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Impact of Link Length Manufacturing Errors on the Positioning Accuracy of Industrial Robots

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Abstract

Industrial robots are increasingly playing a vital and indispensable role in modern manufacturing activities. Their accuracy continues to attract the attention of designers and manufacturers aiming to create optimal machinery in terms of both quality and economic efficiency. This paper presents a study investigating the influence of link dimension errors on the positioning accuracy of robotic arms. The relationship between these two factors is established based on the robot kinematics problem. All possible error scenarios within the permissible tolerance limits of each link are considered. A specialized software developed by the authors is used to process and display the relevant data. Specific illustrations are conducted on two six-degree-of-freedom (6DOF) robots from ABB and Kuka. The survey results highlight the extent to which errors in different links affect the positioning accuracy of the end-effector. Notably, the group of errors with the most significant impact are those related to links simulating the key dimensions of a human worker. This finding provides valuable insights for robot designers and manufacturers.

Keywords: Manufacturing Tolerance, Industrial Robot, Robot Link, Kinematics, Positioning Accuracy

1. Introduction

Industrial robots are indispensable machinery in modern industrial production activities. With their diverse and extensive applications, they not only replace human workers in repetitive, monotonous tasks and hazardous environments but also excel in tasks requiring high flexibility and precision that are challenging for humans to perform.

The accuracy of a robot determines the quality and cost of the tasks it undertakes. In terms of quality, higher precision leads to better task performance, which is a desired outcome for users. However, as the operational error of the robot decreases, the technical and technological requirements for its components become more stringent and sophisticated, significantly increasing the overall manufacturing cost of the robotic arm. Therefore, designing and manufacturing robots to meet the required accuracy is a challenging, time-consuming, and labor-intensive task for designers.

The positional accuracy of the robot's end-effector remains a critical issue in high-precision industrial applications. This is particularly true for assembly tasks, which are among the most important industrial tasks, as the positional accuracy of the gripper is almost always greater than the tolerance of the parts to be assembled. Thus, improving the positional accuracy of the end-effector is a crucial issue that needs to be addressed [1]. Numerous global studies have shown that the initial error at the robot's end-effector is primarily due to geometric and kinematic errors, with other factors having minimal impact [2, 3, 4]. Approximately 75% of the initial error in a new machine is attributed to manufacturing and assembly [5].

Several researchers have conducted studies to evaluate the tolerances of various parameters (geometric tolerances, kinematic parameters, manufacturing tolerances, joint deviations, etc.) to identify which parameters have a greater impact on the end-effector's deviation (robot reliability). Some studies on the effects of joint clearance include: R. Weill and B. Shani [6], who developed a model to assess the impact of geometric errors of components on the positional and orientational errors of the robot's end-effector, concluding that joint angle errors have a significant impact. The model was implemented using computer programming (using SILICON-GRAPHIC with the C language). Similarly, Y. H. Andrew Liou and colleagues [7] identified which joint clearances have a greater impact on the positional and orientational accuracy of the end-effector using the Taguchi experimental design method, comparing the process with Monte Carlo simulation techniques. Ting Kwun-Lon and colleagues [8, 9] analyzed and evaluated the effects of joint clearance on the positional and orientational deviations of individual links and the robotic arm. The authors used the N-bar rotatability laws to investigate planar kinematic chains.

Jeong Kim *et al*. [10] used the advanced first-order second moment (AFOSM) method to determine the influence of link tolerances and joint deviations on the repeatability of positional and orientational accuracy at the robot's end-effector, with validation through Monte Carlo simulation. Dao Duy Son and Kazem Abhary [11] employed the Taguchi Design of Experiment (DOE) method to investigate the effects of link mass tolerances, motor inertia tolerances, link length tolerances, and joint clearance tolerances on robot accuracy. Statistical analysis revealed the parameters with the most to least significant impact on the robotic arm's accuracy. Similarly, Fattah Hanafi Sheikhha [12] applied the Taguchi method to determine which parameters most affect the end-effector accuracy of a 3-PSP parallel robot. The study found that tool length tolerance significantly impacts the end-effector tolerance of parallel robots.

