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An Approximate Solution of Fingero-Imbibition Phenomenon under Magnetic Field Effects

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Abstract
This paper represents an approximate solution of the fingero-imbibition phenomenon under magnetic field effects. The fingero-Imbibition phenomenon arising in two immiscible phases (water with magnetic particles and oil) flow through homogeneous porous media with the effect of variable magnetic field in a vertical downward direction which may arise during the secondary oil recovery process on account of the simultaneous occurrence of two important phenomenon viz, Instability (fingering) and Imbibition. The effect of the variable magnetic field is chosen in such an imbibition. The effect of the variable magnetic field is chosen in such a way that it increases the velocity of the injected conductive fluid. The Mathematical formulation leads to the non-linear partial differential equation. An approximate solution is obtained by using the finite difference Schmidt method and appropriate initial and boundary conditions. The solution of the problem is shown in tables with graphs by using MATLAB.

Keywords: Fingero-Imbibition, Magnetic Field, Porous Media, Double Phase, Schmidt Method, Immiscible Fluids

1. Introduction

In this paper, we investigated the flow of two immiscible fluids within a homogeneous porous medium, influenced by a fluctuating magnetic field [1]. Verma highlights that in this particular case, the injected liquid (I), which is water, has a lower viscosity and preferentially wets the porous medium. Additionally, the magnetization (M) is directly proportional to the magnetic field intensity (H). Consequently, the macroscopic behaviour of the fingers is governed by a statistical approach [2].

In the secondary recovery phase, the native fluid is pushed by the injected fluid, which causes fingering or water drive. Verma [3, 4] referred to these simultaneous occurrences as finger-imbibition. It is common knowledge that there will be an unintentional flow of the resident fluid out of the medium and a spontaneous flow of the wetting fluid into the media when two fluids that selectively wet the medium and a porous medium filled with one fluid come into contact. An imbibition is the term for this phenomena [5].

Instead of the entire front moving in a regular manner when one fluid inside a porous medium is replaced by another with a lower viscosity, protuberances or fingers may form and shoot through the media at comparatively high speeds. Fingering is the term for this phenomenon [5].

Finger imbibition phenomenon refers to the simultaneous occurrence of fingering and imbibition in a displacement process involving two immiscible fluids under specific conditions. In the context of petroleum technology and hydrogeology, the phenomena of fingering and imbibition—whether they occur separately or simultaneously during the displacement process have gained significant attention. Many authors have examined these phenomena from a variety of angles, including Grahm and Richardson [6], Scheidegger [7, 8, 9], Verma [3, 4, 10, 2, 11] Mehta [12], Mishra [13], Rijik [14] and M.R.Tailor [15] Here, the finger-imbibition phenomenon through a homogeneous porous medium with a layer of magnetic fluid (Fig 1) involved in the injected phase is discussed numerically. The two fluids are assumed to be immiscible. The injected fluid has a lower viscosity and preferential wetting behaviour towards porous materials and capillary pressure. These fundamental presumptions form the basis of the current inquiry. Using the finite difference Schmidt method a numerical solution to the governing non-linear partial differential equation has been found.

The graph of space and time versus saturation of injected fluid of numerical results have been obtained which indicates the stabilization of fingers.

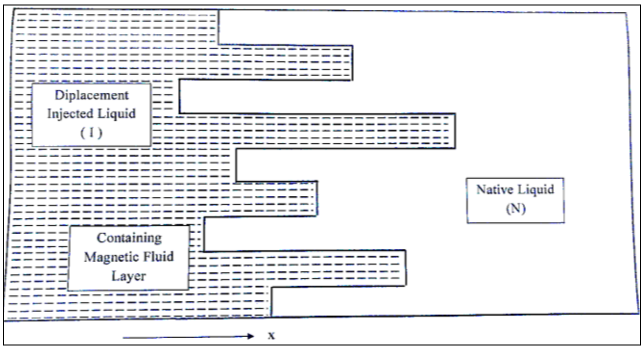

Fig 1: Schematic representation of Fingers at level 'x'

2. Statement of the problem

A finite cylindrical section of a porous medium, saturated with native liquid (N), is entirely enclosed by an impermeable boundary (x = 0). This boundary is adjacent to a formation where injected liquid (I) is introduced, incorporating a thin layer of magnetic fluid. It is assumed that the latter fluid, which is less viscous, is preferentially wetting. This setup initiates a displacement process, where the injection of fluid (I) starts with imbibition, leading to the displacement of the native liquid (N) and the emergence of protuberances, also known as fingers. This phenomenon illustrates a one-dimensional process of finger imbibition.

