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Numerical Calculation of the Angle of Displacement of Star Images during Solar Eclipses

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Abstract
A finite differences procedure is presented for calculating the angle of displacement of star images during solar eclipses. It is based on Schiff's paper in 1960 in which he obtained results using a purely analytical method. A key feature of both methods is the assumption that light follows a strictly straight-line trajectory on its way between a given star and the Earth's surface, in contradiction to the claim of Einstein's General Relativity Theory (GRT) that it follows a curved trajectory instead. The calculations described herein make use of a quantity $S(r) = (1 + Gm_0/c^2r)^{-1}$ (G is the Universal Gravitation Constant, m_0 is the Sun's mass, c is the speed of light in free space and r is the distance separating the current position of the light from the Sun's center of mass). The speed of light varies in direct proportion to S (r). The result is that of two light rays moving parallel to the Sun, the one that is farther away moves at a higher average speed and travels further during the same time period. The ratio of the increased distance of travel ΔX to the corresponding distance ΔY separating the two light rays is the displacement angle Θ of the star images from their normal position. The finite differences result for Θ is shown to be in quantitative agreement with Schiff's analytical value, which in turns agrees very well with the result inferred from the GRT closed formula for Θ. The main point of Schiff's paper was to show that his method is much easier to apply than GRT, and this assertion is verified by the present finite differences approach. It is also noted that the angle of precession of Mercury's perihelion, another of Einstein's great successes, is also predicted to be in quantitative agreement with the GRT value by extending Schiff's scaling procedure to the acceleration due to gravity g.

Keywords: Angle of Displacement of Start Images Θ, Schiff's Scaling Method, Straight-line Trajectory of Light, Finite Differences Procedure, General Theory of Relativity (GRT)

1. Introduction

One of the key predictions of Einstein's General Theory of Relativity (GRT) [1] is the displacement of star images which is observed during solar eclipses. In recent work [2], it has been shown that the same value for the angle of displacement can be obtained in a relatively simple manner using a method of finite differences. This approach has been suggested by Schiff in 1960 [3] in which his results were obtained analytically.

The present work gives a detailed account of how such a finite differences calculation can be carried out in practice. It should be noted at the outset that a basic assumption [4] in this approach is that *light travels in a straight line* on its way from a star to the Earth's surface and beyond. The same assumption was used by Schiff. This assumption contrasts sharply with the GRT view that light follows a curved path [1].

2. Step-by-Step Execution of the Numerical Method

The first step in the calculation is to compute the value of the ratio Gm_0/c^2; G =6.6743x 10^{-11} m^3/s^2kg is the Universal Gravitation Constant, m_0 = 1.9891×10^{30} kg is the gravitational mass of the Sun and c = 299792458 m/s is the speed of light in free space. It has a value of 1477. 137 m. The origin of the coordinate system is located at the Sun's center of mass. A line is drawn along the y axis to a position Y_s which is separated from the Sun's corona by a distance of 696340 m. Next a line is drawn among the x axis to a distance of X_0. In the original calculation [2] carried out with this method, it has a value of 10^{12} m. At the outset, the star is assumed to be located at this point in space, i.e., with the coordinates (X_0, Y_s).

The next stage in the calculation is to be repeated with a succession of x values. To start with, the value of the distance separating this point from the Sun's center of mass is equal to r= $(X_0^2 + Y_s^2)^{0.5}$. Next the angle $\alpha = \cos^{-1}(Y_s/r)$ is computed. The key parameter S is then computed as S= $(1 + Gm_0/c^2r)^{-1}$. It is next necessary to resolve the light's velocity vector into its radial and transverse components. The radial component is equal to c sin α while the corresponding transverse component is equal to c cos α. According to Schiff's scaling procedure [3], the radial component is to be multiplied with S^2 and the transverse with S [2]. The speed of light is then computed on this basis to be Sc $(\cos^2 \alpha + S^2 \sin^2 \alpha)^{0.5}$.

The next step is to bring time slices into the computation. The idea is to compute the distance (Δx) travelled by the light during this period of time (Δt) at the current speed given above, i.e. $\Delta x = Sc\, \Delta t\, (\cos^2 \alpha + S^2 \sin^2 \alpha)^{0.5}$. This value is added on to the starting position of the light in this cycle. In the present study, the same value for Δt (0.01s) and starting position (1.0×10^{12} m = X_0) will be used as in the original computations [2].

