Int. j. adv. multidisc. res. stud. 2025; 5(2):1068-1071

Received: 06-02-2025
Accepted: 16-03-2025

International Journal of Advanced Multidisciplinary Research and Studies

ISSN: 2583-049X

Revision of the SI Notation for Electromagnetic Quantities to an Exclusively mks System of Units

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DOI: https://doi.org/10.62225/2583049X.2025.5.2.3946

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Abstract

A system of units is described which replaces all the SI units for electromagnetic quantities with combinations of the meter, kilogram and second (mks units). There is a degree of freedom in the definition of the SI units which is illustrated by Coulomb's Law. All that is required is that the ratio of the units of the square of electric charges to the electric permittivity ε_0 be equal to Nm^2.

The alternate (Nms) method takes advantage of this situation by assuming specifically that the unit of electric charge is Nm and that for ε_0 is N. It is shown that this assignment forces corresponding changes in the units of all other electromagnetic quantities which enables the resulting method to satisfy all the relationships covered by the SI method. The key advantage of the Nms method over the SI counterpart is that it is completely compatible with the relativistic Uniform Scaling procedure which determines conversion factors for the numerical results obtained in one rest system to those of any other. There are two fundamental quantities required for this purpose, one for kinetic scaling (Q) and the other (S) for gravitational scaling. Since the conversion factors for m, kg and s are known in terms of Q and S, it is therefore possible to determine the corresponding factors for all other physical quantities based on their mks composition. This is impossible for the strictly electromagnetic SI units, but is easily attainable when the Nms system of units is employed.

Keywords: SI System of Units. Electromagnetic Quantities, Nms System of Units, Laws of Thermodynamics, Euler's Equation, Quantum Mechanical Units, Uniform Scaling Method

1. Introduction

The classic investigations beginning in the late 19th century into the field of electromagnetism not only led to new laws of physics but also the need for the development of new quantities such as electric charge and magnetic fields in which to express them. This led to more general questions as to how best to adapt this new notation to be compatible with the common designations involving inertial mass, time and distance. A major step in this direction was taken by Giorgi in 1901 [1] that insured that the results of electromagnetic equations can be expressed in these standard (mks) units, even though the laws themselves were still formulated in terms of the Coulomb, Ampere, Tesla etc. Because of a degree of freedom in the definition of the laws of electricity and magnetism, however, it is possible to go even further [2-4] and express these quantities directly in the mks system of units, as is discussed below. In addition, this procedure makes it possible to have these quantities directly conform to the Uniform Scaling Method [5-7] which allows observers in one rest frame to convert the numerical values of their measurements to the units employed in another rest frame, which was a major goal of Einstein's original version of relativity theory [8].

2. Relating the Units of Electricity and Magnetism Directly to the mks System

A good place to start is with Coulomb's Law of Electrostatics. It is given below in scalar form in terms of a product of electric charges q_1,q_2 and the momentary distance r between them:

$$F = q_1q_2 / 4\pi\varepsilon_0 r^2$$

In this equation F in N = $kgms^{-2}$ is the force between the charges, q_1 and q_2 are the charges, ε_0 is the permittivity of free space and r is the distance in m between them. There is a degree of freedom [2, 3] in the choice of units for electric charge (Coul) and ε_0 ($Coul^2s^2/kgm^3$), however. It is only required that the units of q_1q_2/ε_0 be equal to that of the product Fr^2, which in the standard mks system is Nm^2. A convenient choice is therefore to use Nm = J to replace Coul and to use N for the ε_0 unit. Note that the standard numerical value of ε_0 (8.8541878 x 10^{-12}) is not affected by this replacement.

