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Stochastic Solutions for two Nonlinear Models of Nonlinear Partial Differential Equations in Mathematical Physics

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Abstract
Nonlinear partial differential equations (NPDEs) in applied science provide an appropriate platform for the creation of novel research in the disciplines of applied mathematics and physical sciences. The development of more effective computer and simulation approaches for analysing these equations is extremely important. Researchers can roughly correctly identify themselves from the process described by solving these equations, enabling them to learn about some realities that are difficult to understand by regular observation. Mathematical and computational modelling have aided in the understanding of real-world phenomena observed in quantum mechanics, optical fiber communications, mechanical engineering, plasma physics, fluid mechanics, etc. A stochastic process is an observation at a specific time that results in a random variable. Brownian motion is a stochastic process that is both a martingale and a Markov process. We expect that recent breakthroughs in stochastic calculus via stochastic partial differential equations (SPDEs) will establish the framework for thoroughly modeling real-world models. Mathematicians, more than anybody else, are most at ease applying SPDEs and stochastic processes to natural models. We will investigate the impact of different stochastic types on the behaviour of the offered solutions. Random effects were found to modify the intensity of the energy wave or the collapse produced by model medium turbulence. Using Matlab software, various profile pictures are given to describe the behaviour of the dynamics for the offered solutions.

Keywords: Nonlinear Partial Differential Equations, Nonlinear Schrödinger Equation, Brownian Process, Solitary Wave Solutions

1. Introduction

Nonlinear partial differential equations (NPDEs) are frequently used in several practical disciplines to portray a broad range of complicated phenomena, such as plasma physics, superfluid, chemical engineering, quantum mechanics, biomathematics, optical fibre communications, and many more [1, 2, 3, 4]. Solving these nonlinear partial differential equations (NPDEs) increases our comprehension of the simulated events. Mathematicians and physicists emphasise developing better techniques for solving nonlinear partial differential equations. These solutions may give a complete understanding of the mechanics behind many physical phenomena and dynamic processes as represented by these NPDE models [5, 6, 7].

The subject of solitary waves is crucial for understanding the complex events associated with many technological fields in applied science [8, 9, 10]. The NPDEs arise when these physical occurrences are understood mathematically. By examining the particular treatments for various NPEDs, we can gain a better understanding of these physical phenomena. Many researchers have studied the dynamics of the solutions during the past few years, employing a range of strong and efficient mathematical methodologies that might represent a broad spectrum of physical phenomena [11, 12, 13].

Several analyses of bifurcations for several nonlinear structures have been used to examine the influence of particle Kappa distributions on the energy and wave picture in fluids [14]. However, in recent decades there has been a significant interest in understanding the energy equation through the existence of dynamical waves in astrophysical space, which includes two charged particles fluids, namely the mesosphere, and cometary tails [14]. In three dusty fluid plasmas, the energy of damped cylindrical soliton features has been examined [15].

In biophysics, it is evident that a number of characteristics of the energy and propagation nerve solitons, such as the reversible emission and reabsorption of heat and the corresponding mechanical, fluorescence, and turbidity changes, cannot be explained by the Hodgkin-Huxley model. The isothermal and isentropic compressive modulus' substantial undershoot and stunning recovery are its most notable characteristics. These characteristics result in lowest soliton velocity and maximum wave amplitude that are close to the myelinated nerves' propagating velocity. Solitons also spread without incurring shape and energy distortions [16].

Stochastic nonlinear partial differential equations (SNPDEs) are widely used in a variety of applications, including engineering, complex networks, epidemics, optical fiber, biology, quantum mechanics, complex networks and many others [17, 18, 19, 20, 21]. A complete knowledge of SNPDE theory requires experience with advanced probability and stochastic processes. Numerous processes rely on particles moving stochastically in random potentials. Brownian motion is a typical stochastic process that is both a martingale and a Markov process [22, 23]. Random application in physical systems has been the focus of much theoretical and analytical research undertaken internationally in the last several decades [24, 25]. Many different mathematical physics techniques have been applied to find new deterministic or stochastic solutions [26, 27, 28, 29, 30]. Consequently, the obtained wave characteristics may be controlled by stochastic noise effects that offer some insight into the morphological and structural alterations brought about by the produced decaying and damping waves. Important phenomena that call for the use of sophisticated theoretical methods, such crystal growth models, for growth descriptions in physics can be produced by mechanical and optical qualities.

Brownian motion is a fundamental concept in calculating random probabilities and a basis for modeling stochastic systems. This process plays a vital role in the creation of SNPDEs. Many real random phenomena can be effectively represented using the Brownian motion process. This process is an essential part of Martingale theory [31].

