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Investigation of Mean Free Path, Macroscopic Cross- Sections, and Total Cross-Sections in Particle-Induced Reactions for Production of Theranostic ^{186}Re Using Geant4

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Abstract
The study of Mean Free Path (MFP), Macroscopic Cross-Section (MCS) and Total Cross- Section (TCS) as a function of incident energy for particle induced reactions is receiving much interest due to their importance in applications, such as medical radionuclide production and optimization, radiotherapy, shielding design and geological dating studies. This paper investigated the MFP, MCS and TCS as functions of alpha, deuteron, proton and neutron particles on ^{186}Os targets in order to model nuclear reactions for the production and optimization of theranostic radionuclides with the use of GEANT4, a C++ based Monte Carlo simulation toolkit that offers visualization, graphical user interface and several options to customize target geometry, size, beam energy as well as irradiation time. The findings reveal the shortest MFP to be 2.9 cm, peak MCS as 0.35 cm^{-1}, and peak TCS was 0.34 b from neutron interactions at 2.5 MeV. For proton, deuteron and alpha particles interactions, their maximum MCS and TCS values manifest at energies in range between 17.5 MeV and 25 MeV which indicate apparent analogy in their behaviour in ^{186}Os at high energies. Further, the results also reveal the excitation function datasets for the ^{186}Os (n.p)^{186}Re reaction which demonstrated good agreement with evaluated nuclear data files (ENDF) and experimental data (EXFOR). The new nuclear data will be invaluable for radiopharmaceutical manufacturers in producing ^{186}Re to sustain the theranostic approach.

Keywords: GEANT4, Mean Free Path, MCS, TCS, Radiotherapy, Theranostic

1. Introduction

Sustainable practice of modern nuclear medicine relies upon consistent, regular and uniform supply of radionuclides in the appropriate physical and chemical states for the formulation of radiopharmaceutical drugs, generators and radiation sources calibration (Uddin *et al.*, 2023) [10]. The application of radionuclides in both diagnostic and therapeutic nuclear medicine has become well established, providing solace to millions of patients in developed and developing countries suffering from complex anomalies such as cancers of different organs as well as pain syndromes emanating from various morbidities (Rösch *et al.*, 2017) [7]. However, the course of development of modern nuclear medicine traversed several approached inundated with operational challenges that characterize the practice right from its inception in 1941. Among the contemporary challenges is the radiation overdose syndrome which emanates from rising absorbed radiation dose caused by the one-dose-fits-all administration of internal radiotherapy (Li *et al.*, 2017) [4]. This brought to light the variation of radiation dose absorption in different patients according to their individual/personal characteristics. In addressing this, administration of SPECT radionuclides as surrogates began in order to acquire pretherapy information on bio distribution and patient dosimetry such that therapy planning will be achieved where the required lethal radiation dose can be delivered to the diseased tissues without harming healthy ones. This individualized information thereafter facilitates the use of higher dose targeted molecular therapy in the same patient which is referred to as 'theranostic approach' (Nagai *et al.*, 2022) [6]. Theranostics is essentially the integration of diagnostic and therapeutic radionuclides on one drug vehicle so that one nuclide diagnoses and the other treats. The approach has several advantages comprising, higher resolution imaging, increased treatment efficacy, reduction treatment time and cost as well as avoidance of other auxiliary procedures (Yordanova *et al.*, 2017) [12]. Theranostic radionuclide application comes in either matched pair, where both the diagnostic and therapeutic components are of the same element such as ^{86}Y/^{90}Y or in mismatched pair where the two components are of different elements such as ^{68}Ga/^{177}Lu.

A few radionuclides qualify as true theranostics going by the theranostic criteria (Türler, 2019) [8] which provide that the half-life has to be between 2 hours to 24 hours and 2 days to 10 days for diagnostic and therapeutic parts respectively. Other prerequisites include low gamma energy emission, production route to be low energy reaction to name a few. Further, the diagnostic part should be a low energy positron emitter suitable for PET/MRI while the therapy part should be a β^- or Auger electron emitter capable of being produced in large quantities in GBq and in no carrier added form. This perhaps may apparently be the reason behind paucity of such nuclides making the need to explore non-reactor routes in prospecting for theranostic radionuclides. Nuclear reactions obviously are the means to generate such nuclides and they involve particle – target interactions that lay among the purviews of this work.