Some studies have focused on designing tolerances for link lengths and joint clearances to ensure the required robot positioning accuracy. Wu and Rao [13] transformed the tolerance design problem into a constrained optimization model, where the objective function is positional error and the constraint is manufacturing cost. Hu and colleagues [14] proposed a tolerance allocation scheme considering joint motion and test errors, with repeatability as a constraint. Additionally, Kim [15] proposed pseudo-Boolean programming for a cost-constrained tolerance model, bounded by positional and orientational error variances, to improve computational efficiency, applicable to linearly constrained optimization problems. Peng Huang and colleagues [16] proposed a tolerance design method for robots using an Improved Genetic Algorithm (IGA) to select optimal kinematic parameter tolerances, with manufacturing cost as a constraint, potentially minimizing the failure probability of positioning accuracy.

Despite these achievements, challenges remain, necessitating further research into optimizing link and joint errors to ensure robots operate optimally, efficiently, and economically. Therefore, continued research into the effects of kinematic errors in industrial robots will contribute to the development of enhanced accuracy solutions. Thus, studying the impact of manufacturing errors on the positioning accuracy of industrial robots is essential.

The remainder of this paper is organized as follows: Section 2 presents the research foundation – the relationship between link length manufacturing errors and robotic arm positioning accuracy. Section 3 details the illustrative investigation conducted on two 6-degree-of-freedom (6DOF) industrial robots. Conclusions are presented in Section 4..

2. Theoretical Basis

2.1 The Relationship Between Link Length Error and Robot Positioning Accuracy

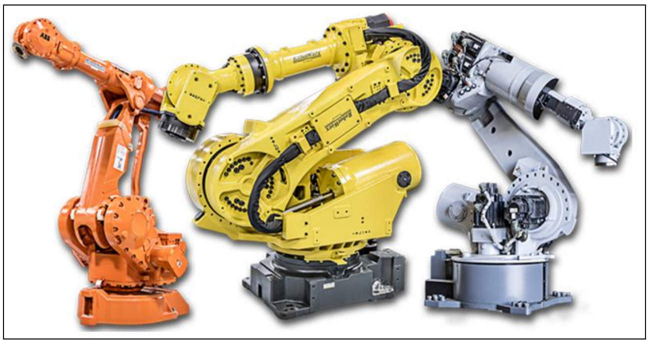


Fig 1: Six-degree-of-freedom serial robot

An industrial robotic manipulator is a serial chain of links and joints (Figure 1). At the end of this chain is the end-effector (gripper), which directly interacts with the object to perform the required operations and tasks of the robot. During operation, two key parameters must be satisfied: The positioning and orientation (pose) of the gripper within the workspace. The kinematic parameters of the manipulator that directly contribute to determining this position and orientation include:

- ai, di represents the link dimensions of the robot, where a is the link length and d is the link offset.
- θ_i (qi) is the generalized coordinates (joint variables) of the robot.;
- αi The twist angle between two joint axes;

The kinematics problem of the robot establishes the relationship between these parameters (Denavit-Hartenberg (D-H) parameters) and the end-effector pose. The general form of the kinematic equation, assuming ideal links with no dimensional errors, can be written as follows:

$$f(d_i, a_i, q_i, \beta_i) = p_j; i = 1, \dots, n \qquad (1)$$

Where n is the number of degrees of freedom of the considered robot (n =6 for a 6-DOF robot, n=5 for a 5-DOF robot, and n=4 for a 4-DOF robot);

a_i, d_i is the link dimensions of the robot; (a: Link length; d: Link offset)

q_i is the generalized coordinate (joint variable) of the robot.;

β_i is the twist angle between two joint axes;

p_j is the actual position of the robot end-effector at the evaluated position j in the workspace;

Model (1) is an ideal model, whereas in reality, dimensional errors exist due to the manufacturing and assembly processes. According to technical requirements, these errors must remain within the tolerance limits specified in the design drawings. For instance, if a robot link has a nominal length of d_1 = 335 ±0,167 with a tolerance of ± 0.167 mm, its actual manufactured length will range from 334.833 mm to 335.167 mm. These deviations directly affect the positioning accuracy of the robot and must be considered in the kinematic model.

