allowed $2p_{3/2}(n)$ to $2p_{3/2}(p)$ [5.31 min], allowed $2p_{1/2}(n)$ to $2p_{1/2}(p)$ [6.96 min], and first forbidden $1g_{9/2}(n)$ to $1f_{5/2}(p)$ [39.0 min]). For ^{71}Cu, the allowed $2p_{1/2}(n)$ to $2p_{3/2}(p)$ [19.5 s] transition is the limiting beta decay mode.

Table 1: Calculated Single-Particle and Experimental Decay Properties of Copper Nuclei with 70 ≤ A ≤ 95

Nuclide /neutron shell	a(A)	Half-Life (Decay Mode)	
		Experiment [a,b,c]/Theory [d]	This Work
^{70}Cu H 1g9/2	-0.0319	44.5 s [c]	44.2 s (β^-) [e]
^{71}Cu F 1g9/2	-0.0346	19.5 s [c]	19.5 s (β^-) [e]
^{72}Cu F 1g9/2	-0.0330	6.63 s [c]	6.60 s (β^-) [e]
^{73}Cu G 1g9/2	-0.0362	4.2 s [c]	4.20 s (β^-) [e]
^{74}Cu E 1g9/2	-0.0312	1.63 s [c]	1.63 s (β^-) [e]
^{75}Cu F 1g9/2	-0.0346	1.224 s [c]	1.22 s (β^-) [e]
^{76}Cu F 1g9/2	-0.0306	641 ms [c]	642 ms (β^-) [e]
^{77}Cu F 1g9/2	-0.0318	467.9 ms [c]	467 ms (β^-) [e]
^{78}Cu E 1g9/2	-0.0313	330.9 ms [c]	331 ms (β^-) [e]
^{79}Cu E 1g9/2	-0.0310	240.0 ms [c]	240 ms (β^-) [e]
^{80}Cu F 2d5/2	-0.0172	113.6 ms [c]	114 ms (β^-) [e]
^{81}Cu F 2d5/2	-0.0098	73.2 ms [c]	73.2 ms (β^-) [e]
^{82}Cu C 2d5/2	0.0019	18.8 ms [d]	44.3 ms (β^-) [e]
^{83}Cu C 2d5/2	0.0125	12.2 ms [d]	29.1 ms (β^-) [e]
^{84}Cu C 2d5/2	0.0231	10.1 ms [d]	20.0 ms (β^-) [e]
^{85}Cu C 2d5/2	0.0337	7.52 ms [d]	14.2 ms (β^-) [e]
^{86}Cu C 1g7/2	0.0443	6.30 ms [d]	10.4ms (β^-) [e]
^{87}Cu C 1g7/2	0.0549	4.40 ms [d]	7.79 ms (β^-) [e]
^{88}Cu C 1g7/2	0.0655	[f]	5.98 ms (β^-) [e]
^{89}Cu C 1g7/2	0.0761	[f]	4.67 ms (β^-) [e]
^{90}Cu C 1g7/2	0.0867	[f]	3.70 ms (β^-) [e]
^{91}Cu C 1g7/2	0.0973	[f]	2.98 ms (β^-) [e]
^{92}Cu C 1g7/2	0.1079	[f]	2.42 ms (β^-) [e]
^{93}Cu C 1g7/2	0.1185	[f]	2.00 ms (β^-) [e]
^{94}Cu C 3s1/2	0.1291	[f]	1.66 ms (β^-) [e]
^{95}Cu C 3s1/2	0.1397	[f]	1.40 ms (β^-) [e]

[a]Ref. 38. [b]Ref. 39. [c]Ref. 40. [d]Japanese data Compilation calculation.
[e]Allowed $2p_{1/2}(n)$ to $2p_{3/2}(p)$ beta decay transition. [f]No data provided in Refs. 38 - 40.

The model predicts the proper decay mode for the known 81 ≥ A ≥ 70 copper systems [38-40]. As noted in Table 1, the model half-lives are also consistent with data [38-40].