Several studies have been performed to explore the production of ^{186}Re. Uccelli *et al.* (2022) [9] reported ^{186}Re production from the ^{185}Re (n,g)^{186}Re reaction which, at 15.9 MeV, an integral yield of 1.25 MBq/A/h was achieved and could be increased with enrichment (Uccelli *et al.*, 2022) [9]. However, despite the advancements in nuclear reaction theories and the expansion of global nuclear reaction databases, the need for new nuclear data remains pressing (Usman & Ahmad, 2022) [11]. The current understanding of nuclear reaction mechanisms for the production of medical radionuclides has been outpaced by their demand, resulting in an apparent production data rift. This situation precipitates paucity of low energy nuclear data needful for the production of radionuclides for theranostic applications which bring about their indubitable scarcity thereby manifesting inaccessibility of theranostic advantages for the patient. To surmount this hurdle henceforth, need exists for exploring low energy reaction routes for optimal and sustainable production of medical radionuclides to sustain theranostic applications.

This paper examined the interactions of alpha particles, deuterons, neutrons and protons with ^{186}Os target nuclei, to evaluate MFP, MCS, TCS and their variation characteristics in low energy nuclear reactions, for the production of ^{186}Re isotope.

2. Theoretical Background

The transport of incident particles in nuclear target involves several parameters which are affected by factors such as energy, size, charge of the particle etc. Among the transport parameters are:

Mean Free Path (MFP) is the average distance a particle travels before interacting with a nucleus. It is a measure of how a particle can travel in a medium before colliding with a nucleon. Mathematically, it is expressed as:

$$\lambda = \frac{A}{\sqrt{2}\,\pi d^2 \rho} \tag{1}$$

Where λ is MFP, A is the atomic mass, d is the diameter of the nucleus and ρ is the density of the target (Luoni *et al.*, 2021) [5]. In addition, mean free path is a crucial parameter in chain reaction management, adjusting therapeutic dose in tissues as well as in radiation shielding design.

The macroscopic cross-section (Σ) indicates the probability of interaction per unit length. It relates to the MFP as it's reciprocal. The Total reaction Cross Section (σ_t) - combines all possible reaction channels (elastic, inelastic, capture and fission processes). For a transparent sphere target, it is expressed as:

$$\sigma_t = \pi R^2 \left(1 - \frac{\left[1 - (1 + 2\sum R)e^{-2R\Sigma}\right]}{2R^2 \sum^2}\right) \tag{2}$$

where R is the radius of the nucleus as

$$R = R_0 \left(A_p^{\frac{1}{3}} + A_T^{\frac{1}{3}}\right) \tag{3}$$

where R_0 is reduced nuclear radius parameter, A_p and A_T are the mass numbers of projectile and target, respectively.

Measurements of **Cross-section (σ)** for low energy reactions (< 100 MeV) display strong dependence on incident particle energy which translates to specific radionuclide yield dependence on bombarding energy and is referred to as Excitation function (Zubaida & Ahmad, 2019). This parameter provides an instrumental guide in the selection of optimum irradiation conditions that will maximize production of desired radionuclide while minimizing that of unwanted contaminant radionuclide.

$$\sigma = \pi\left(R_p^2 + R_t^2\right) ln\left[1 + \frac{A_p A_t\, \sigma_t}{\pi\left(R_p^2 + R_t^2\right)}\right] \tag{4}$$

where σ is inelastic cross section R_p and R_t are projectile and target radius respectively.

3. Method

This work employed GEometry ANd Tracking version 4 (**GEANT4) toolkit**, CMake 3.3, Visual Studio 2022, **Microsoft Excel, ENDF and EXFOR database libraries** to model alpha particle, deuteron, neutron and proton interactions with ^{186}Os targets. The selected physics list was QGSP_BERT_HP which combines three physics processes namely Quark Gluon String Parton, Bertini cascade and High Precision processes, to simulate different reaction stages, from initial nucleon interactions to final de-excitation. **The target geometry was** modeled as cubical slab to minimize particle self-absorption effects. A monoenergetic particle beam (protons, neutrons, deuterons or alpha) was configured with energies ranging from 0 to 25 MeV. The maximum energy selection was based on the criteria that theranostic candidate nuclides must be produced using low energy reactions achievable in common accelerators/cyclotrons affordable to hospitals. For the investigation of MFP, MSC and TSC to evaluate nuclear reactions involved in theranostic nuclides production, equation (1) and Equation (2) have been used. QGSP_BERT_HP physics list was used to evaluate the reaction channels and the reaction cross sections for theranostic isotopes production.