The set of all kinematic parameters of a robot's links and joints consists of actual dimensions that contain errors. These errors accumulate, causing the end-effector to reach an actual position that deviates from the desired position in the workspace.

The kinematic model of the robot, when accounting for link, joint, and twist angle deviations, is given by:

$$f(a_i \pm \delta a_i, d_i \pm \delta d_i, q_i \pm \delta q_i, \beta_i \pm \delta \beta_i) = p_j \pm dri = 1, \dots, n \tag{2}$$

Where, δa_i, δd_i: Link length deviations;

δq_i: Joint angle deviations (joint clearance);

$\delta \beta_i$: Twist angle deviation, which represents the error in the angle between joint axes, arising from the assembly process.

In the mathematical model above, theoretically, all parameters play an equivalent role, and a comprehensive analysis of the deviations δa_i, δd_i δq_i and $\delta \beta_i$ is entirely feasible. However, due to the significant computational workload required for a full analysis, this study focuses on solving the problem with deviations in the link dimensions, while the joint deviation δq_i and twist angle deviation $\delta\beta_i$ are not considered. In other words, in the kinematic equation,, q_i and β_i are taken at their nominal values. The kinematic equation then simplifies to:

$$\boldsymbol{f(d_i \pm \delta d_i, a_i \pm \delta a_i, q_i, \beta_i) = p_j \pm \delta r; i = 1, \dots, n} \tag{3}$$

In this context, $\pm\delta d_i, \pm\delta a_i$ represent the tolerances of the links—the D-H parameters form a spatial dimension chain depending on the robot's configuration at the corresponding position p_j. If the standard kinematic parameters q_i from model (1) are used to control the actual model (3), the end-effector position will deviate by an amount as described in equation (3), resulting in $p_j \pm \delta r$ instead of the expected p_j from equation (1). The relationship between link deviations and robot positioning accuracy is expressed through the kinematic equation above.

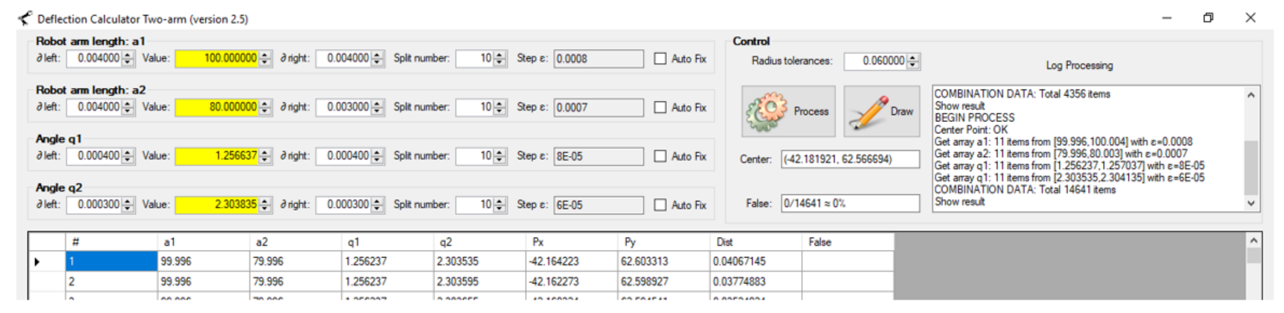
To analyze the influence of individual link errors, the study investigates the effect of each deviation by assigning tolerances to the examined link while keeping all other links at their nominal dimensions. The output value δr will indicate which link deviation has the greatest impact on the robot's accuracy.

2.2 Specialized Software

he evaluation process is conducted using a mathematical model of robot kinematics and statistical analysis. To ensure accuracy and efficiency, the study is carried out on multiple robots with different kinematic configurations. For each robot, the evaluation is performed at various points within the workspace. At each evaluation point, thousands of cases within the tolerance range of each link are examined and recorded. This entire process is executed using specialized software developed by the research team.