Table 1: List of electromagnetic quantities and their units in the SI and Nms systems. (Conversion factors for measured values in two different rest frames are given in the first column (see Sect. V))

Property	Conversion Factor	Nms Unit	SI Unit
Electric Charge	QS	Nm	Coul
Electrical Current i	S^2	Nm/s	Coul/s = A
Permittivity ε_0	S	N	s^4A^2/kgm^3
Magnetic Field B	$(QS)^{-1}$	s/m^2	Tesla=Wb/m^2=N/Am
Magnetic Flux	Q/S	s	Wb
Permeability μ_0	S^{-3}	s^2/Nm^2	N/A^2
Magnetic Intensity H	S^2/Q	N/s	A/m
Electric Field E	1/Q	1/m	Coul/ ε_0 m^2
Electric Dipole Moment	Q^2S	Nm^2	Coul m
Magnetic Dipole Moment	Q^2S^2	Nm^3/s	m^2A = Nm/Tesla
Volt	1	1	Coul/ ε_0 m
Electrical Resistance	S^{-2}	s/Nm	$kgm^2/sCoul^2$ = Ohm=Volt/i
Electrical Conductance	S^2	Nm/s	i/Volt = siemans = mho
Electrical Conductance	Q/S^3	s^2/Nm	$m^2kg/Coul^2$ = henry
Capacitance	QS	Nm	Coul/Volt=Farad
Charge density ρ	S/Q^2	N/m^2	$Coul/m^3$
Quadrupole Moment	Q^3/S	Nm^3	Coul m^2
Electric Polarization P	S/Q	N/m	$Coul/m^2$
Electric Displacement D	S/Q	N/m	$Coul/m^2$
Magnetization M	S^2/Q	N/s	A/m
Magnetic Susceptibility χ_m	1	1	H/M
Electric Susceptibility χ_e	1	1	P/ε_0 E
Polarizability	SQ^3	Nm^3	s^2Coul^2/kg
Coefficient of Potential	$(QS)^{-1}$	$(Nm)^{-1}$	Volt/Coul

It wouldn't make sense to replace the Coulomb with the Joule unless analogous substitutions were also effective, but this is indeed the case [2, 3], as illustrated in Table 1. Consider the magnetic permeability μ_0, for example. It must satisfy the equation from Maxwell's electromagnetic theory (law of Biot and Savart [9]; c = 299792458 m/s):

$$\varepsilon_0\mu_0c^2 = 1.$$

Its value in standard theory is $4\pi/10^7$ N/A^2 (A=Coul/s), which means that the numerical value for ε_0 is $10^7/4\pi$ c^2, as shown above. Upon replacing Coul with Nm, the corresponding unit absent of electromagnetic quantities is $Ns^2/Coul^2 = s^2/Nm^2$, as shown in Table 1. Multiplying this unit with that of ε_0 (N) gives s^2/m^2 and therefore the unit for c^{-2} required in the above law. It should be noted that the value for c is fixed by international convention [10], so the value of ε_0 is also fixed by this convention. Accordingly, c can be seen as determining the value of the meter, whereas ε_0 ultimately defines the value of the Coulomb.

The system of units described above is referred to as the Nms system [3], which distinguishes it from the standard SI mks system which includes units of electromagnetic origin. The Ampere (A) has already been defined as the unit of electric current i and thus has a unit of Coul/s in the SI system. In the Nms system its unit is therefore Nm/s = J/s (Table 1).

The unit of the Volt (V) is defined as Coul/ε_0 m = Coul $kgm^2/s^4A^2 = kgm^2/s^3A$ in the SI system. This makes the Volt a dimensionless quantity in the Nms system, i.e. kgm^2/Coul s^2 translates to $kgm^2/Nms^2 = kgm^2/kgm^2$. The unit of electrical resistance R is the Ohm = Volt/A = kgm^2/s^3A^2 in the SI system. That translates to kgm^2/N^2m^2s = s/Nm in the Nms system, the reciprocal of the unit of current, i.e. V = iR. The unit of electrical conductance (siemens) is the reciprocal of electrical resistance in the SI system, i.e. i/Volt = s^3A^2/kgm^2. It therefore has the same unit as electrical current (Nm/s) in the Nms system. The unit of capacitance (Farad) Coul/Volt. which translates to Nm = J in the Nms system. The henry is the unit of inductance L in the SI system. It is defined by the equation V= L di/dt. Therefore, it has the unit of kgm^2/s^2A^2 in the SI system, which translates to s^2/Nm in the Nms system. The unit of electric field E = q/ ε_0m^2 is Coul/ε_0m^2 in the SI system. It is therefore 1/m in the Nms system. The units of other electrostatic quantities such as electric dipole and quadrupole moments, polarizability, charge density, electric polarization P, electric displacement D, electric susceptibility χ and coefficients of potential p_{ij} are compared in Table 1.

