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Geometric Techniques of Local Geoid Determination

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Abstract
Local geoid determination employing geometric techniques harnesses positional data from GNSS and conventional leveling. Such methodologies ranging from polynomial interpolation to least-squares collocation provide centimeter-level accuracy for local geoid undulations, crucial for conversion of ellipsoidal heights to orthometric heights. This study reviews core methods, exemplifies their application, and assesses their performance in various regions.

Keywords: Local Geoid Determination, Geometric Techniques, Least-Squares Collocation, Polynomial Fitting, Interpolation Methods

1. Introduction

Geometric techniques for local geoid determination play a pivotal role in converting ellipsoidal heights obtained via Global Navigation Satellite Systems (GNSS) into orthometric heights that reference the Earth's gravity field. The foundational relationship governing this conversion follows the equation:

$$N = h - H \tag{1}$$

Where N represents the geoid undulation, h the ellipsoidal height, and HHH the orthometric height, each measured at GNSS-leveling control points (Sisman *et al*., 2023) [6]. Accuracy of the computed geoid relies significantly on the appropriate selection of interpolation and estimation methodologies, considering factors such as control-point density, covariance modelling, and incorporation of global models.

Polynomial surface fitting through ordinary least squares (OLS) has emerged as a prevalent geometric method due to its computational simplicity and effective performance over uniformly sampled terrains. Empirical studies demonstrate reliable accuracy of 2–3 cm when second- or third-order polynomials are employed, though irregular sampling may necessitate higher-order fitting to accommodate data heterogeneity (Phinyo *et al*., 2021). Selection of polynomial degree proves critical; inappropriate complexity may introduce overfitting or elevated uncertainties (Tusat & Mikailsoy, 2018) [8]. Example applications such as that conducted in Nigeria's Federal Capital employed nine-coefficient polynomials to model undulations and achieved root-mean-square errors on the order of a few centimeters (Oluyori *et al*., 2018) [3].

Least-squares collocation (LSC) offers an advanced alternative by incorporating stochastic covariance functions to optimize interpolation under Gaussian assumptions (Phinyo *et al*., 2021). For instance, application of LSC with bias and tilt parameters alongside a Gaussian covariance model in Thailand reduced the error distribution from ±5.8 cm to ±3.7 cm a marked improvement in elevation determination (Phinyo *et al*., 2021). Comparatively, standard LSC and geopotential models like EGM2008 yielded ±3.9 cm and ±10.5 cm, respectively, underscoring the value of parameter-augmented collocation (Phinyo *et al*., 2021).

Iran's IRG2018 local geoid model illustrates further sophistication of LSC, by applying regional covariance structures combined with Remove–Compute–Restore workflows and integrated global geopotential and terrain data. That study obtained estimated accuracies within 20 cm when assessed against independent GNSS-leveling control points (Ramouz *et al*., 2019) [5]. In Kuwait, hybrid approaches integrating OLS-polynomial fitting with digital terrain information and global geopotential data

achieved accuracies of 2–3 cm (De Gruyter, 2022) [1]. Similar advancements have been reported in Turkey, where researchers implemented conditional adjustment with unknown design-matrix errors to obtain precise polynomial geoid surfaces (Sisman *et al*., 2023) [6].

Deterministic interpolation techniques such as local kriging, radial basis functions, inverse-distance weighting, and moving least squares have also demonstrated utility; research suggests that kriging may outperform basic polynomial fitting in certain contexts (Discovery Researcher, 2025) [2]. Application of radial basis function collocation in "strip-area" projects has been shown to yield improvements in surface smoothness and undulation accuracy (SciDirect, 2015) [7]. Generalized regression neural networks (G-RNN) and multiquadric functions represent emerging methods whose efficiencies have been evaluated using large GNSS-leveling datasets, particularly in eastern Turkey, where comparative assessments suggest potential advantages over traditional LPI techniques (Aydın *et al*., 2020).

Selecting optimal interpolation techniques depends upon empirical model selection criteria such as root-mean-square error and mean absolute error; selection methods guided by these criteria consistently perform better than arbitrary polynomial selection, especially in heterogeneous datasets (Tusat & Mikailsoy, 2018) [8] Accurate covariance modelling is essential within LSC frameworks; studies dividing geoid computation domains into sub-areas for tailored covariance modeling often report improved outcomes relative to uniform covariance models (Ramouz *et al*., 2019) [5].

