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On ∝ - Soft Sets and ∝ - Soft Continuous Mappings

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Abstract
In this search we will define ∝ - soft set, restriction of ∝ - soft set, ∝ - soft map, study the continuity on ∝ - soft map and the restriction of ∝ - soft continuous map.

Keywords: Soft Sets, ∝ - Soft Sets ∝ - Soft Map ∝ - Soft Continuous Map Restriction of ∝ - Soft Continuous Map

1. Introduction

Soft set was introduced in [4] by Molodtsov. Maji and Roy [3] introduced operations on soft sets, new operations in soft set was introduced in [2] by Irfan and Feng, in [1, 6, 7, 8] properties of soft topological spaces has been studied. Soft mapping defined in [9] by Wardowski, soft continuous mapping types study in [5].
In this search we will define the restriction of ∝ - soft set in two ways,
Then, define the restriction mappings over an ∝ - soft set, define and study the restriction of ∝ - soft continuous mapping on ∝ - soft open, ∝ - soft close maps, we would like to mention that all ∝ - soft sets in this search are on the same set of parameters P, denoted by $S(M)_P$, through this search symbols (A,P) equals to $\tilde{A}$ and (F,P) equals to $\tilde{F}$.

Definition 1.1 [5, 7]

1. The soft set (F,P) over M is said to be a (null soft set) denoted by $\tilde{\Phi}$, if $\forall e \in P$, $F(e) = \Phi$, (Φ the null set).
2. The soft set (F,P) over M is said to be an (absolute soft set) denoted by $\tilde{M}$ if $\forall e \in P$, $F(e) = M$.
3. Let (F,A), (G,B) be two soft sets in $S(M)_P$, we say that (F,A) is a soft subset of (G,B) denoted (F,A) $\tilde{\subseteq}$ (G,B) if (i) $A \subseteq B$ and (ii) $F(e) \subseteq G(e)$ $\forall e \in A$.
4. The soft difference (H,P) of two ∝ - soft sets (F,P) and (G,P) over M denoted by (F,P)$\tilde{\setminus}$ (G,P) is defined as H(e) = F(e) \ G(e) for all $e \in P$.
5. The union of two soft sets of (F,A) and (G,B) over the common universe M is the soft set (H,C), where C= A∪B, and ∀ $e \in$ C we write (F,A) $\tilde{\cup}$ (G,B)= (H,C),

$$H(e) = \begin{cases} F(e) & \text{if } e \in A \setminus B \\ G(e) & \text{if } e \in B \setminus A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

6. The intersection of two soft sets (F,A) and (G,B) over a common universe set M is the soft set (H,C), where $C = A \cap B$ and $\forall e \in C$, $H(e) = F(e) \cap G(e)$, we write (F,A) $\tilde{\cap}$ (G,B) = (H,C).

Definition 1.2 [10]
Let $\tilde{\tau}$ be the collection of soft sets over M then $\tilde{\tau}$ is soft topology on M if:

1. $\tilde{\Phi}$, $\tilde{M}$ belong to $\tilde{\tau}$.
2. The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
3. The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The triple (M,$\tilde{\tau}$, P) is called a soft topological space over M.
$\tilde{\tau}$o-set for soft open sets and $\tilde{\tau}$c-set for soft closed sets.

Definition 2.1
The set $\tilde{F}$ (or denoted (F, P)) in soft topological space (M,$\tilde{\tau}$, P) is called

1. (∝o-soft set) if $\tilde{F} \subseteq$ int [cl (int ($\tilde{F}$))].
2. (∝c-soft set) if: Cl [int (cl ($\tilde{F}$))] $\subseteq \tilde{F}$.

Remarks 2.2
1. int ($\widetilde{B}$)) is the largest soft open set contained in $\widetilde{B}$.
2. cl ($\widetilde{B}$)) is the smallest soft closed set contain $\widetilde{B}$.
3. The complement of ∝o-soft set is ∝c- soft set and vice versa
4. From definition every o- soft set is ∝o-soft set but the converse is not true in general.