The article's remainder is organized as follows. Sec. 2 introduces the basic ingredients of the presented analysis, such as travelling wave solutions, stochastic process. Sec. 3 presents the Monte Carlo method. Sec. 4 presents stochastic solutions for the nonlinear Schrödinger equation (NLSE) and modified Korteweg-de Vries (mKdV) equation. Sec. 5 introduces the interpretation of the presented results. Finally, conclusion is provided in Sec. 6.

2. Preliminary

Here, we present the basic notion of the presented analysis in our study. This section is also interesting in it self. Travelling wave solutions are a fundamental aspect of nonlinear wave phenomena in applied sciences. These solutions are critical in understanding the dynamics of wave propagation in various physical contexts, including nuclear ethers.

2.1 Travelling wave solution

A traveling wave solution is a stable form solution that moves at a constant rate. Different forms of traveling waves appear in a variety of scientific domains, including shallow water waves, electromagnetic theory, chemistry, biology, and plasma physics. We describe a few unique types of traveling waves in the following sense [32, 33]:

Definition: The travelling wave solution to a partial differential equation (PDE) is a solution that can be expressed in the form:

$$\psi(x,t) = \Psi(\xi),$$

$\xi = x - v\,t$ is the traveling wave, and v denotes the speed of the wave.

2.1.1 Solitary Waves

A solitary wave is one that, when observed in the reference frame moving at the wave's group velocity, propagates without any temporal evolution in shape or size. It is a travelling wave whose transition from the asymptotic state at $\xi = -\infty$ to the other asymptotic state at $\xi = \infty$ is localized in ξ, where $\xi = x - \nu\,t$, and ν is the wave speed [33].

2.1.2 Solitons

A soliton is a nonlinear solitary wave with the extra feature of retaining its permanent structure after interacting with another soliton. When two solitons propagate in different directions, for instance, they can effectively pass through one another without colliding [32, 33]. Solitons are a type of solution to model equations such as the KdV and NLS equations. The following are the characteristics of a soliton:

i. has a permanent form.

ii. is a localized within a region.

iii. may powerfully combine with various solitons while maintaining its identity.

iv. is produced via a careful balancing act between dispersive and nonlinear effect.

2.1.3 Kink Waves

Kink waves are travelling waves that rise or fall from one asymptotic state to another and achieve a fixed value at infinity [33]. Burger's equation is a well-known equation, which produces kink wave solutions.

2.1.4 Periodic Waves

Periodic waves are travelling waves that exhibit periodicity, such as $\cos(x - t)$ [33]. The periodic solutions, for example, are the travelling wave solutions of the wave equation $\psi_{tt} = \psi_{xx}$.

2.2 Stochastic process

A stochastic process is a mathematical representation of the potential emergence of a random phenomenon at any given moment in time after its first occurrence [31]. This represents the time progression of a random phenomenon. The Brownian motion process is an example of a stochastic process that operates in continuous time. Brownian motion is commonly used in dispersion systems. The financial pricing mechanism is considered a physical application of Brownian motion. The primary characteristics of Brownian motion, represented by $\{W(t)\}_{t\geq 0}$ are:

(a) $W(t)$, $t \geqslant 0$ represent continuous functions of time t with $W(t) \sim N(0, t)$.

(b) For $s < t < r < p$, $W(t) - W(s)$; $W(p) - W(r)$ are independent.

(c) $W(t) - W(s)$ follows a normal distribution with zero mean and variance $t - s$, i.e.

$W(t) - W(s) \sim \sqrt{t - s}\, N(0, 1)$, $N(0, 1)$ represents a standard normal distribution.

3. Monte Carlo method

Monte Carlo simulation is a technique used in mathematical and statistical modeling to estimate the results of complex systems. This technique relies on generating large numbers of random samples to simulate physical, financial or engineering processes. Here's a detailed explanation of Monte Carlo simulation:

Definition of Monte Carlo simulation

Monte Carlo simulation is a computational method used to estimate the possible outcomes of a complex system by generating large numbers of random samples. These samples are used to estimate the distribution of possible outcomes and analyze the effect of stochastic factors on the system.

Monte Carlo simulation steps

1. **Model Selection**: Identify the mathematical or statistical model that represents the system to be analyzed.
2. **Random sampling** generation: Using random number generators to generate samples of random variables in the model.
3. **Simulation execution**: Execute the model using random samples and record the results.
4. **Results Analysis**: Analyze results to estimate the distribution of possible outcomes and identify important statistics such as mean and standard deviation.