Integration of global geopotential models (e.g., EGM2008, EIGEN6C4) and digital terrain models through Remove–Compute–Restore workflows offers both background surfaces and systematic bias correction. Aligning local geoid models with global reference surfaces utilizing Helmert transformations or polynomial trend adjustments further reduces datum inconsistencies and enhances absolute accuracy, as shown in Kuwait and Iran (De Gruyter, 2022; Ramouz *et al*., 2019) [1, 5].

2. Methodological Framework

Precise determination of a local geoid via geometric techniques involves multiple methodological stages. These stages include data acquisition, interpolation modeling, stochastic estimation through collocation, and hybrid integration with global geopotential models.

2.1 Acquisition and Preprocessing of Control Points

Control points established via GNSS and spirit leveling deliver essential data: ellipsoidal heights (hhh) and orthometric heights (HHH). The foundational geoid height NNN is computed directly via the relationship $N = h - H$ at each control point (Phinyo *et al*., 2021; Sisman *et al*., 2023 [6]). Data quality assessment, including outlier detection and datum consistency checks, ensures reliability for subsequent modeling.

2.2 Polynomial Surface Fitting via Least Squares

Deterministic modeling typically employs polynomial surfaces of varying degrees. Commonly used models include first- to third-degree polynomials:

$$N(x, y) = a0 + a1x + a2y + \cdots + aijxiyj \quad (2)$$

with the coefficients aija_{ij}aij obtained by minimizing residuals using ordinary least squares (Sisman *et al*., 2023) [6]. Polynomial order selection must be empirically driven, avoiding both overfitting and underfitting; studies recommend polynomial degree selection through statistical criteria such as root-mean-square error (Tusat & Mikailsoy, 2018; Sisman *et al*., 2023) [8, 6]. Case studies from Turkey's Samsun province (Sisman *et al*., 2023) [6] to Nigeria's Maiduguri area (Uzodinma, 2005) demonstrate the effectiveness of higher-order polynomials (third-order or greater), particularly when control points exceed the minimum required by the polynomial equation (third-order requires ten or more points).

2.3 Least Squares Collocation (LSC)

Least-squares collocation (LSC) provides a statistically optimal method for estimating geoid undulations by modeling the stochastic behavior of residual height differences through spatial covariance functions. The foundational theory of LSC views observed geoid heights as a combination of a true signal and random measurement noise. This decomposition enables the separation of systematic variations from random errors, thereby enhancing the quality of the estimated geoid surface (Moritz, 1978; Ruffhead, 1987) [10, 13].

The mathematical framework of LSC relies on modeling the spatial autocorrelation between geoid heights through empirical or analytical covariance functions. These functions, such as Gaussian, exponential, or spherical models, describe how the similarity between observations decreases with distance. The fitting of these covariance models is critical, as the structure of spatial variability directly influences the interpolation accuracy. Once the covariance structure is defined, the generalized collocation equations are used to estimate geoid values at unsampled locations by minimizing the estimation variance. The formulation is expressed as:

$$\widehat{N} = C_{NP}(C_{PP} + C_{\epsilon})^{-1}(N_{\text{obs}} - N_{\text{global}}) \quad (3)$$

Where N is the predicted geoid undulation, CNP is the cross-covariance between the prediction and known points, CPP is the auto-covariance among the known points, and $C\epsilon$ represents the covariance of measurement noise (Forsberg & Tscherning, 2008) [9].

Implementation of LSC in geoid modeling is often structured through the Remove–Compute–Restore (RCR) strategy. In the first phase, a global geopotential model (e.g., EGM2008 or EIGEN6C4) is subtracted from the observed geoid data to isolate the short-wavelength residuals. During the compute phase, these residuals are interpolated using LSC. Finally, the global model's contribution is restored, yielding the full geoid surface. This sequence ensures that long-wavelength features are retained from the global model, while local variations are modeled from high-resolution data.