2. The restriction of ∝ - soft set on A ⊂ P (P is the set of parameters)
Definition 2.1
Let M the universal set, (F,P) is ∝ - soft set where F:P→P(M) the restriction of (F,P) with respect to the set of parameters A ⊂ P is a soft set (G,P) where G:P\A→P(M), F(e) = G(e) ∀ e ∈ A.

Example 2.2
Let D = {Wight, Read, Green} represent three colors that each box b_1, b_2 and b_3 consist, P the set of parameters represent the set of three boxes = { b_1, b_2, b_3},
the ∝ - soft set (F,P) represent the scientific information's that each box consist
(F,P) = {(b_1, {Wight}), (b_2, {Wight, Read}), (b_3, {D})}
Means that the box1 contains Wight only, b_2 contains Wight and Read, b_3 contains all Wight, Read and Green. Let A = { b_1, b_2} ⊂ P, k:P\A→ P(D).
Then, (k,A) = {(b_1, {Wight}), (b_2, { Wight, Read})}is the restriction of (F,P) with respect to A which represent only two boxes b_1, b_2.

Corollary 2.5
Let M be the universal set, for a given ∝- soft set the restriction ∝- soft set (with respect to A ⊂ P) is a special case of ∝ - soft subse.
Proof: Let (F,P) is ∝ - soft set where F:P→P(M), (G,A) is the restriction of (F,P) with respect to the set of parameters
A ⊂ P where G:P\A→P(M), since (i) A ⊆ P and (ii) G(e) ⊆ F(e) ∀ e ∈ A. Then, (G,A) ⋐ (F,P).

Propositions 2.6
Let M be the universal set, the ∝ - soft union of a ∝ - soft set with restriction ∝ - soft set (with respect to A ⊂ P) is an ∝ - soft set.
Proof: Let (F,P) is ∝ - soft set where F:P→P(M), (G,A) is the restriction of (F,P) with respect to the set of parameters A ⊂ P where k:P\A→P(M),
since x∈A ∩ P and F(x) ∪ k(x) = F(x). Then, (F,P) ⋓ (k, A) = (F,P) which is an ∝ - soft set.

Proposition 2.7
Let M be the universal set, the ∝ - soft intersection of ∝ - soft set with the restriction ∝ - soft set (with respect to A ⊂ P) is the restriction ∝ - soft set.
Proof: Let (F,P) is ∝ - soft set where F:P→P(M), (G,A) is the restriction of (F,P) with respect to the set of parameters A ⊂ P where G:P\A→P(M), since F(e) = G(e) ∀ e ∈ A. Then, (F,P) ⋒ (G,A) = (G,A) which is an ∝ - soft set.

3. The restriction of ∝ - soft set with respect to A ⊂ M (M is the universal set)
Definition 3.1
Let M be a universal set let (F,P) be a ∝ - soft set where F:P→p(M) (p = power of a sets) the restriction of ∝ - soft set (F,P) with respect to a subset A of a universal set M is a soft set (G,P) where G: P → F(e) ∩ A ∀ F(e) ∈ p(M) where G(e) = F (e) ∩ A, ∀ e∈ P.

Example 3.2
Let M = {a, b, c} be a set of three cars, P the set of parameters represent the set of three colors = {w_1, w_2, w_3} $\tilde{\tau} = \{\tilde{\Phi}, \widetilde{M}, (F,P)\}$, where (F,P) = {($w_1${a, b, c}), ($w_2${a, c}), ($w_3$,{ Φ, Φ, Φ})}is ∝ - soft set
So (G,P) = {(w_1, {a, b}), (w_2, {a, Φ, Φ}), (w_3, {Φ, Φ, Φ})} is a restriction of ∝ - soft set (F,P) with respect to A = {a,b}.

Proposition 3.4
Let M be the universal set, for a given ∝ - soft set the restriction ∝ - soft set (with respect to A ⊂ M) is a soft subset.
Proof: Let (F,P) is ∝ - soft set where F:P→P(M), (G,P) is the restriction of (F,P) with respect to A ⊂ M where G : P → F(e) ∩ A , ∀ F(e) ∈ P(M), since (i) P ⊆ P and (ii) G(e) ⊆ F(e) ∀ e ∈ A. Then, (G,A) ⋐ (F,P).