Monte Carlo Simulation Apps

1. **Financial:** Used to estimate the value of financial options and analyze risk.
2. **Engineering:** Used to analyze the reliability of engineering systems and estimate the life of components.
3. **Physics:** Used to simulate physical processes such as particle propagation.

4. The stochastic solutions

In this section we consider some physical models of nonlinear partial differential equations in the presence of Brownian motion process.

4.1 Nonlinear Schrödinger equation (NLSE)

Through the Brownian motion mechanism, the solitary wave for the NLSE explains a number of fascinating complex events. Optical fiber communications, deep water, plasma physics, quantum mechanics, superfluid, condensed matter physics, and many more fields depend heavily on these phenomena. A very effective method for handling a variety of real-world random occurrences is the Brownian process. A important component of stochastic calculus and the foundation for modeling stochastic systems is Brownian motion. The appropriate travelling wave solution converts the stochastic NLSE problem into nonlinear ordinary differential equations. In particular, we analyse the NLSE model using the Brownian motion technique. The general form of a nonlinear Schrödinger equation is:

$$i\,\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} + |u|^2 u = 0. \tag{3.1}$$

By adding the effect of random noise from Brownian motion, the equation becomes:

$$i\,\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} + |u|^2 u = \sigma\, W(t,x), \tag{3.2}$$

where $\sigma W(t,x)$ Represents random noise that represents the effect of Brownian motion. This effect is important for understanding how noise can affect the stability of wave solutions.

Steps to solve using Ito calculus:

Preliminary analysis:

When dealing with the nonlinear Schrödinger equation (NLSE), wave propagation in a nonlinear medium is analysed taking into account that random noise can lead to changes in wave stability. The nature of these changes depends on the strength of the noise.

Convert the equation to Ito format:

To apply numerical methods, the Ito calculus formula is used, which helps deal with the random effect of noise. The modified form of the equation is:

$$du = \left(i\,\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} + |u|^2 u\right)t + \sigma\, dW(t,x). \tag{3.3}$$

The first part represents the inevitable interaction between waves, while the second part reflects the random effect of Brownian motion.

Numerical rounding using finite differences:

Calculation of spatial derivatives:

To calculate spatial derivatives using the following finite differences formula:

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2}. \tag{3.4}$$

Calculation of time derivatives:

To calculate time derivatives using finite differences formula:

$$\frac{\partial^2 u}{\partial x^2}\;\frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} \approx i\left(\frac{1}{2}\frac{\partial^2 u}{\partial x^2} + |u|^2 u\right) + \sigma\xi, \tag{3.5}$$

where $\sigma\xi$ is the random distribution of Brownian motion.

Monte Carlo simulation:

Step 1: Generate random noise

At each time step, random noise is generated using:

$$\Delta W(t,) = \Delta t \cdot N(0,1). \tag{3.6}$$

Step 2: Update the numerical solution:

The solution is updated at each time step using the following equation:

$$u^{n+1} = u^n + \Delta t\left(i\,\frac{1}{2}\frac{\partial^2 u}{\partial x^2} + |u|^2 u\right)t + \sigma\,\Delta W(t,x). \tag{3.7}$$

Step 3: Calculate the average solutions using the Monte Carlo method:
To calculate the final effect, the Monte Carlo simulation is used to calculate the average solutions after performing iteration across different noise paths:

$$u_{average}^{n+1} = \frac{1}{N}\sum_{k=1}^{N} u_k^{n+1}. \quad (3.8)$$

Noise impact analysis:
In case of weak noise: Weak noise leads to slight fluctuations in solutions. The waves remain largely stable with some small vibrations, see Fig 1.
In case of strong noise: When the noise increases, there is a significant disturbance in the stability of the solutions, resulting in the collapse of the wave or a significant change in its shape. The behaviour of solutions varies depending on the intensity of random noise, see Fig 2.