At this stage, it should be noted that that the values of both S and sin α (α = π/2 and cos α =0) are very close to 1, which means that the value of the light speed in this cycle is c and the distance travelled is c Δt. As time goes by, the values of S and α steadily decrease, so the value of Δx gradually decreases as well. In order for the light to reach a position of $x=-X_0$ below the Earth's surface, an elapsed time of roughly $2X_0/c$ = 6600 s will be required. Since Δt = 0.01 s, this means that N = 660000 cycles must be executed. The value of Δx will be insignificantly less than 0.01 c in at least one third of the cycles. None of this is a problem for computers, but it is highly recommended that the computations be carried out in quadruple precision, as was done in the original study [2]. in order to avoid significant accumulation of error.

3. Evaluation of the Deflection Angle

In order to compute the angle of deflection, it is necessary to repeat the above procedure for another light ray located farther away from the first one, but lying parallel to it. The distance separating the two rays is referred to as Δy. The starting point for the second light ray along the x axis is the same as for the first (X_0), but its y coordinate is $Y_s + \Delta y$. The elapsed time is the same as before (0.01 s). The above procedure is then carried out for N cycles, the same number as before. Since the value of S is greater in each case than in the computations for the first ray, the average speed of light is greater for the second one. As a result, the second ray travels farther than the first in the N cycles. The distance separating the two is referred to as ΔX, i.e. $X(Y_s + \Delta y) - X(Y_s)$. The deflection angle Θ in radians is then defined to be the ratio ΔX/Δy.

According to Schiff's analytical procedure [3], $\Theta = 4Gm_0/c^2R$, the same formula as Einstein obtained using GRT.[1] Substituting the values for G, m_0 and R given in Sect. II gives a value of Θ = 1".75019. The corresponding value obtained in the earlier treatment [2] based on numerical integration is Θ = 1".75176. The latter result has been obtained using slightly different values for m_0 and R, however. If these values are used instead in the analytical formula, the result is Θ = 1".75104. Since G is only known to four significant figures, the latter two results for Θ can be regarded as perfectly consistent. In other words, the numerical approach gives effectively the same result for Θ as do both GRT and Schiff's analytical method. Finally, it should be noted that the finite differences approach should be carried out at the limit as Δt approaches zero. In the original study [2], only a single value of 0.01 s was actually considered, so further applications with smaller values would be useful in order to more accurately compare with the result obtained using Einstein's closed formula for this quantity.

4. Detailed Analysis of the Results

It has been assumed in both Schiff's analytical calculation [3] and the finite differences method [2] that light has a constant speed c throughout its motion from the star. Schiff based this assumption on the fact that the local speed of light is the same at each gravitational potential. There is also another argument [4] in favor of using a constant value of the speed of light which is based on Newton's gravitational theory. The question that needs to be considered is whether light is affected by the gravitational pull of the Sun. The fact that the gravitational mass of light is equal to zero does not in itself justify the conclusion that light is not affected by gravity. It is true that the local value of the acceleration due to gravity g is not equal to zero, but the actual value that needs to be applied for light is subject to a scaling procedure which does lead to the conclusion that the Sun does not exert force on the light waves. The local value of g must be multiplied with a factor of $\gamma(v)^{-2}$ to account for the effects of the speed of the object [5-7]. Since the speed of light is equal to c, the value of γ(v) is infinite in this case, which means that the appropriate scale factor is equal to zero. This means that light is not affected by gravity. It is also true according to Newton's theory that light cannot exert a force on other bodies because of its null gravitational mass. This is an example of his Third Law of Kinetics, which asserts that for every action there is an equal reaction. Were it not for the scaling of the local value of g, this law would be violated.