In real-world manufacturing and assembly, the actual values of link lengths and joint angles cannot be measured in detail for every individual case. Instead, these parameters are only known to lie within the designed tolerance range. Therefore, the developed software is designed not only to verify the actual position of the end-effector for specific link and joint values but also to simulate all possible variations in actual dimensions within the given tolerance range. This exhaustive search method ensures that all possible dimensional deviations within the permissible limits are considered when forming the spatial kinematic chain. This approach is particularly valuable in mass production of industrial robots, where interchangeability occurs, and it is essential to ensure that the required accuracy of dimensional chains and the end-effector position is maintained.

Based on the Denavit–Hartenberg convention, the forward kinematics problem defines the orientation and position of the end-effector in terms of kinematic parameters: Link length a, link offset d, joint variable q and twist angle β. Given these parameters, the position and orientation of the end-effector are uniquely determined within the workspace. The end-effector position coordinates (x,y,z) are used to assess deviations from the desired position (without errors). The evaluation results include the coordinates of all deviation points, the number of occurrences, and detailed information about each point, which are fully compiled into an Excel-format table within the software. Figure 2 presents the software interface running the simulation for a 2DOF robot.



Robot arm length: a1
∂left: 0.004000 Value: 100.000000 ∂right: 0.004000 Split number: 10 Step ε: 0.0008 ☐ Auto Fix

Robot arm length: a2
∂left: 0.004000 Value: 80.000000 ∂right: 0.003000 Split number: 10 Step ε: 0.0007 ☐ Auto Fix

Angle q1
∂left: 0.000400 Value: 1.256637 ∂right: 0.000400 Split number: 10 Step ε: 8E-05 ☐ Auto Fix

Angle q2
∂left: 0.000300 Value: 2.303835 ∂right: 0.000300 Split number: 10 Step ε: 6E-05 ☐ Auto Fix

Radius tolerances: 0.060000
Center: (-42.181921, 62.566694)
False: 0/14641 ≈ 0%

```
COMBINATION DATA: Total 4356 items
Show result
BEGIN PROCESS
Center Point: OK
Get array a1: 11 items from [99.996,100.004] with ε=0.0008
Get array a2: 11 items from [79.996,80.003] with ε=0.0007
Get array q1: 11 items from [1.256237,1.257037] with ε=8E-05
Get array q1: 11 items from [2.303535,2.304135] with ε=6E-05
COMBINATION DATA: Total 14641 items
Show result
```

#	a1	a2	q1	q2	Px	Py	Dist	False
1	99.996	79.996	1.256237	2.303535	-42.164223	62.603313	0.04067145	
2	99.996	79.996	1.256237	2.303595	-42.162273	62.598927	0.03774883	

Fig 2: Software interface for verification of a two-degree-of-freedom robot

3. Illustrative survey with six-degree-of-freedom robots.

A six-degree-of-freedom serial robot with biomimetic first three joints (replicating the functionality of a human arm) is responsible for positioning, while the last three joints function similarly to a spherical joint, determining the end-effector's orientation. With six degrees of freedom, the robot ensures sufficient flexibility and complexity in both positioning and orientation tasks.

Specifically, the kinematic chain of the robotic arm is divided into two groups. The first group consists of links with dimensions d_1, a_2, d_4 which simulate key dimensions of a worker's arm to achieve the desired reach. The second group includes links with dimensions a_1, a_3, d_6 – which serve as auxiliary dimensions in the kinematic chain.

Despite their diversity in types and quantities, the majority of industrial robots adopt this six-degree-of-freedom structure. This configuration is considered the most common and representative among industrial robots (Figure 1). The study is conducted on two such robots: The ABB IRB 2600-20 (12)/1.65 and the KUKA KR6, with their dimensional parameters listed in Table 1.

Table 1: Dimensional Parameters of ABB IRB 2600-20 (12)/1.65 and KUKA KR6 Robots

6DOF Robot	d_1 (mm)	a_1 (mm)	a_2 (mm)	a_3 (mm)	d_4 (mm)	d_6 (mm)
ABB IRB2600-20 (12)/1.65	445	150	700	115	795	85
Kuka kr6	335	75	270	90	295	80