3. Fundamental equations

Assuming that the flow of two immiscible liquids is governed by Darcy's law, we can express the seepage velocities of the injected liquid (I) and native liquid (N) as follows:

$$V_i = -\frac{K_i}{\delta_i} K \left(\frac{\partial P_i}{\partial x} + \alpha H \frac{\partial H}{\partial x}\right) \tag{3.1}$$

$$V_n = -\frac{K_n}{\delta_n} K \left(\frac{\partial P_n}{\partial x}\right) \tag{3.2}$$

Where $\alpha = \frac{\mu}{4\pi} = \mu_0 \lambda + \frac{16\pi\mu_0\lambda^2 r^3}{g(1+2)^3}$ and

V_i = filtration velocity of the injected liquid,
V_n = filtration velocity of the native liquid,
K_i = relative permeability of the injected liquid,
K_n = relative permeability of the native liquid,
K = permeability of the porous medium,
δ_i = viscosity of the injected liquid,
δ_n = viscosity of the native liquid,
P_i = pressure of the injected liquid,
P_n = pressure of the native liquid,
H = magnetic field intensity in the X-direction,
λ = constant parameter,
$\mu = \mu_0$ = permeability of the magnetic field,
g = accelerations due to gravity.

The equation of continuity for injected liquid (neglecting the variation into phase densities) are given by

$$P\left(\frac{\partial S_i}{\partial t}\right) + \left(\frac{\partial V_i}{\partial x}\right) = 0 \tag{3.3}$$

$$P\left(\frac{\partial S_n}{\partial t}\right) + \left(\frac{\partial V_n}{\partial x}\right) = 0 \tag{3.4}$$

Where,
P = Porosity of the medium,
S_i = Saturation of the injected liquid (water),
S_n = Saturation of the native liquid (oil).

The definiteness of phase saturation is that

$$S_i + S_n = 1 \tag{3.5}$$

The analytical condition of governing imbibition [5] is given by

$$V_i + V_n = 0 \tag{3.6}$$

A capillary pressure P_c defined as the pressure discontinuity between the phases across their common interface, is a function of the phase saturation, we may write as

$$P_c = -\beta g(S_i) \text{ due to Muskat [1].} \tag{3.7}$$

and

$$P_c = P_n - P_i \tag{3.8}$$

$\beta =$ Constant capillary pressure coefficient.

For definiteness, the following important relationships between the saturation S_i and S_n and the fictitious relative permeability K_i and K_n have been assumed

$$K_i = S_i = S_i^3 \tag{3.9}$$

$$K_n = S_n = 1 - S_i = 1 - \alpha' S_i \tag{3.10}$$

Where $\alpha' =$ 1.111
For the statistical treatment of the fingers due to Scheidegger and Johnson [16].
Here, it is considered that the effect of variable magnetic field is in only X – direction, it is also assumed that the conducting fluid is flowing in horizontal X – direction and non-conducting fluid is steady-state. The effect of the variable magnetic field is to increase the pressure of the injecting fluid by quantity $\frac{\mu H^2}{8\pi}$. Recently Verma [10] has considered the relationship between the magnetic field H as

$$H = \frac{\lambda}{x^n} \tag{3.11}$$

Where λ is constant parameter and n is constant.