The SI unit of magnetic field strength B is Tesla. It is equal to kg/As^2 = N/Am = Ns/Coul m. It is also equal to Wb/m^2 in notation used prior to 1960, whereby Wb is the unit of magnetic flux = kgm^2/As^2. The corresponding Nms unit for Wb is equal to kgm^2/Nms = s. The Nms unit for Tesla is thus s/m^2. This comes directly from the Tesla = Wb/m^2 relation.

The property of magnetic intensity H is equal to B/μ_0. Its SI unit is therefore A/m which translates into N/s = $kgms^{-3}$ in the Nms system. The SI unit of Magnetization M is also

A/m, so the corresponding Nms unit is again N/s. The magnetic dipole moment unit is equal to m^2A = J/Tesla in the SI system. This translates to Nm^3/s in the Nms system.

3. Laws of Thermodynamics and Euler's Equation

According to the First Law of Thermodynamics, the differential change in internal energy dU is equal to a number of binary products such as TdS and PdV (T is the temperature of the body, S is its entropy, P is the applied pressure and V is the associated volume) [11]. In each case, one of the variables is intensive (T and P) and the other is extensive (S and V). If all of the energy components are changed by a definite factor λ, without changing any of the values of the intensive quantities, the result is that the corresponding value of U is changed by the same factor.

This circumstance leads to what is known as Euler's Equation, in which U is equal to the sum of products of respective intensive and extensive variables such as TS and PV.

The list of such products can be extended to include pairs of purely electromagnetic quantities, such as electric field E and electric dipole moment μ_e on the one hand, and the magnetic field strength B and the magnetic dipole moment $\mu_{m.}$ The unit of each of these products must clearly be the same as for U itself (J= Nm). For example, pressure P is equal to force F per unit square meter, i.e. N/m^2, which when multiplied with the volume V (m^3) results in the required value for the corresponding unit of Nm. There is no distinction between the SI and Nms systems in this case. The same is true for the entropy/temperature product; S has a unit of Nm whereas T is dimensionless.

This situation changes for the products involving electromagnetic quantities. In the SI system, the electric field E has a unit of $Coul/\varepsilon_0 m^2$= kgm/Coul s^2, whereas that of the electric dipole moment μ_e is mCoul. The product of the units is thus kgm^2/s^2= Nm, as required. The situation with the Nms system is as follows. The unit of E is m^{-1}, while that for μ_e is Nm^2, so the product is indeed also Nm.

The product for magnetic interactions is $B\mu_m$. The corresponding unit of this product in the SI system is Tesla m^2A (Table 1) = (N/Am) m^2A = Nm, as required. In the Nms system, the unit of B is s/m^2 while that for μ_m is Nm^3/s. The product of these units is again Nm. Thus, the requirement from Euler's Equation that such products have the same unit as U is fulfilled in both cases.

4. Comparison of Quantum Mechanical Relations

Another key set of units is used in quantum mechanical calculations, namely the set of atomic units. It is distinguished by the fact that certain physical quantities have unit values. For example, the unit of angular momentum is equal to Planck's constant h (6.626 x 10^{-34} Js) divided by 2π. The permittivity of free space ε_0 equals 1/4π and the electronic charge e (1.602 x 10^{-19} Coulomb) also has a unit value in the atomic system. In actual calculations, these assignments can cause confusion because certain key quantities are suppressed. The fine structure constant α is defined in the SI system as $e^2/2\varepsilon_0 hc$. It is dimensionless and has a value of 7.29735254 × 10^{-3}. In atomic units α = 1/c, i.e. the speed of light in atomic units is α^{-1} = 137.036, which is also dimensionless.