The assumption that light always travels along a straight line is clearly in conflict with GRT [1], which claims that light must travel a curved path through space. This result arises because of Einstein's conclusion that gravitational calculations must be carried out in curvilinear space rather than in the standard Euclidian coordinate system employed in both Schiff's and the finite difference approach considered above. It is normal in solving differential equations that a change in variables allows for an easier path to solution, but it needs to be emphasized that such procedures can have no effect on the actual solution. Einstein claimed that the changeover to curvilinear coordinates is essential, which clearly contradicts the above principle. In other words, if one makes the reverse transformation to Euclidean coordinates, one should obtain the same solution from GRT as found in the other approaches, namely a straight-line path for light, thereby rendering Einstein's curved trajectory conclusion to be simply an artifact of the use of a non-Euclidian coordinate system.

A basic requirement of both Schiff's and the numerical methods is the existence of two parallel rays of light coming from a given star's location. The separation of these rays

(Δy) is infinitesimal in Schiff's analytical calculation [3]. and 1000 m in the original numerical study [2].

As has been discussed in Sect. III, the result obtained for the deflection angle Θ is the same for both. What is most interesting, however, is that although it is assumed in the calculations that the observer is located very close to the Sun's corona, the actual experiments which have confirmed the effect were carried out on the Earth's surface, i.e. very far from the Sun's corona.

This state of affairs is at least consistent with Einstein's closed form expression [1, 3] for Θ ($4Gm_0/c^2R$) because it indicates quite clearly that the only quantities which are essential in the formula are the mass of the Sun and its radius. There is no mention of the position of the observer making the actual measurements. This therefore indicates that the same value for $\Theta = \Delta X/\Delta y$ is obtained independent of where the observer is located. In other words, if the two parallel light rays in question are assumed to be separated by a distance of Δy, the corresponding value for ΔX will be equal to $\Theta\ \Delta y$. There is thus a *scalene triangle with an acute angle Θ* stretching from the position of the Sun's corona out to infinity whose characteristics are determined solely by the mass of the Sun and its radius. By extension the same result should apply to any star which emits light, i.e. it will have its own fixed value for a deflection angle based exclusively on its mass and radius.

Finally, the line connecting the ΔX points can be looked upon as a wavefront of the light emitted by the star (see Fig. 1 of Ref. 2). It is rotated by an angle Θ relative to a line connecting the Sun's center of mass with its corona and stretching outward. A straightforward interpretation[2] is that when light is received by an observer, he assumes that its origin lies backward along the normal to the wavefront. This explains why the star's image appears to be deflected relative to its position in the absence of a solar eclipse. This situation can be simulated in an experiment with light refraction, whereby the origin of the light is perceived by an observer who is located within the refractive medium (see Fig. 3 of Ref. 2). The value of the group refractive index n_g of the medium can be obtained directly on this basis.

5. Conclusion

In his 1960 paper [3], Schiff pointed out that a scaling procedure for the speed of light leads to quantitatively the same result for the displacement of star images during solar eclipses as Einstein's far more complicated General Theory of Relativity (GRT) [1]. The adaptation of his method using a fine differences approach outlined in detail in the present work bears out Schiff's conclusion. It is recommended that students taking a course in GRT follow the detailed instructions given herein to prove to themselves that the scaling method does lead to the same result as GRT in a far simpler manner. Schiff steered away from making a more general statement about the capabilities of his method because he was unable at that time to apply it successfully to another prominent test of GRT, namely the precession of Mercury's perihelion.

It was subsequently shown [8] that by applying an additional scaling of the acceleration of gravity g, however, this objective can also be achieved using his simplified method. These developments clearly raise the question of whether GRT can be replaced by the Uniform Scaling Method [7, 9].

The two methods do not always agree, however. Schiff and others [10, 11] suggested an experiment to test GRT which would distinguish between the two theories. It was noted that application of Thomas precession [12] for satellites leads to the following prediction, namely that the rate of precession ω_T of a component of the spin in the plane of the satellite's orbit will vary as (M is the gravitational mass of the planet/Earth and **r** is the satellite's radial distance from the planet's center of mass):

$$\omega_T = (GM/2c^2r^3)\ \mathbf{v} \times \mathbf{r}.$$

By contrast, a GRT calculation [10, 11] gives to a good approximation

$$\omega_T = -3\ GM/2c^2r^3)\ \mathbf{v} \times \mathbf{r}.$$

i.e. an effect which is 3 times larger and in the opposite sense. The adapted Schiff method is in quantitative agreement with the former result for ω_T, so it is critical that the proposed experiment be carried out.

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