4. Mathematical Formulation of Problem
The value of V_i and V_n from the equations (3.1) and (3.2) respectively, substituting into the equation (3.6), we get,

$$\frac{K_i}{\delta_i} K\left(\frac{\partial P_i}{\partial x} + \alpha H \frac{\partial H}{\partial x}\right) + \frac{K_n}{\delta_n} K \frac{\partial P_n}{\partial x} = 0 \tag{4.1}$$

From the equation of capillary pressure (3.8), the equation (4.1) will reduce

$$\frac{\partial P_i}{\partial x} = -\frac{\left(\frac{K_n}{\delta_n}\frac{\partial P_c}{\partial x} + \frac{K_i}{\delta_i}\alpha H \frac{\partial H}{\partial x}\right)}{\left(\frac{K_i}{\delta_i} + \frac{K_n}{\delta_n}\right)} \tag{4.2}$$

Equation (4.2) together with equation (3.1) produces,

$$V_i = \frac{K_i K_n \cdot K}{K_i \delta_n + K_n \delta_i}\left(\frac{\partial P_c}{\partial x} - \alpha H \frac{\partial H}{\partial x}\right) \tag{4.3}$$

Substituting equation (4.3) into equation (3.3)

$$P\left(\frac{\partial S_i}{\partial t}\right) + \frac{\partial}{\partial x}\left[\frac{K_i K_n \cdot K}{K_i \delta_n + K_n \delta_i}\left(\frac{\partial P_c}{\partial x} - \alpha H \frac{\partial H}{\partial x}\right)\right] = 0 \tag{4.4}$$

This equation (4.4) is the desired non-linear differential equation of finger-imbibition phenomenon with the effect of a magnetic field for the flow of two immiscible phases through the porous media, The present investigations involve water (less viscous) and oil, therefore, according to Scheidegger [16], we use the following approximation;

$$\frac{\left(\frac{K_I}{\delta_I}\frac{K_{II}}{\delta_{II}}\right)}{\left(\frac{K_I}{\delta_I}+\frac{K_{II}}{\delta_{II}}\right)} \approx \frac{K_{II}}{\delta_{II}} \tag{4.5}$$

Putting this equation (4.5) into equation (4.4), we obtain

$$P\left(\frac{\partial S_I}{\partial t}\right)+\frac{\partial}{\partial x}\left[K\frac{K_{II}}{\delta_{II}}\left(\frac{\partial P_c}{\partial x}-\alpha H\frac{\partial H}{\partial x}\right)\right]=0 \tag{4.6}$$

At this stage for definiteness, we may use the relationship equation (3.7), (3.9) and (3.10). In the light of these equations, the equation (1.4.6) can be rewritten as

$$P\left(\frac{\partial S_I}{\partial t}\right)+\frac{\partial}{\partial x}\left[K\frac{K(1-S_I)}{\delta_{II}}\left(-\beta\frac{\partial S_I}{\partial x}-\alpha H\frac{\partial H}{\partial x}\right)\right]=0 \tag{4.7}$$

For the sake of simplicity, substituting

$$1-S_i(x,t)=S(x,t) \tag{4.8}$$

And then

$$P\left(\frac{\partial S}{\partial t}\right)-\frac{K\beta}{\delta_{II}}\frac{\partial}{\partial x}\left(S\frac{\partial S}{\partial x}\right)+\frac{\alpha K}{\delta_{II}}\frac{\partial}{\partial x}\left(S\,H\frac{\partial H}{\partial x}\right)=0 \tag{4.9}$$

Considering the magnetic field H in the X – direction only we may write equation (3.11) as

$$H=\frac{\lambda}{x^n} \tag{4.10}$$

This equation (4.9) is rewritten as

$$P\left(\frac{\partial S}{\partial t}\right)-\frac{K\beta}{\delta_{II}}\frac{\partial}{\partial x}\left(S\frac{\partial S}{\partial x}\right)-\frac{\alpha K\lambda^2 n}{\delta_{II}}\frac{\partial}{\partial x}\left(\frac{S}{x^{2n+1}}\right)=0 \tag{4.11}$$

A set of suitable initial and boundary conditions associated with the problem (4.11) are given by

$$S(0,t)=S_0 \quad at\ x=0\ and\ t\geq 0 \tag{4.12}$$

$$S(1,t)=S_1 \quad at\ x=1\ and\ t\geq 0 \tag{4.13}$$

$$S(x,0)=\theta_\epsilon << 1 \quad at\ 0<x<1\ ;t=0 \tag{4.14}$$

$$\left.\frac{\partial S}{\partial x}\right|_{x=0}=0\ , \quad \left.\frac{\partial S}{\partial x}\right|_{x=1}=0\ ; \quad t\geq 0 \tag{4.15}$$