^{70}Cu – ^{79}Cu nuclei occupy the $1g_{9/2}$ neutron shell. These systems decay through allowed $2p_{1/2}(n)$ to $2p_{3/2}(p)$ beta transitions. Model predictions for the beta decay half-lives of ^{70}Cu – ^{79}Cu are within 0.7% of the experimental values [40]. In addition beta decay is the predicted decay mode in agreement with Ref. 40.

The ^{80}Cu – ^{81}Cu nuclei partially fill the $2d_{5/2}$ neutron shell. These systems decay through allowed $2p_{1/2}(n)$ to $2p_{3/2}(p)$ beta transitions. Only ^{80}Cu and ^{81}Cu have measured half-lives. The half-life values of the ^{80}Cu and ^{81}Cu systems are within 0.4% of the data [40]. Model calculations also predict the correct decay mode of these $2d_{5/2}$ copper nuclei.

5.2 95 ≥ A ≥ 82 Copper Isotopes without Experimental Half-Life Data

The a(A) values for 82 ≥ A ≥ 95 copper isotopes were obtained from a linear fit based on the half-lives of ^{79}Cu – ^{81}Cu. The resulting a(A) values are listed in Table 1.

^{82}Cu – ^{85}Cu complete the $2d_{5/2}$ neutron shell, and decay through allowed $2p_{1/2}$(n) to $2p_{3/2}$(p) beta decay transitions. These copper systems have beta decay half-lives between 14.2 and 44.3 ms. Single-particle calculations are within a factor of 2.5 of the Japanese Data Compilation calculations [40].

^{86}Cu – ^{93}Cu nuclei fill the $1g_{7/2}$ neutron shell, and have beta decay half-lives in the range of 2.00 to 10.4 ms. These systems decay through allowed $2p_{1/2}$(n) to $2p_{3/2}$(p) beta decay transitions. The model results for ^{86}Cu and ^{87}Cu within about a factor of two of the Japanese Data Compilation calculations [40]. ^{87}Cu is the last copper system included in the Japanese Data Compilation calculations [40].

^{94}Cu and ^{95}Cu fill the $3s_{1/2}$ neutron shell, and these systems decay through allowed $2p_{1/2}$(n) to $2p_{3/2}$(p) beta decay transitions. The half-lives of ^{94}Cu and ^{95}Cu are 1.66 and 1.40 ms, respectively.

The copper calculations are terminated at A = 95 to account for the spontaneous neutron emission as the mass increases. Using the methodology of Chowdhury *et al.* [42], spontaneous emission is expected to occur at A = 91 – 93. The Japanese Data Compilation calculations [40] suggest that A = 87 terminates the copper systems. Model calculations were extended to A = 95 to account for uncertainties in estimates of Ref. 40 and 42.

6. Conclusions

Single-particle model calculations readily incorporate alpha, beta, positron, electron capture, and spontaneous fission decay modes. Neutron emission decay modes have short half-lives that are not readily determined by a single-particle model. However, estimates on the neutron decay mode were evaluated using the methodology of Chowdhury *et al*. Using that model, spontaneous neutron emission is predicted to occur in the range of A = 91 – 93. The Japanese Nuclear Data Compilation calculations terminate their calculations at A = 87. Given these results, single-particle model calculations are extended to encompass these values, and were extended to A = 95.

Single particle model calculations predict that A = 82 – 95 neutron excess copper systems form bound systems that have limiting beta decay half-lives in the range of 1.40 – 44.3 ms. Model half-life results for the A = 82 – 87 copper nuclei are within about a factor of two of the predictions of the Japanese Nuclear Data Compilation calculations.

These neutron excess copper systems decay through allowed $2p_{1/2}$(n) to $2p_{3/2}$(p) beta decay transitions. The model likely overestimates the actual half-life values, because it does not explicitly include the short-lived neutron emission decay modes.

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