Proposition 3.5
Let M be the universal set, the ∝ - soft union of a ∝ - soft set with restriction ∝ - soft set (with respect to A ⊂ M) is an ∝ - soft set.

Proof: Let (F,P) is $\propto$ - soft set where F:P$\rightarrow$P(M), (G,P) is the restriction of (F,P) with respect to A $\subset$ M where $G : P \rightarrow F(e) \cap A, \forall F(e) \in P(M)$, since e$\in$ P $\cap$ P and F(e) $\cup$ G(e) = F(e). Then, (F,P) $\breve{\cup}$ (G,P) = (F,P).

Proposition 3.6
Let M be the universal set, the $\propto$ - soft intersection of $\propto$ - soft set with the restriction $\propto$ - soft set (with respect to A $\subset$ M) is the restricted $\propto$ - soft set.
Proof: Let (F,P) is $\propto$ - soft set where F:P$\rightarrow$P(M), (G,P) is the restriction of (F,P) with respect to A $\subset$ M where $D : P \rightarrow F(e) \cap A, \forall F(e) \in P(M)$, since D(e) = F(e)$\cap$A= F(e) $\forall$ e $\in$ A then (D,P) $\tilde{\cap}$ (F,P) = (D,P),$\forall$ e $\in$ P.

4. Restriction of $\propto$ - soft mappings
Definition 4.1
Let (F,P),(G,P) are soft sets in $S(M)_P$, A soft relation $\tilde{f} \tilde{\subseteq}$ (F,P) $\tilde{\times}$ (G,P) is called a soft mapping from (F,P) to (G,P) [denoted by $\tilde{f}$:(F,P) $\Rightarrow$ (G,P)] if the following two conditions are satisfied:
(i) for each soft element $\tilde{x} \tilde{\in}$ (F,P), there exists only one soft element $\tilde{y} \tilde{\in}$ (G,P) such that $\tilde{x} \tilde{f} \tilde{y}$ which will be noted as $\tilde{f}(\tilde{x}) = \tilde{y}$,
(ii) for each empty soft element $\tilde{x} \tilde{\in}$ (F,P), $\tilde{f}(\tilde{x})$ is an empty soft element of (G,P).

Definition 4.2
Let $(M,\tilde{\tau},P)$ be an $\propto$ - soft topological space over M, (F,P) be a non-empty $\propto$ - soft subset of $(M,\tilde{\tau}, P)$. Then,
$\tilde{\tau}_{\tilde{F}} = \{(F, P)^* = (F,P) \tilde{\cap} (G, P), \forall (G,P) \tilde{\in} \tilde{\tau}\}$ is the $\propto$ - soft relative topology on (F,P) and $((F, P), \tilde{\tau}_{\tilde{F}}, P)$ is called $\propto$ - soft subspace topology of $(X,\tilde{\tau},P)$.

Definition 4.3 [3]
The mapping $\tilde{f} : (M, \tilde{\tau}_M, P) \Rightarrow (Y, \tilde{\tau}_y, P)$ is $\propto$ - soft open ($\propto$ - soft close) mapping between two $\propto$ - soft topological spaces if the image of each $\propto$ - soft open ($\propto$ - soft close) set over a universal set M is a $\propto$ - soft open set ($\propto$ - soft close set).

Theorem 4.4
The restriction of $\propto$ - soft open ($\propto$ - soft close) map over a $\propto$ - soft open ($\propto$ - soft close) set is $\propto$ - soft open ($\propto$ - soft close) map.
Proof: Let $\tilde{f}$: $(M, \tilde{\tau}_M, P) \Rightarrow (Y,\tilde{\tau}_y, P)$ be a $\propto$ - soft open mapping between two $\propto$ - soft topological spaces, (A,P) be a $\propto$ - soft open set in $(M, \tilde{\tau}_M, P)$.
To prove $\tilde{f} | (A,P),\tilde{\tau}_{\tilde{A}}, P):((A,P),\tilde{\tau}_{\tilde{A}}, P) \Rightarrow (Y,\tilde{\tau}_Y, P)$ is $\propto$ - soft open map, (G, P) be a $\propto$ - soft open subset in (A, P) then (G, P) $\tilde{\cap}$ (A, P) = (G, P) is $\propto$ - soft open set in $\tilde{\tau}_M$ by $\propto$ - soft relative topology definition, since $\tilde{f}$ is $\propto$ - soft open map then
$\tilde{f}((G,P))$ is $\propto$ - soft open in$\tilde{\tau}_Y$ so prove $\tilde{f} | ((A,P), \tilde{\tau}_{\tilde{A}}, P)$ is $\propto$ - soft open map, similarly for $\propto$ - close map.