Advancements in LSC have introduced bias and tilt parameters into the design matrix to correct for systematic offsets between the global geoid and local height systems. These parameters account for vertical shifts (bias) and planar inclinations (tilt) that arise due to differences in geodetic datums or vertical control networks. Incorporating these adjustments leads to improved alignment between

computed and observed geoid surfaces. For instance, in Thailand, applying a Gaussian covariance model together with bias and tilt corrections improved the local geoid accuracy from ±3.9 cm (standard LSC) to ±3.7 cm, compared to a deviation of approximately ±10.5 cm when only the EGM2008 model was used (Phinyo *et al*., 2021).

In practice, the selection and calibration of covariance models are tailored to the regional terrain and data configuration. Some studies have demonstrated the effectiveness of partitioning the study area into sub-regions, each with distinct covariance functions. This approach addresses spatial non-stationarity in geoid residuals and enhances modeling precision, particularly in geophysically heterogeneous zones (Ramouz *et al*., 2020) [5].

Moreover, the implementation of parameter-augmented LSC has proven valuable for national and regional geoid models. In the United States, for example, the GEOID18 model employed LSC with planar corrections to generate a geoid surface with a national average root-mean-square deviation of approximately 1.3 cm when validated against GNSS/leveling control points. Such outcomes highlight the efficacy of combining collocation with auxiliary trend modeling techniques to accommodate inconsistencies in observational data and reference surfaces.

Therefore, LSC stands as a central methodology in high-accuracy geoid modeling, especially when integrated with data-driven enhancements such as empirical covariance fitting, parameter augmentation, and remove–compute–restore workflows. These refinements enable LSC to model geoid undulations with greater fidelity, making it suitable for precise height transformation and geodetic applications

2.4 Auxiliary Interpolation Methods

Alternate interpolation methods such as kriging, inverse-distance weighting (IDW), moving least squares (MLS), and radial basis functions (RBF) provide adaptable frameworks for local geoid determination, particularly when control points are non-uniformly distributed or terrain features exhibit significant spatial heterogeneity.

Kriging, a geostatistical interpolation technique, estimates unknown values by leveraging spatial autocorrelation structures derived from variogram models. Unlike deterministic methods, kriging not only predicts values but also quantifies the associated prediction uncertainty. This makes it particularly valuable in areas where data quality or spacing is variable. In several empirical studies, kriging has demonstrated lower residual errors than polynomial interpolation when applied to complex terrain. For example, kriging outperformed traditional regression-based polynomial surfaces in geoid modeling over coastal and mountainous areas where terrain variability introduced spatial complexity beyond the scope of deterministic models.

Inverse-distance weighting, though computationally efficient, assigns weights inversely proportional to the distance between the target point and known data points. Its simplicity makes it useful for quick approximations, especially in regions with moderately homogeneous data. However, its performance diminishes in cases where spatial correlation is non-linear or anisotropic, often leading to larger residuals compared to kriging or RBF methods.

Moving least squares (MLS) offers an advanced local approximation scheme wherein a weighted polynomial surface is fitted over neighborhoods of points, with weights diminishing as distance increases. Unlike ordinary least squares, MLS can handle irregular and sparse sampling effectively by adapting the fitting surface to the local data geometry. This method is particularly useful for ensuring smoothness in geoid surfaces without sacrificing local accuracy, especially when the geoid varies smoothly but irregularly across the region.

Radial basis functions (RBF) represent another robust alternative, employing a series of basis functions centered at each control point. Common kernels include Gaussian, multiquadric, and thin-plate splines. These functions create smooth interpolation surfaces by minimizing bending energy or function curvature. In geoid studies over strip areas or regions with anisotropic spatial features, RBFs have yielded geoid models with sub-centimeter residuals when compared against GNSS/leveling benchmarks. The flexibility of kernel selection allows RBFs to adapt to a wide variety of spatial patterns, making them particularly effective in local geoid refinement.

Trend surface analysis and spline methods, though less commonly employed in modern geoid estimation, remain suitable in environments where data is well-structured and variations follow predictable patterns. In such cases, global polynomials or surface splines can efficiently model the underlying trend with high computational efficiency.

Overall, while polynomial interpolation remains a foundational method in geoid modeling, alternative interpolation techniques offer significant improvements in spatial accuracy, particularly in geophysically complex areas. Kriging excels in data-rich, variable terrains due to its geostatistical rigor. RBF methods are preferred when smoothness and adaptability are priorities. MLS provides reliable local fitting in irregular datasets, and IDW offers a quick, though often less precise, alternative. The choice among these techniques should be informed by dataset characteristics, terrain variability, and accuracy requirements of the geodetic application.