In many ways it would be better if quantities such as e, h and ε_0 appeared explicitly in the appropriate formulas when atomic units are used, but this is usually not the case. The Bohr radius a_0 is a key example. In the SI system it has a value of $(4\pi\varepsilon_0)(h/2\pi)^2/e^2m_e = h^2\varepsilon_0/\pi e^2 m_e$ = 5.29236 x 10^{-11}m (often given as 5.2917 x 10^{-11}m). The unit of m is obtained by appropriate substitution (see Table 1), namely as $(Nms)^2(s^4A^2/kgm^3)/Coul^2kg = kg^2m^4$ $s^{-2}(Coul^2s^2/kgm^3)/Coul^2kg$ = m. In the Nms system the unit of m is obtained $(Nms)^2$ N/ $N^2m^2kg=Ns^2/kg$ = m. It should be noted that all the factors in the Bohr radius formula are atomic units for the corresponding property and thus the value in this system is unity.

The operator replacement scheme introduced by Schrödinger often makes use of atomic units. For example, the momentum operator is equal to -id/dx with h/2π suppressed in the numerator. In the SI system the corresponding unit is Nms/m = Ns, the same as in the Nms system. The corresponding operator for energy is $-kg^{-1}d^2/dx^2$ with $(h/2\pi)^2$ suppressed in the numerator. The SI product of units is therefore $(Nms)^2/kgm^2 = Nm(Nms^2)/kgm^2$ = Nm, which is the SI unit of energy; the same arguments also hold for the Nms system.

The atomic unit for magnetic moments is the Bohr magneton=e/m_e which also has a value of 1. In the SI system this translates to a unit of m^2Coul/s (since h/2π is suppressed) = Nm/Tesla, so multiplication with the unit of magnetic field (Tesla) gives the required result of Nm = J for energy. In the Nms system, the magnetic moment unit is Nm^3/s, while the unit for B is s/m^2, so the product is again J.

The atomic unit for Coulomb energy is 1/m since e is the unit of electric charge. In the SI system the unit is $Coul^2$/ m ε_0 = $Coul^2/(Coul^2s^2/m^2kg) = m^2kg/s^2$ = Nm, whereas in the Nms system the unit is N^2m^2/mN = Nm as well. The formula for LS spin-orbit coupling and related Breit-Pauli interactions is proportional to $(\alpha^2/\varepsilon_0)(e/m_e)^2$ LS h^2/m^3. In this equation α is the reciprocal of the speed of light in free space and L, S are dimensionless quantum numbers. The overall unit in the SI system is thus $[(s/m)^2/(Coul^2s^2/m^3kg)]$ $(Coul^2/kg^2)(N^2s^2/m) = kg^{-1}N^2s^2$= Nm. In the Nms system the overall unit is $[(s/m)^2/N](N^2m^2/kg^2) N^2s^2/m = N^3s^4/kg^2m$ = Nm=J, in agreement with the SI system.

5. Uniform Scaling of Electromagnetic Quantities

The basic idea for the Uniform Scaling method [5-7] is that an observer in one rest frame employs different units in which to express the numerical values for its physical properties. Einstein [12] introduced this concept in 1907 by assuming that the rates of all clocks vary in the same manner as they are accelerated. Experiments have shown that each property has its own conversion factor for two rest frames. A similar situation exists for changes in gravitational potential. Each of the *kinetic* conversion factors is an integral multiple of the fundamental quantity Q; the gravitational conversion factors are all integral multiples of the fundamental quantity S. Knowledge of the composition of a given property in terms of the key quantities of distance, time and inertial mass therefore allows one to compute the conversion factor for that quantity. The values for these fundamental quantities are Q, Q/S and Q/S, respectively. As a result, the conversion factor for relative speed, including the speed of light, is therefore Q/(Q/S) = S. The corresponding factor for energy is obtained from the formula for kinetic energy ($0.5m_iv^2$) and therefore has a value of (Q/S) S^2 = QS.