Example 4.5
Let M = $\{h_1, h_2\}$, C = $\{c_1, c_2\}$, P= $\{e_1$, Blue$\}$, the $\propto$ - soft map
$\tilde{f}$:$(M,\tilde{\tau}_d,P) \Rightarrow (C,\tilde{\tau}_d,P)$ defined as follow:
$\tilde{f}((e_i,\{h_j\})) = ((e_i,\{c_j\})), \forall i,j = 1,2$ is $\propto$ - soft open and $\propto$ - soft closed map.

Definition 4.6 [11]
$\tilde{f}$: $(M,\tilde{\tau}_M,P) \Rightarrow (Y,\tilde{\tau}_Y,P)$ is a $\propto$ - soft continuous mapping if for each $\propto$ - soft open set (G,P) over Y, $\tilde{f}^{-1}((G,P))$ is a $\propto$ - soft open set over M.

Theorem 4.7
The restriction of $\propto$ - soft continuous mapping over the $\propto$ - soft open set ($\propto$ - soft close set) is $\propto$ - soft continuous.
Proof: Let $\tilde{f}$: $(M,\tilde{\tau}_M,P) \Rightarrow (Y,\tilde{\tau}_Y,P)$ be a $\propto$ - soft continuous mapping between two $\propto$ - soft topological spaces, (A,P) be a $\propto$ - soft open set in $(M,\tilde{\tau}_M,P)$, the restriction of $\tilde{f}$ on (A,P) is the mapping
$\tilde{f} |((A,P),\tilde{\tau}_{\tilde{A}}, P): ((A,P),\tilde{\tau}_{\tilde{A}}, P) \Rightarrow (Y,\tilde{\tau}_Y, P)$, to prove
$\tilde{f} |((A,P),\tilde{\tau}_{\tilde{A}}, P)$ is $\propto$ - soft continuous map, let (G,P) be a $\propto$ - soft open set in $\tilde{\tau}_Y$.
Then, $(\tilde{f} |((A,P),\tilde{T}_{\tilde{A}}, P))^{-1} ((G, P)) = \tilde{f}^{-1}((G,P)) \tilde{\cap}(A, P)$
since $\tilde{f}$ is $\propto$ - soft continuous then $\tilde{f}^{-1}((G,P))$ is $\propto$ - soft open set in $\tilde{\tau}_M$
by $\propto$ - soft relative topology definition and the intersection of wo $\propto$ - soft open is $\propto$ - soft open so $\tilde{f}^{-1}((G,P)) \tilde{\cap}(A, P)$ is $\propto$ - soft open in $\tilde{\tau}_{\tilde{A}}$,
so $\tilde{f} |((A,P),\tilde{\tau}_{\tilde{A}}, P)$ is $\propto$ - soft continuous map, similarly for close case.

Example 4. 8
Let M = {house1, house2, house3}, P = {Wight, Read} and
$\tilde{\tau} = \{\tilde{\Phi}, \tilde{M}$, {(Wight, {house2}), (Read,{house1}) }, {(Wight,{house1, house3}), (Read,{house2, house3}) },{(Wight, {house2}), (Read, M)}, {(Wight,Φ), (Read, {house1})}, {(Wight, {house1, house3}), (Read, M)},{(Wight,Φ),(Read,{house2, house3})},{ (Wight,Φ), (Read, M)}} and
$\tilde{\tau}' = \{\tilde{\Phi}, \tilde{M}$, {(Wight,{house2}),(Read,{house1})},{(Wight,{house2, house3}), (Read,{house1, house2})}, {(Wight,{house1, house2}), (Read, M)},{(Wight, {house2}), (Read,{house1,house2})}}, let $\tilde{f}$: $(M,\tilde{\tau}, P) \Rightarrow (M,\tilde{\tau}',P)$ be a mapping defined as
$\tilde{f}$((Wight, house λ)) = (Wight, house1), λ = 1,2,3 and
$\tilde{f}$((Read, house λ)) = (Read, house1), λ = 1,2,3. Then, $\tilde{f}$ is a $\propto$ - soft continuous.