2.5 Hybrid Integration with Global Geopotential and Terrain Models

Integrated geoid modeling approaches have evolved to incorporate both global geopotential models and high-resolution digital terrain models (DTMs) within the framework of the Remove–Compute–Restore (R–C–R) strategy. This methodology enhances the precision of local geoid determination by combining the strengths of long-wavelength global gravity information with localized spatial interpolation techniques.

In the initial "Remove" phase, global geopotential models such as the Earth Gravitational Model 2008 (EGM2008) or EIGEN6C4 are used to account for the low-frequency components of the Earth's gravity field. These models provide a coarse but globally consistent representation of geoid undulations, incorporating satellite altimetry and terrestrial gravity data. By subtracting the global model from the observed GNSS-derived geoid undulations, a residual surface is obtained that captures the short-wavelength and high-frequency variations specific to the local geoid field.

The "Compute" phase involves interpolation of these residuals using geometric techniques such as polynomial surface fitting or stochastic methods like least-squares collocation (LSC). These residuals often reflect local gravitational anomalies influenced by terrain features, geological discontinuities, or data inconsistencies. The use

of high-resolution DTMs in this stage enables the accurate representation of terrain-induced gravity effects, particularly important in mountainous or topographically complex regions. Techniques such as residual terrain modeling (RTM) are sometimes applied here to quantify and compensate for gravitational contributions of terrain masses not captured in global models.

The final "Restore" step involves recombining the interpolated residuals with the original global geopotential model to yield a full-resolution local geoid model. This integrated model benefits from the global consistency of the reference geopotential surface while simultaneously accommodating detailed local variations.

To ensure consistency between the global model's vertical datum and the local height system, datum transformation techniques are typically implemented. Among these, the seven-parameter Helmert transformation is commonly used to correct for systematic differences including scale, rotation, and translation between geodetic datums. Application of such transformations is critical in aligning the ellipsoidal heights from GNSS measurements with orthometric heights derived from national leveling networks. Empirical implementations of this integrated approach have demonstrated significant improvements in geoid accuracy. In Kuwait, the use of EGM2008 combined with LSC and terrain corrections led to geoid model accuracies within the range of 2–3 centimeters when validated against independent GNSS/leveling control points. Similarly, in Iran, regional geoid modeling using EIGEN6C4 in conjunction with local gravity and elevation data achieved accuracies within 20 centimeters, a notable refinement considering the geophysical complexity of the region.

These outcomes underscore the value of hybrid geoid modeling frameworks that harness the complementary strengths of global and local datasets. By anchoring on the long-wavelength gravity field of global geopotential models and refining with terrain-sensitive interpolation of residuals, the R–C–R strategy enables the generation of high-precision geoid models suitable for modern geodetic applications, including orthometric height transformation, national height modernization programs, and geophysical interpretation.

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2.6 Model Validation and Error Assessment

A rigorous evaluation of geoid model accuracy necessitates the application of objective statistical validation metrics that quantify the agreement between modeled geoid undulations and observed reference values. Among the most commonly utilized measures are the Root Mean Square Error (RMSE), standard deviation, mean absolute error (MAE), and maximum residual. These indicators provide insights into both the central tendency and dispersion of residuals, enabling researchers to assess the reliability, consistency, and predictive capacity of the geoid surface across the spatial domain.

RMSE, in particular, is frequently employed as a principal indicator of accuracy. It represents the square root of the average of squared differences between predicted geoid undulations and actual GNSS/leveling-derived values at independently selected control points. A lower RMSE reflects a closer fit to the reference data and indicates higher geometric fidelity of the geoid surface. Standard deviation, on the other hand, measures the variability of the residuals and complements RMSE in evaluating model stability.

Empirical studies underscore the efficacy of these metrics in real-world geoid modeling scenarios. For example, the THAI17G geoid model of Thailand, developed using least-squares collocation enhanced with bias and tilt parameters and a Gaussian covariance function, achieved an RMSE of approximately ±3.7 cm when validated against an independent set of GNSS/leveling stations. This result demonstrated a significant improvement over both standard LSC implementations and direct usage of global geopotential models such as EGM2008, which exhibited deviations exceeding ±10 cm in the same region. The incorporation of parametric corrections and optimal covariance fitting was instrumental in achieving this level of precision (Phinyo *et al*., 2021).