The output displayed the MFP, MSC, TCS and other parameters. Multiple runs were conducted for each configuration to ensure reproducibility and minimize random fluctuations. Output data was processed using MS Excel and plot the graphs of MFP, MSC and TSC as a function of particle energy. Simulated cross-sections were compared against experimental datasets EXFOR and evaluated datasets ENDF from Nuclear Data Section (NDS) of the IAEA, for comparison and validation of simulation accuracy.

4. Results and Discussion

The observed trends highlighting differences of MFP, MCS, and TCS with proton, deuteron, neutron and alpha particles in ^{186}Os target are presented in Figures 1-3

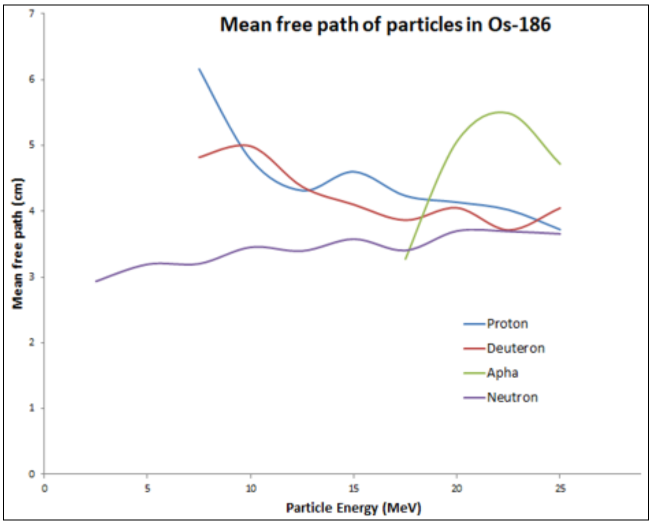

Fig 1: Mean free path of alpha, deuteron, neutron and proton interaction with Os-186

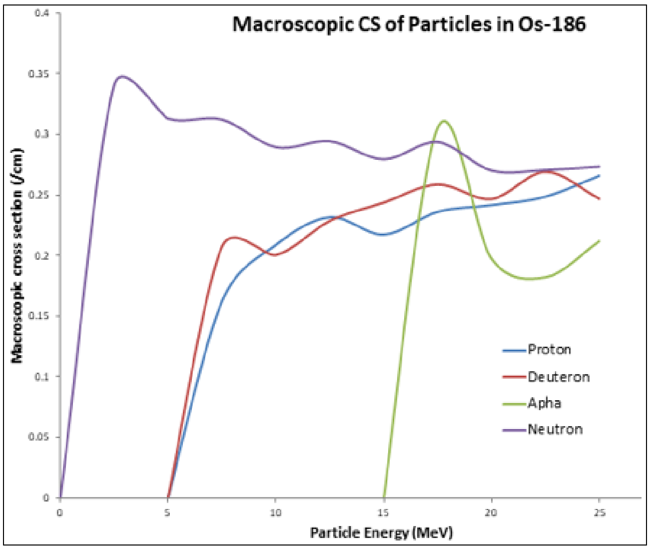

Fig 2: Macroscopic cross-section of alpha, deuteron, neutron and proton interaction with Os-186

Fig 1 shows the MFP function in ^{186}Os where neutrons trended slightly undulating MFP throughout the reaction implying apparent immunity from rising energy effects. Protons and deuteron displayed decline in MFP as they converge at 12.5 MeV. Both particles then steadily declined onwards to converge with neutrons at 22.5 MeV to 25 MeV range clearly denoting achieving analogous interaction rates in this range. Alpha exhibited minimal MFP hence rising interactions up to 17.5 MeV then steadily drops to converge with other incident particles at 22.5 MeV. This suggests that high energies propel particles into uniform rate of interactions irrespective of type.