Equation (4.11) is a non-linear partial differential equation of second order governing the fingero – imbibition phenomenon of two imbibition phenomenon of two immiscible liquids through the homogenous porous medium with magnetic field effect. This equation is very complicated to get its solution therefore for definiteness, choose $n=-\frac{1}{2}$ in it Mehta [12]. Equation (4.11) can be rewritten as,

$$\left(\frac{\partial S}{\partial t}\right)-\alpha_1 S\frac{\partial^2 S}{\partial x^2}-\alpha_1\left(\frac{\partial S}{\partial x}\right)^2+\alpha_2\frac{\partial S}{\partial x}=0 \tag{4.16}$$

Where

$$\alpha_1=\frac{K\beta}{\delta_{II}P}\ ;\ \alpha_2=\frac{\alpha K\lambda^2}{2\delta_{II}P} \tag{4.17}$$

5. Solution of the problem
by the finite difference Schmidt method [17]

$$\frac{\partial S}{\partial t}=\frac{S_{i,j+1}-S_{i,j}}{k}+o(k) \tag{5.1}$$

$$\frac{\partial S}{\partial x}=\frac{S_{i,j}-S_{i-1,j}}{h}+o(h) \tag{5.2}$$

$$\frac{\partial^2 S}{\partial x}=\frac{S_{i-1,j}-2S_{i,j}+S_{i+1,j}}{h^2}+o(h^2) \tag{5.3}$$

From the equation (4.16), we get,

$$S_{i,j+1}=\left(1-\frac{k\alpha_2}{h}\right)S_{i,j}+\frac{k}{h^2}S_{i,j}\{\alpha_1(S_{i+1,j}-S_{i-1,j})\}-\frac{k\alpha_1}{h^2}S_{i,j}^2+\frac{k\alpha_1}{h^2}S_{i-1,j}^2+\frac{k\alpha_2}{h}S_{i-1,j} \tag{5.4}$$

Let $\alpha_1=0.125, \alpha_2=0.137, k=0.025, h=0.25$

The equation (5.4) rewrite as

$$S_{i,j+1}=0.9863\,S_{i,j}+0.4S_{i,j}\{0.125\,(S_{i+1,j}-S_{i-1,j})\}-0.05S_{i,j}^2+0.05S_{i-1,j}^2+\qquad 0.0137S_{i-1,j} \tag{5.5}$$

With boundary conditions

$$S(0,t)=S_0=0.01;\quad x=0\ and\ t\geq 0 \tag{5.6}$$

$$S(1,t)=S_n=0.9;\quad x=1\ and\ t\geq 0 \tag{5.7}$$

$$S(x,0)=0.1;\quad 0<x<1\ and\ t=0 \tag{5.8}$$

This expression represents the finite difference Schmidt method for the governing partial differential equation. The solution of the expression in the form of tables and graphs is obtained using MATLAB.

6. Graphical and Numerical representation

Table 1: The numerical value of the saturation of injected liquid

x	t=0	t=0.025	t=0.05	t=0.075	t=0.1	t=0.125	t=0.15	t=0.175	t=0.2
0	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.25	0.1	0.101872	0.10376685	0.10573634	0.10785199	0.11021504	0.11297341	0.11635121	0.12070268
0.5	0.1	0.1	0.10124966	0.10413131	0.1092153	0.11737374	0.12997703	0.14924142	0.17885441
0.75	0.1	0.132	0.1734304	0.22686123	0.29536939	0.38248277	0.49196137	0.62727421	0.79053777
1	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9

Table 2: The numerical value of the saturation of injected liquid

t	x=0	x=0.25	x=0.5	x=0.75	x=1
0	0.01	0.1	0.1	0.1	0.9
0.025	0.01	0.101872	0.1	0.132	0.9
0.05	0.01	0.10376685	0.10124966	0.1734304	0.9
0.075	0.01	0.10573634	0.10413131	0.22686123	0.9
0.1	0.01	0.10785199	0.1092153	0.29536939	0.9
0.125	0.01	0.11021504	0.11737374	0.38248277	0.9
0.15	0.01	0.11297341	0.12997703	0.49196137	0.9
0.175	0.01	0.11635121	0.14924142	0.62727421	0.9
0.2	0.01	0.12070268	0.17885441	0.79053777	0.9