Similarly, in Samsun province of Turkey, a comprehensive evaluation of geometric geoid modeling was conducted using third-order polynomial surface fitting in conjunction with a conditional adjustment method incorporating unknown parameters. When compared to GNSS/leveling observations across a network of distributed checkpoints, the geoid model achieved residuals within the centimeter range, highlighting the appropriateness of polynomial models in relatively structured and uniformly distributed datasets (Sisman *et al*., 2023) [6]. The study further revealed that model performance was sensitive to both the spatial configuration of the control points and the order of the fitted surface, affirming the necessity of tailored statistical validation.

Beyond these regional examples, many national geoid modeling programs now mandate extensive validation protocols involving not only RMSE and standard deviation but also cross-validation techniques. These include leave-one-out validation, where each control point is temporarily excluded from the modeling process and then predicted, allowing for assessment of prediction performance across the entire dataset. Such methods provide robust evidence of model generalization capability.

In high-precision geodetic applications, achieving centimeter or sub-centimeter residuals through such validation techniques is critical, particularly in engineering surveying, topographic mapping, and hydrological modeling. Thus, comprehensive statistical accuracy assessment remains an indispensable component of the geoid determination process, guiding methodological choices and supporting the operational deployment of geoid surfaces in national and regional geospatial infrastructures.

3. Conclusion

Geometric Geometric techniques for local geoid determination represent a practical and effective approach for deriving geoid undulations, particularly in regions where dense terrestrial gravity data is unavailable or insufficient. When implemented with methodological precision ensuring robust data preprocessing, optimal model selection, and adequate spatial coverage these techniques can yield high-quality geoid surfaces suitable for a wide range of geodetic and engineering applications.

Among the geometric approaches, polynomial surface fitting remains one of the most widely used due to its conceptual simplicity, computational efficiency, and ease of implementation. This method is particularly effective in relatively small, topographically homogeneous areas where the geoid exhibits a smooth spatial variation. Polynomial models, especially of second or third order, have

demonstrated consistent performance in GNSS/leveling-based local geoid modeling, often producing residuals within a few centimeters when applied to well-distributed control networks. The accuracy of polynomial fitting, however, depends heavily on the selection of appropriate model order, control-point density, and spatial uniformity of observations.

For applications demanding higher precision, especially in areas with complex topography or variable data quality, more advanced techniques such as least-squares collocation (LSC) provide a statistically optimal solution. LSC incorporates stochastic modeling through spatial covariance functions, allowing for the estimation of geoid values with minimized prediction error. Furthermore, it enables the inclusion of measurement noise and systematic biases, offering a refined interpolation framework capable of addressing non-stationary geoid behavior.

An even more robust strategy involves the integration of geometric techniques with global geopotential models, digital terrain data, and regional gravity datasets through hybrid approaches. The Remove–Compute–Restore (R–C–R) framework exemplifies this integration, where long-wavelength components from global models are removed, local residuals are estimated using polynomial or collocation methods, and the final surface is restored by reintroducing the global signal. These hybrid methods have proven highly effective in enhancing geoid accuracy, as they combine the strengths of global consistency and localized adaptation.

As the demand for more accurate and spatially adaptive geoid models continues to grow, recent studies have begun to explore the potential of soft computing and machine learning algorithms in geoid modeling. Techniques such as artificial neural networks (ANN), support vector machines (SVM), and radial basis function networks have shown promise in modeling complex spatial patterns that traditional deterministic or stochastic methods may not fully capture. These methods offer the capability to learn non-linear relationships and adaptively refine geoid surfaces based on data characteristics, terrain complexity, and geophysical heterogeneity.

Future research may focus on developing hybrid machine learning frameworks that combine geodetic principles with data-driven learning architectures. Such frameworks could automatically adjust model parameters based on the local geophysical environment, optimize interpolation in data-sparse regions, and integrate multi-source datasets including satellite altimetry, airborne gravimetry, and terrestrial leveling. Furthermore, advancements in high-performance computing and cloud-based geospatial processing may facilitate the operational deployment of these intelligent modeling systems on a national or continental scale.

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