A certain amount of manufacturing tolerance for the links is assumed to conduct the study. Let δd_i (δa_i) be the tolerance of link i with length d_i (a_i). To ensure a reasonable balance in widening or tightening the tolerance range for different link sizes, the tolerances of the remaining dimensions are determined using the following formula:

$$\delta d_j = \delta d_i \frac{d_j}{d_i} \qquad (4)$$

Accordingly, the tolerances of the link dimensions for each robot are shown in Table 2:

Table 2: Tolerances of the links for the surveyed robots

	Δd_1 *(mm)*	δa_1 *(mm)*	δa_2 *(mm)*	δa_3 *(mm)*	δd_4 *(mm)*	δd_6 *(mm)*
ABB IRB2600-20 (12)/1.65	0.25	0.084	0.393	0.065	0.447	0.048
Kuka kr6	0.167	0.037	0.135	0.045	0.147	0.04

For each robot, the survey is conducted at 12 randomly selected points within the workspace. At each point, one specific link is assigned a tolerance while the other links remain at their nominal dimensions, and this process is repeated for each link. A comprehensive sweep is performed across the entire tolerance range for the selected link. Specifically, within the defined tolerance range, the interval is divided into 10,000 segments, resulting in 10,001 possible end-effector positions in the workspace due to random variations in that link's dimension. The entire dataset is generated with the support of custom-developed specialized software.

Thus, at each survey point, for a 6DOF industrial robot with six link dimensions under investigation, a total of 60,006 coordinate data points of the end-effector position with deviations are recorded. Across 12 survey points, the dataset for analysis comprises 720,072 entries.

Table 3 illustrates 10 out of 10,001 cases of end-effector position deviations in the workspace when analyzing the impact of tolerance in dimension d_1 for the ABB IRB2600-20 (12)/1.65 robot.

Table 3: Illustration of 70 out of 10,001 possible end-effector positions corresponding to the tolerance of link 1 height on the IRB2600-20 (12)/1.65 robot

No.	d1	a1	a2	a3	d4	d6	q1	q2	q3	q4	q5	q6	Px	Py	Pz	Deviation
1	444.75	150	700	115	795	85	1.57	1.34	1.75	0.87	0.18	2.15	-11.504	159.7782	635.3439	**0.25**
2	444.7501	150	700	115	795	85	1.57	1.34	1.75	0.87	0.18	2.15	-11.504	159.7782	635.344	**0.24995**
3	444.7501	150	700	115	795	85	1.57	1.34	1.75	0.87	0.18	2.15	-11.504	159.7782	635.344	**0.2499**
4	444.7502	150	700	115	795	85	1.57	1.34	1.75	0.87	0.18	2.15	-11.504	159.7782	635.3441	**0.24985**
5	444.7502	150	700	115	795	85	1.57	1.34	1.75	0.87	0.18	2.15	-11.504	159.7782	635.3441	**0.2498**
6	444.7503	150	700	115	795	85	1.57	1.34	1.75	0.87	0.18	2.15	-11.504	159.7782	635.3442	**0.24975**
7	444.7503	150	700	115	795	85	1.57	1.34	1.75	0.87	0.18	2.15	-11.504	159.7782	635.3442	**0.2497**
8	444.7504	150	700	115	795	85	1.57	1.34	1.75	0.87	0.18	2.15	-11.504	159.7782	635.3443	**0.24965**
9	444.7504	150	700	115	795	85	1.57	1.34	1.75	0.87	0.18	2.15	-11.504	159.7782	635.3443	**0.2496**
10	444.7505	150	700	115	795	85	1.57	1.34	1.75	0.87	0.18	2.15	-11.504	159.7782	635.3444	**0.24955**

Exploring the impact of manufacturing errors on other links is conducted in a similar manner. Figure 3 illustrates the influence of positional errors of the gripper in the workspace due to the manufacturing tolerances of each link.

