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Kinematics of a Planet in Space Surrounding a Rotating Star

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Abstract
It is shown that the center of mass of a star without rotation creates a gravitational field in which inertial motion occurs along a large circle of the sphere determined by the initial conditions. A rotating star is represented mathematically as an axis of rotation passing through the center of mass. The trajectories of the planets pass along a second-order surface that envelops the axis of rotation without intersecting with it. Such a surface is an one-sheet hyperboloid. The standard equation of such a hyperboloid is known. It is shown that only the section of the hyperboloid by a plane perpendicular to the axis of rotation is an ellipse whose parameters depend on the parameters of the hyperboloid and the distance of the section plane from the center of mass. If the section plane contains the axis of rotation, then the trajectory of the planet in this plane is a hyperbola. It is concluded that the stationary closed orbit of the planet is an ellipse in a plane perpendicular to the axis of rotation.

Keywords: Axis of Rotation, One-sheet Hyperboloid, Section of Hyperboloid by Plane, Planet Trajectories, Hyperbola, Ellipse

1. Introduction
The problem of a star and a planet revolving around it has been solved many times both within the framework of Newton's theory of gravitation and in Einstein's general theory of relativity. But all these solutions could not explain, for example, why the orbits of the planets all lie near a plane called the ecliptic plane. Applying Einstein's theory of relativity to questions concerning gravity and motion leads to unsolvable equations for complex quantities. It is possible to transform Euclidean four-dimensional space into Minkowski space by declaring the measure of one of its axes to be an imaginary quantity. It is also possible to transform Euclidean four-dimensional space into Riemannian space by introducing curvilinear coordinates and imposing a Riemannian metric on it. But in this case, all curvilinear coordinates must remain real. The author published several works in 2022 - 2024, in which a number of contradictions of the well-known theory are removed on a new basis. The proposed article consistently presents the main content of the previous works, corrects some errors made in them and on this basis proposes a solution to the problem formulated in the title of the article. The second section of this article considers the geometry of the four-dimensional space-time continuum determined by gravity. The third section describes the surface that encloses the fixed axis of rotation of the star. Sections through this surface determine the properties of the planet's trajectories. The conditions under which the planet's trajectory is an ellipse are determined. Under other conditions, the planet's trajectory must be unclosed. These issues were also considered by the author in work [1], but the assumptions made in this work should be considered unsuccessful.

2. Geometry of the four-dimensional space-time continuum determined by gravity
The four-dimensional continuum that unites space and time is a composite space, the metric tensor of which must be a cellular matrix:

$$\|g\| = \begin{Vmatrix} g_{00}(r) & g_{0\alpha} \equiv 0 \\ g_{\alpha 0} \equiv 0 & \|g_{\mu\nu}(r)\| \end{Vmatrix} \quad (1)$$

(From here on, indices designated by Greek letters take the values 1, 2, 3, and designated by Latin letters take the values 0, 1, 2, 3).

This means that three-dimensional space on which a Riemannian metric is defined is a hypersurface in four-dimensional space. Three-dimensional Riemannian space, compared to four-dimensional space, has features that are essential for physics. When reducing a symmetric metric tensor of the third rank to diagonal form, only one of the roots of the cubic characteristic equation remains a real function of the point for any values of the coordinates [2]. Therefore, the metric tensor of three-dimensional space can be represented as:

$$\|g_{\mu\nu}(\mathbf{r})\| = [1+G(\mathbf{r})] \cdot \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad (2)$$

We will call the function $G(\boldsymbol{r})$ the gravitational potential. The quadratic differential form of three-dimensional space in the point $\boldsymbol{r}$ has the form:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = [1+G(\mathbf{r})]\left[\left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2\right]. \quad (3)$$

From it follows that three-dimensional space is always conformally Euclidean.

The curvature of three-dimensional space is described by the bivalent symmetric Ricci tensor. If the metric tensor is reduced to the diagonal form (2), then the Ricci tensor in locally Euclidean coordinates is expressed through the second derivatives of the gravitational potential and the connectivity coefficients are equal to zero.

$$R_{\mu\nu} = \frac{1}{2}\left[\{1-\delta_{\mu\nu}\}\left(\frac{\partial^2 G}{\partial x^2 \partial x^3} + \frac{\partial^2 G}{\partial x^1 \partial x^3} + \frac{\partial^2 G}{\partial x^2 \partial x^1}\right) + \delta_{\mu\nu} \sum_{\alpha=1}^{3} \frac{\partial^2 G}{\left(\partial x^\alpha\right)^2}\right]. \quad (4)$$