Fig 2 shows variations in ^{186}Os target, with peak MCS for neutron particles at 2.5 MeV. 7.5 MeV was threshold for deuteron and proton particles but they exhibit peak MCS at 7.5 MeV and 12.5 MeV respectively. MCS of these particles show responsive rise with incident energy increase and appear to converge at 22.5 – 25 MeV indicating analogy of interaction rates of the particles. Alpha is the only particle with high threshold of 15 MeV and its MCS reached peak of 0.32b at 17.5 MeV which plummets slightly before rising to converge with other particles at 25 MeV

In Fig 3 neutron shows peak TCS at 2.5 MeV threshold with steady decline while proton and deuterons with similar threshold and rising TCS up to mid energies, then show convergence of their values at 22.5 – 25 MeV. This indicates they perform analogous interactions. Alpha threshold of 17.5 MeV trended small TCS rise up to 20 MeV before stabilising at 25 MeV with the lowest TCS in the simulation.

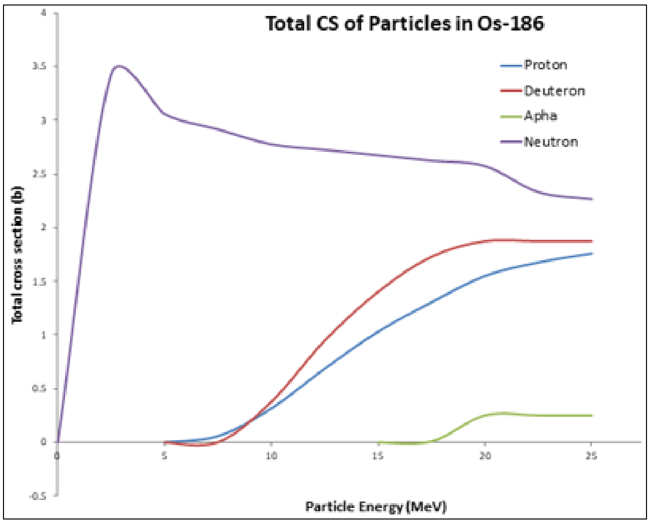

Fig 3: Total cross-section of alpha, deuteron, neutron and proton interaction with Os-186

Excitation Functions for the Production of Theranostic Radionuclides

Calculated excitation functions for the reactions that produce theranostic radionuclides are presented below along with their EXFOR/ENDF comparisons for validation in Fig 4.

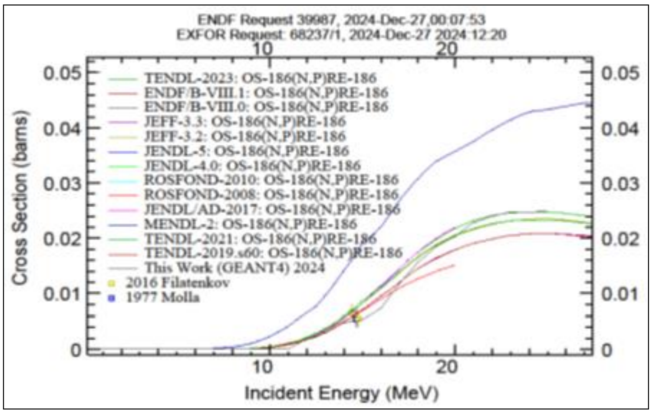

Fig 4: Excitation function of Os-186(n,p)Re-186 reaction

Production of **^{186}**Re from ^{186}Os (n,p) reaction is given in Fig 4, it shows excitation function of ^{186}Os (n,p)^{186}Re reaction where a good agreement is notable with other evaluated unclear data files such as TENDL 2023, ENDF/B VIII.I, JEFF-5, JENDL-4, ROSFOND (2010), MENDL-2 and TENDL 2019 and 2021. GEANT4 simulation data appears though to be lower than other evaluated datasets. JENDL-5 shows much higher values but this simulation indicates similar threshold to most of evaluations. Thus with the good agreement of nuclear evaluations this simulation can be regarded as reliable. Re-186 has a half-life of 3.7 days and emits moderate β^- at energies 1.07 MeV and 0.939 MeV (22%). Its low energy gamma emission of 137 keV allows *in vivo* tracking of radiolabelled biomolecules and for calculations of dosimetric estimates. The radionuclide is usually paired with I-124 for theranostic applications.