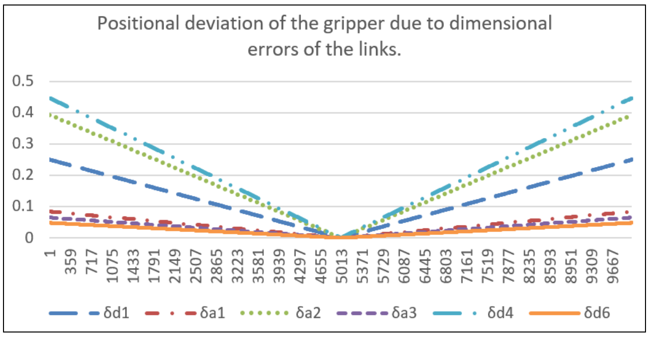


Fig 3: Positional deviation of the ABB IRB2600-20 (12)/1.65 robot in the workspace due to the influence of individual link errors within the allowable tolerance range

The survey process was repeated for the remaining 11 points, resulting in a total of 720,072 positional deviations of the end-effector. The collected data was analyzed to determine the maximum positional error of the end-effector caused by link tolerances. Based on the statistical data, the positioning accuracy of the end-effector in the workspace is most affected by manufacturing errors in the links in the following order: d_4, a_2, d_1, a_1, a_3, and d_6. The error in d_4 causes the greatest positional deviation of the end-effector, followed by a_2, while d_6 has the least impact.

A similar survey was conducted for the Kuka KR6 robot. The statistical data illustrating the impact of link errors on the final position is visually represented in Figure 4.

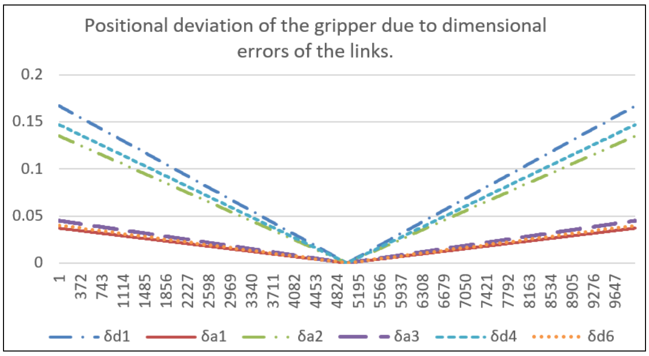


Fig 4: Positional deviation of the Kuka KR6 robot in the workspace due to the influence of individual link tolerances within the allowable range

From the statistical data above, the positional accuracy of the gripper in the workspace is significantly affected by the manufacturing tolerances of the links in the following order: d_1, d_4, a_2, a_3, a_1, and d_6. The tolerance of d_1 causes the greatest positional deviation of the gripper, followed by d_4 and a_2, while the least affected group includes a_3, a_1, and d_6.

4. Conclusion

Robots are mechanical devices that play a crucial role in modern society, particularly in industrial manufacturing. Improving the quality of robotic arms, especially their positioning accuracy, is essential. Based on the forward kinematics problem, the relationship between link length errors and robotic arm positioning accuracy is established. A specialized software tool was developed by the research team to facilitate fast, convenient, and synchronized data processing for large datasets.

The study investigates the impact of manufacturing tolerances on robotic arm accuracy, specifically for two industrial 6-degree-of-freedom (6DOF) serial robots: The ABB IRB2600-20 (12)/1.65 and the Kuka KR6. The survey results, derived from analyzing 2 × 720,072 positional deviations within the workspace, indicate that the primary dimension group has the most significant impact on gripper positioning errors. This group includes link dimensions d_1, a_2, and d_4, which simulate the fundamental and essential dimensions of a worker to achieve the desired reach while working (d_1 represents the upper body length from the waist to the shoulder, a_2 represents the upper arm length, and d_4 represents the forearm length). Depending on the robot manufacturer and version, the dimension with the greatest length (d_1, a_2, or d_4) has the most substantial influence on robotic arm accuracy.

The second group, consisting of auxiliary dimensions a_1, a_3, and d_6, has a lesser impact on end-effector positioning errors.

These findings offer valuable insights for robot designers and manufacturers when deciding whether to tighten or loosen the tolerances of specific links in the robotic arm. This balance ensures both the desired accuracy and cost-effectiveness in manufacturing.

Due to the complexity and vast amount of data in this study, the research team has focused solely on assessing the impact of link errors on the positioning accuracy of the robotic arm. However, this research topic holds further potential. Future studies may extend the analysis to other error sources, such as joint clearance, and examine their effects on the orientation accuracy of the gripper within the workspace.

5. Acknowledgments

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