Let $\widetilde{A}$={(Wight,Φ), (Read, {house1})} be a $\propto$ - soft subset of $\tilde{\tau}$.
Then, $\tilde{\tau}_{\widetilde{A}}$ = {(Wight,Φ), (Read, Φ), {(Wight,Φ), (Read, {house1})}} = { $\widetilde{\Phi}_A$, $\widetilde{A}$ }
$\widetilde{K}(\tilde{x}) = \tilde{f}(\tilde{x})$ $\forall$ $\tilde{x}$ in ((A,P),$\tilde{\tau}_{\widetilde{A}}$, P).
Then, $\widetilde{K} = \tilde{f}$ | ((A,P),$\tilde{\tau}_{\widetilde{A}}$,P): ((A,P),$\tilde{\tau}_{\widetilde{A}}$,P) $\Rightarrow$ (X,$\tilde{\tau}'$,P) is $\propto$ - soft continuous map since
$\widetilde{K}^{-1}(\widetilde{\Phi}_A) = \widetilde{\Phi}_A \in \tilde{\tau}_A$, $\tilde{g}^{-1}(\widetilde{M}) = \tilde{g}^{-1}$ ({(Wight,M), (Read, M)}) = {(Wight,M), (Read, M)} $\widetilde{\cap}$ $\widetilde{A} = \widetilde{A} \in \tilde{\tau}_A$
$\widetilde{K}^{-1}$ ({(Wight, {house2}), (Read, {house1})}) = {(Wight,Φ), (Read, M)} $\widetilde{\cap}$ $\widetilde{A} = \widetilde{A} \in \tilde{\tau}_A$
$\widetilde{K}^{-1}$ ({(Wight, {house2 house3}), (Read, {house1 house2})}) = {(Wight, Φ), (Read, {house1})} $\widetilde{\cap}$ $\widetilde{A} = \widetilde{A} \in \tilde{\tau}_A$
$\widetilde{K}^{-1}$ ({(Wight, {house1 house2}), (Read, M)}) = {(Wight, {house1}), (Read, {house1})} $\widetilde{\cap}$ $\widetilde{A} = \widetilde{A} \in \tilde{\tau}_A$
$\widetilde{K}^{-1}$ ({(Wight, {house2}), (Read, {house1 house2})}) = {(Wight, Φ), (Read, {house1})} $\widetilde{\cap}$ $\widetilde{A} = \widetilde{A} \in \tilde{\tau}_A$

5. Conclusion

Any $\propto$ - soft set (F,P) can be restriction in two ways: First method - The restriction of $\propto$ - soft set (F,P) on A $\subset$ P (P is the set of parameters), Second method-The restriction of $\propto$ - soft set (F,P) with respect to A $\subset$ M (M is the universal set) For a $\propto$ - soft map $\tilde{f}$:(X,$\tilde{\tau}_X$, P) $\Rightarrow$ (Y,$\tilde{\tau}_Y$, P) and [((A,P),$\tilde{\tau}_{\widetilde{A}}$, P) is subspace of (X,$\tilde{\tau}_X$, P)], the restriction of $\tilde{f}$ with respect to (A,P) is the map: $\widetilde{K} = \tilde{f}$ | ((A,P),$\tilde{\tau}_{\widetilde{A}}$, P): ((A,P),$\tilde{\tau}_{\widetilde{A}}$, P) $\Rightarrow$(Y,$\tilde{\tau}_Y$,P), we prove that the restriction of $\propto$ - soft open map over a $\propto$ - soft open set is $\propto$ - soft open map, similarly for $\propto$ - soft close map over a $\propto$ - soft close set.

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