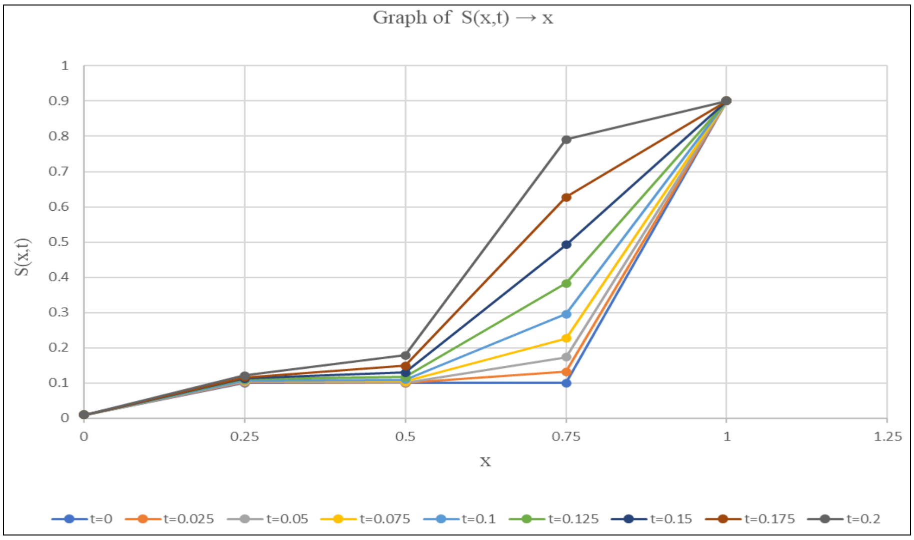

Fig 2: The graph of S(x,t) → x

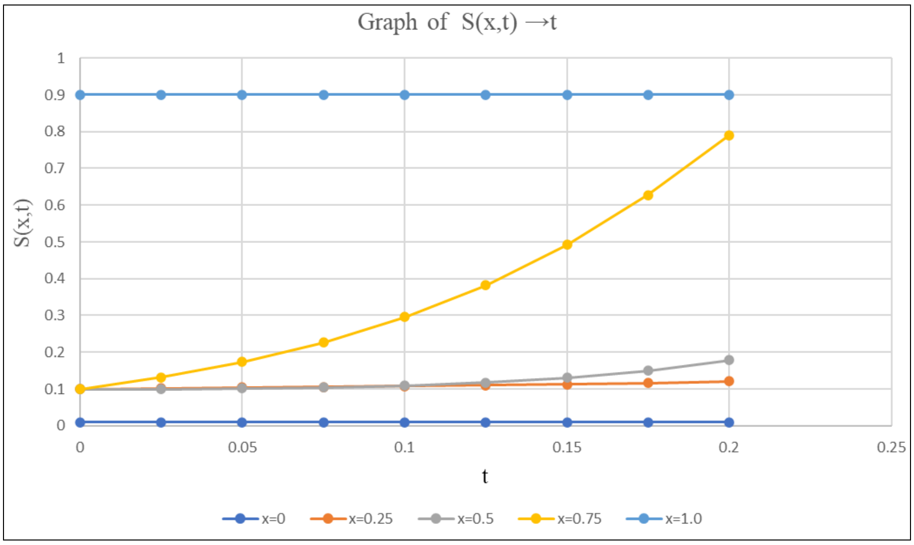

Fig 3: The graph of S(x,t) → t

7. Conclusion

Here, we have discussed a numerical solution of a mathematical model of the fingero-imbibition phenomenon in a homogenous porous medium involving a magnetic field by using the finite difference Schmidt method. Equation (5.5) represents the finite difference Schmidt method for the governing partial differential equation.

For the sake of convenience in mathematical analysis, some standard relations of saturation and capillary pressure are assumed, which are consistent with physical problems. The solution of injected liquid is obtained under the assumption that the average cross-section area is occupied by the fingers. The numerical as well as graphical illustrations are given by Table 1, Table 2, Fig 2 and Fig 3.

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