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Fog Formula

John H Jennings
Jennings Research & Editing, 2530 Hillegass Ave. #307, Berkeley, CA 94704, USA

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Corresponding Author: **John H Jennings**

Abstract
Zakarova wrote a 1971 article on a math model for fog. Here, the author will attempt to write a paper putting some of Zakharov's mathematical ideas together. The article was translated by AIR FORCE CAMBRIDGE RESEARCH LABORATORIES in Massachusetts. This article is concerned with the equations up to (7a) on page 6 in the 21 pages, which is the following.

$$\partial \mathbf{S} / \partial \mathbf{t} = \partial / \partial \mathbf{z}\, \mathbf{k}_x\, \partial \mathbf{S} / \partial \mathbf{z} + \partial (\delta\, |\mathbf{v}|) / \partial \mathbf{z} \qquad (7a)$$

The start is due to assuming two equations of motion and one equation of heat transfer.
Fog happens when the relative humidity is 100%, so the assumption is that the water vapor in fog is in a saturated state. Various equations result in (7a) so this author is here to present the argument. There will be NOMENCLATURE and reasoning behind the mathematical procedure.

Keywords: Fog, Mathematical Model, Relative Humidity, Water Vapor, Saturated State

1. Introduction

In meterology, the x-axis is directed along the direction of the geostrratic wind and formulae predicting radiation fog are the following (Vager and Zilitinkevich, 1968; Lebedev, 1963; Lushev and Matveev, 1964) [5, 1, 2].
Here are the equations of motion.

$$\partial u/\partial t = fv + \partial/\partial z\, k\, \partial u/\partial z \qquad (1)$$

$$\partial v/\partial t = - f\,(u - G) + \partial/\partial z\, k\, \partial v/\partial z \qquad (2)$$

u and v are the projections of the wind on the x and y axes, respectively. z is the depth into the ground.

2. Results

After some algebra, Zakharova writes.

$$\partial q/\partial t = \partial/\partial z\, k_x\, \partial q/\partial z - m / \rho \qquad (3)$$

Difficulties in determining m may be avoided by introducing the specfic moisture content, S, from Lushev and Matveev (1967) [2] and Matveev (1961, 1965) [3, 4], where $S = q + \delta$, where q is specific humidity and δ is specific water content.

$$\partial \mathbf{S} / \partial \mathbf{t} = \partial / \partial \mathbf{z}\, \mathbf{k}_x\, \partial \mathbf{S} / \partial \mathbf{z} - (1/\rho)\, \partial Q_k / \partial \mathbf{z} \qquad (4)$$

$$Q_k = - (4/3)\, \pi\, \rho_k\, N \int[0 \text{ to } \infty]\, r^3\, f(r, z, t)\, v(r)\, dr \qquad (5)$$

where v (r) is falling velocity of droplets with radius r, f(r, z, t) is the function of the calculated distribution of droplets according to size, ρ_k is density of the water droplets and N the total number of droplets in 1 cm^3 of cloudy air, related to the water content δ by the relation.

$$N = (3\ \delta\ \rho)/(4\ \pi\ \rho_k) / \int[0 \text{ to } \infty]\ r^3\ f(r, z, t)\ dr \quad (6)$$

Going further

$$Q_k = -\rho\ \delta \int[0 \text{ to } \infty]\ r^3\ f(r, z, t)\ v(r)\ dr / \int[0 \text{ to } \infty]\ r^3\ f(r, z, t)\ dr = -\ \rho\ \delta\ |\ v(z, t)\ | \quad (7)$$

So, using Eq. (7), Eq. (4) is rewritten to Eq. (7a), the fog formula.

$$\partial\ \mathbf{S} / \partial\ \mathbf{t} = \partial / \partial\ \mathbf{z}\ \mathbf{k}_x\ \partial\ \mathbf{S} / \partial\ \mathbf{z} + \partial\ (\delta\ |\mathbf{v}|) / \partial\ \mathbf{z} \quad (7a)$$

3. Discussion

Eq. (7a) is the fog formula in ZAKHAROVA'S paper up to the end of page 6, so we need to understand what this means. S = q + δ, the specific water content, the amount of water vapor and water droplets 1 gram of air. When the condition is fog, the relative humidity = 100% and we have.

$$S = q_m + \delta \quad (8)$$

q_m is what q becomes, the specific humidity of saturated water vapor. Given all the quantities of (7a) are known, we can get S for fog. S is the desired quantity. x and y are the wind at the surface and z is the distance into the ground.

4. Conclusion

The author here is sharing Zakharov's reasoning in the first part of the translated article. The object is to get a mathematical expression for the specific water content in fog.

5. Ackowledgments

Let me mention Father Pius Youn, OP of NEWMAN HALL, Berkeley, California who brought me the Eucharist when I was sick.

Table 1: Nomenclature

α_x	Parameter
D	Mass of dried sample in ground
f	Coriolis constant
G	Velocity modulus of the geostratic wind
k	Parameter
k_x	$\alpha_x k$
m	Nonlinear functional of temperature and temperature gradient
N	Total number of droplets in 1 cm^3 of cloudy air
q	Specific humidity
q_m	Specific humidity of the saturated vapor
S	$q_m + \delta$ specific moisture content
t	Time
u	Falling velocity of droplets with radius r on x axis
v	Falling velocity of droplets with radius r on y axis
|v|	Weighted mean velocity of the falling droplets
W	Mass of original sample in ground
z	Depth into ground
δ	Specific water content = (W – D)/W x 100
ρ	Air density
ρ_k	Density of the water droplets

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