Let us calculate the scalar curvature of the space under consideration. To do this, we need to contract the covariant tensor $\|\boldsymbol{R}\|$ with the contravariant metric tensor whose components are $g^{\mu\nu} = \delta^{\mu\nu}/[1+G(\mathbf{r})]$. We obtain:

$$R = \frac{1}{2[1+G(\mathbf{r})]} \Delta G(\mathbf{r}). \quad (5)$$

Then the equation for determining the potential is similar to the Poisson equation in electrostatics $\Delta G = -4\pi\rho$, where ρ is the mass density. Solution of the problem of the gravitational potential in the vicinity of a star, whose center of mass is the origin and whose volume V_a, is:

$$G(\mathbf{r}) = \int_{V_a} \frac{\rho(\mathbf{a})}{\mathbf{r}-\mathbf{a}} d\mathbf{a} \quad (6)$$

at every point $\boldsymbol{r}$ in three-dimensional space. The equation $G(\boldsymbol{r})=P$, where $P = \text{const}$, defines the equipotential surface on which the point $\boldsymbol{r}_0$, chosen as the initial point, is located. On such a surface, one can represent a quadratic form in four-dimensional space-time:

$$dS^2 = -c^2(dt)^2 + (1+P)\left[\left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2\right] \quad (7)$$

Let's move on to velocities and clarify the physical meaning of non-zero potential.

$$\frac{dV_S}{\sqrt{1+P}}=\mathrm{i}\sqrt{\frac{c^2}{1+P}-\left[\left(\frac{dx^1}{dt}\right)^2+\left(\frac{dx^2}{dt}\right)^2+\left(\frac{dx^3}{dt}\right)^2\right]}=\mathrm{i}\sqrt{\frac{c^2}{1+P}-\left[\left(v^1\right)^2+\left(v^2\right)^2+\left(v^3\right)^2\right]} \quad (8)$$

Here V_S is the speed of movement along the world line. When $G(\boldsymbol{r})=P=0$ formula (8) turns into the well-known formula of the Special Theory of Relativity.
If a material point is located at a point $\boldsymbol{r}_0$ and a velocity vector $\boldsymbol{v}$ tangent to the equipotential surface is defined, then it will move along a geodesic line of this surface with a constant speed.
Then the inertial motion can be defined by the law: "A material point maintains a state of rest or motion along a geodesic line of an equipotential surface with a constant speed until an external force act on it."
Equipotential surfaces of a fixed star cannot explain the characteristic features of the structure of planetary systems.

3. Surface that envelops the axis of rotation

If a rectilinear axis and a starting point on it are defined in three-dimensional space, then a one-sheet hyperboloid the center of which is located at the center of mass is a second-order surface enclosing the axis. Further we will consider the axis of rotation as the coordinate axis of the variable z and denote it OZ. The center of mass is point of intersection of the OZ axis with the plane perpendicular to it. It determines the origin of coordinates $z=0$. This plane is coordinate plane for OX and OY axes. These three axis's determinate Descartes coordinate system.
The canonical equation of a one-sheet hyperboloid in the coordinate system described above has the form:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1. \quad (9)$$

All three axes differ in parameters a, b and c. Let us consider the lines of section of the hyperboloid by coordinate surfaces. The sections by surfaces XOZ and YOZ have the form:

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}=1, \quad \frac{y^2}{b^2}-\frac{z^2}{c^2}=1. \quad (10)$$

These are equations of hyperbolas.
To describe a section by a plane $z^2=h^2$ let's transform the equation (9):

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1+\frac{h^2}{c^2}=Z^2>1; \quad \frac{x^2}{Z^2a^2}+\frac{y^2}{Z^2b^2}=1 \quad (11)$$

Equation (11) describes an ellipse. The diameters of the ellipse increase proportionally to $\sqrt{1+(h/c)^2}$.

4. Kinematics of a planet in a space that encloses the axis of rotation

As shown above, the trajectory of a planet in the field of a massive rotating star can be a hyperbola or an ellipse. The type of trajectory is determined by the initial conditions. To do this, it is necessary to draw a plane perpendicular to OZ through the initial point. If the initial velocity is directed at an angle to this plane, then the trajectory is a hyperbola to which the initial velocity is tangent. The velocity will increase if the planet approaches the origin, that is, the center of mass, or decrease if it moves away. The extremum of the velocity is at the point of intersection of the trajectory with the plane $z=0$.
If the initial velocity lies in a plane perpendicular to OZ, then the trajectory is an ellipse described in formula (11). In this case, the large diameter of the ellipse is $2Za$, and the small one is $2Zb$. The eccentricity of the ellipse is $\varepsilon=\sqrt{1+b^2/a^2}$. Recall that a $Z=\sqrt{1+h^2/c^2}$. Continuous motion along a closed trajectory must be described by a quantity that retains its value determined in the initial conditions. Such a quantity is the modulus of the velocity vector.

The main diameters of the ellipse are perpendicular to each other. Therefore, the square of the differential of the arc of the ellipse can be represented as:

$$ds^2 = d(Za)^2 + d(Zb)^2 \tag{12}$$

Let us introduce time t and components of speeds $V_l = \frac{dR_l}{dt}$. Then from (12) it follows:

$$V_s = \sqrt{V_a^2 + V_b^2} = \sqrt{\frac{d(Za)^2}{dt} + \frac{d(Zb)^2}{dt}}. \tag{13}$$

The velocity vector of movement along the trajectory at each point is directed along the tangent to the trajectory. But when moving along a closed trajectory, the velocity modulus is preserved. Therefore, kinetic energy $\frac{mV_s^2}{2}$ is also conserved.

5. References

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