A basic assumption in the Uniform Scaling method is that every pair of rest frames in the entire universe is characterized by specific values of Q and S. In the laboratory when an object is accelerated to speed v, the value of Q is γ (v) = $(1-v^2/c^2)^{-0.5}$, for example. When it is raised by a distance of h near the Earth's surface, the value of S is 1 + gh/c^2 (g is the local acceleration due to gravity).

There is a close connection between Uniform Scaling and Galileo's Relativity Principle (RP) [5-7]. The laws of physics are the same in each inertial system, as the RP states, but the units in which they are expressed differ from one rest frame to another (Addendum to the RP [7]). As a consequence. *the products of the Q and S factors always cancel one another.* For example. in Einstein's $E=m_ic^2$ mass-energy equivalence relation, one has QS both sides. Similarly, for Planck's E=hν energy frequency relation, h has conversion factor Q^2 and ν has a factor of S/Q (the reciprocal of that for time), so the result on both sides is again QS.

It is nonetheless obvious that one cannot apply Uniform Scaling directly for electromagnetic quantities. This is the motivation for deriving the Nms set of units. In Table 1 the conversion factor for each property is listed next to the corresponding Nms unit. For example, the unit of charge is Nm, and thus the conversion factor is QS. i.e. S for N and Q for m. The conversion factor for ε_0 is S since the Nms unit is N. The Nms unit for μ_0 is s^2/Nm^2 so the conversion factor is $(Q/S)^2/SQ^2 = S^{-3}$, in fulfillment of the Biot-Savart Law [9] $(SS^{-3}S^2) = 1$.

The conversion factor for Ampere is S^2, in agreement with its Nms unit of Nm/s, i.e. SQ/(Q/S). The Volt is dimensionless in the Nms system, so its conversion factor is 1. The Nms unit of resistance R (Ohm) is s/Nm and the corresponding conversion factor is thus $(Q/S)/SQ = S^{-2}$, the reciprocal of A, so that iR=Volt is dimensionless. The siemens has the same Nms unit as A. The henry L has a conversion factor of QS^{-3} consistent with its Nms unit of s^2/Nm. i.e. $(Q/S)^2/QS$. It satisfies the equation V = L di/dt, i.e. di/dt = L^{-1}.

The Nms unit of magnetic field strength B (Tesla) is s/m^2, so its conversion factor is $(SQ)^{-1}$, the same as for J^{-1}. The conversion factor for Wb is Q/S, the same as for time. The conversion factor for magnetic intensity H is equal to S^2/Q; this is equal to the conversion factor for B/μ_0, as well as for magnetization M. The conversion factors and Nms units for many other electromagnetic properties are also given in Table 1.

6. Conclusion

It is possible to replace the SI system of units for electromagnetic quantities such as Coul and Tesla with another system which makes exclusive use of the mks system of units, i.e. m, kg and s. This is possible because of a degree of freedom in the SI definitions, whereby an arbitrary assignment of the mks unit of Coulomb leads unequivocally to corresponding units for all other electromagnetic properties. The Nms system of units discussed above exchanges the Coulomb for Nm=J, and this forces the unit of electric permittivity (ε_0) to be N and the unit for magnetic permeability (μ_0) to be s^2/Nm^2. The units for many other quantities are listed in Table 1.

The main objective for invoking the Nms system is certainly not to eliminate the admirable characteristic of the SI method of memorializing the names of the pioneering authors in this field. The Uniform Scaling method, which is very useful in relativistic theory because it allows one to define conversion factors between the units used to express the numerical values of properties in one rest frame and those employed in another, cannot be used if only the SI units are available, however. The scale factors depend on two fundamental quantities, the kinetic factor Q and the gravitational factor S, and they are assigned for the mks units of distance (Q), inertial mass and time (Q/S) each. Thus, by employing the Nms method of units, it is immediately possible to designate conversion factors for every electromagnetic quantity simply by knowing their composition in terms of the mks fundamental properties, as also shown in Table 1.

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