5. Conclusion

In this paper the particle – energy cross section variations for proton, deuteron, alpha and neutron particles have been investigated using GEANT4 simulation toolkit for the production of theranostic radionuclides. The findings show the shortest MFP of 2.9 cm for neutron interaction at 2.5 MeV which clearly reveal the high interaction affinity for neutron particles due to its neutral charge which excludes Coulomb repulsion hence the low neutron threshold. Peak MCS of 0.35 cm^{-1} was also realised for neutron interactions at 2.5 MeV. Proton deuteron and alpha particles exhibit their peak MCS values at range between 17.5 to 25 MeV implying their trend analogy at higher energies. TCS similarly showed remarkable analogy in maximum values for deuteron proton and alpha within 20 MeV to 25 MeV, while neutrons exhibited highest value of 3.5b at

2.5 MeV threshold. This implies that the neutron is the most interactive particle for ^{186}Os target in 0-25 MeV energy range. This paper has also generated new nuclear data set for the production of ^{186}Re using ^{186}Os (n, p) which will be of invaluable importance for radiodrug manufacturers and will also serve as a good index for excitation function measurements where such data is summarily not existing.

6. References

1. Choiński J, Łyczko M. Prospects for the production of radioisotopes and radiobioconjugates for theranostics. Bio-Algorithms and Med-Systems. 2021; 17(4):241-257. Doi: https://doi.org/10.1515/bams-2021-0136
2. Koning AJ, RD. Modern Nuclear Data Evaluation with the TALYS Code System. Journal of Nuclear Physics. 2011; 12:560-571.
3. Kumar K, Ghosh A. Radiochemistry, production processes, labeling methods, and immunopet imaging pharmaceuticals of iodine-124. Molecules. 2021; 26(2). Doi: https://doi.org/10.3390/molecules26020414
4. Li T, Ao ECI, Lambert B, Brans B, Vandenberghe S, Mok GSP. Quantitative imaging for targeted radionuclide therapy dosimetry - Technical review. Theranostics. 2017; 7(18):4551-4565. Doi: https://doi.org/10.7150/thno.19782
5. Luoni F, Horst F, Reidel CA, Quarz A, Bagnale L, Sihver L, *et al*. Total nuclear reaction cross-section database for radiation protection in space and heavy-ion therapy applications. New Journal of Physics. 2021; 23(10):101201. Doi: https://doi.org/10.1088/1367-2630/ac27e1
6. Nagai Y, Kawabata M, Hashimoto S, Tsukada K, Hashimoto K, Motoishi S, *et al*. Estimated Isotopic Compositions of Yb in Enriched 176 Yb for Producing with High Radionuclide Purity by 176 Yb (d, x) 177 Lu. 2022; 044201:1-10. Doi: https://doi.org/10.7566/JPSJ.91.044201
7. Rösch F, Herzog H, Qaim SM. The beginning and development of the theranostic approach in nuclear medicine, as exemplified by the radionuclide pair 86Y and 90Y. Pharmaceuticals. 2017; 10(2). Doi: https://doi.org/10.3390/ph10020056
8. Türler A. International Year of the Periodic Table 2019: Elements important for Life Sciences. 2019; 73(11):947-949. Doi: https://doi.org/10.2533/chimia.2019.947
9. Uccelli L, Martini P, Urso L, Ghirardi T, Marvelli L, Cittanti C, *et al*. Rhenium Radioisotopes for Medicine, a Focus on Production and Applications. Molecules. 2022; 27(16):1-19. Doi: https://doi.org/10.3390/molecules27165283
10. Uddin MS, Basunia MS, Spahn I, Spellerberg S, Khan R, Uddin MM, *et al*. Cross sections and calculated yields of some radionuclides of yttrium, strontium and rubidium formed in proton-induced reactions on enriched strontium-86: Possibility of production of 85gSr, 83Rb and 82mRb in no-carrier-added form. Radiochimica Acta. 2023; 111(2):81-90. Doi: https://doi.org/10.1515/ract-2022-0086
11. Usman AR, Ahmad AA. Evaluation of 67Ga Cross-sections Using Exifon Code for Medical Applications. FUDMA Journal of Sciences (FJS). 2022; 6(3):113-118.
12. Yordanova A, Eppard E, Kürpig S, Bundschuh RA, Schönberger S, Gonzalez-Carmona M, *et al*. Theranostics in nuclear medicine practice. OncoTargets and Therapy. 2017; 10(January):4821-4828. Doi: https://doi.org/10.2147/OTT.S140671