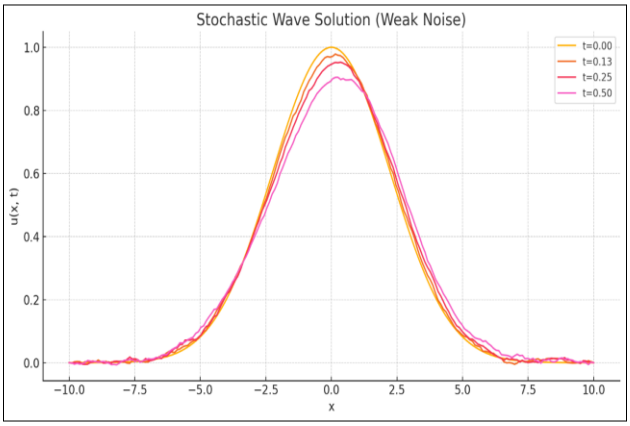

Fig 1: The behaviour of wave solution of NLSE in case of weak noise

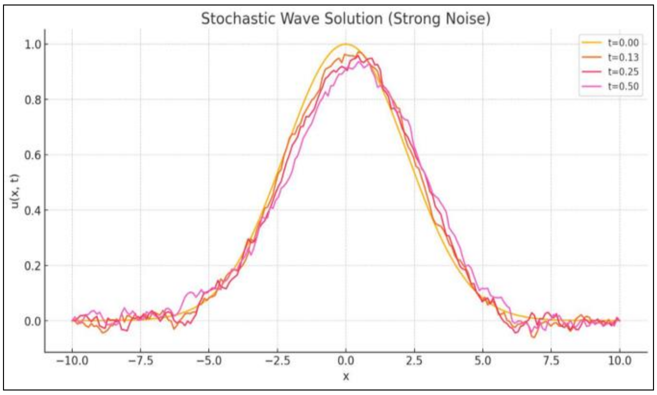

Fig 2: The behaviour of wave solution of NLSE in case of strong noise

4.2 Modified Korteweg-de Vries (mKdV) equation
The modified Korteweg-de Vries (mKdV) equation is a nonlinear partial differential equation that describes the propagation of solitons but with a cubic nonlinearity rather than the quadratic nonlinearity found in the traditional KdV equation. It is often used to model different physical phenomena, including shallow water waves and ion acoustic waves in plasmas.

Equation without Brownian motion effect:
The mathematical representation of the modified KdV equation without the Brownian motion effect is:

$$\frac{\partial u}{\partial t} + 6u^2\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial^3 x} = 0. \quad (3.9)$$

When introducing the stochastic component, specifically the Brownian motion, the equation becomes:

$$\frac{\partial u}{\partial t} + 6u^2\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial^3 x} = \sigma\, W(t,x), \quad (3.10)$$

where $\sigma W(t,x)$ represents the stochastic forcing term due to Brownian motion, adding random perturbations to the system.

Numerical Solution Steps:
Initial Analysis:
The equation describes nonlinear wave propagation. Adding the stochastic term $\sigma W(t,x)$ introduces randomness that can affect the stability and behavior of the wave.

Transformation to Ito Form:
The stochastic differential form of the mKdV equation can be written as:

$$du = \left(-6u^2\frac{\partial u}{\partial x} - \frac{\partial^3 u}{\partial^3 x}\right)dt + \sigma dW(t,x). \quad (3.11)$$

Finite Difference Approximation:
Spatial Derivatives:

$$\frac{\partial u}{\partial x} \approx \frac{u_{i+1} - u_{i-1}}{2\Delta x}, \quad (3.12)$$

$$\frac{\partial^3 u}{\partial x^3} \approx \frac{u_{i+2} - 2u_{i+1} + 2u_{i-1} - u_{i-2}}{\Delta x^3}. \quad (3.13)$$

Temporal Derivative:

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = -6u_i^2\frac{\partial u}{\partial x} - \frac{\partial^3 u}{\partial^3 x} + \sigma\xi, \quad (3.14)$$

where $\sigma\xi$ is the random distribution of Brownian motion.

Monte Carlo Simulation:
Step 1: Generating Random Noise:

$$\Delta W_i^n = \sqrt{\Delta t} \cdot N(0,1), \quad (3.15)$$

where $N(0,1)$ represents a normally distributed random number.

Step 2: Numerical Solution Update:

$$u_i^{n+1} = u_i^n + \Delta t\left(-6u_i^2\frac{\partial u}{\partial x} - \frac{\partial^3 u}{\partial^3 x}\right) + \sigma\Delta W_i^n. \quad (3.16)$$

Step 3: Computing the Average Solutions:

$$\bar{u}_i^{n+1} = \frac{1}{N}\sum_{k=1}^{N} u_{i,k}^{n+1} \quad (3.17)$$

The final average of solutions is computed after repeatedly calculating N different Brownian paths.

Effect Analysis:

Weak Noise: Small perturbations may cause slight oscillations in the wave amplitude, see Fig 3.

Strong Noise: Significant disturbances can lead to wave collapse or severe deformation of the wave shape. see Fig 4.

This comprehensive approach provides insights into how random perturbations influence nonlinear wave dynamics in systems described by the modified KdV equation.

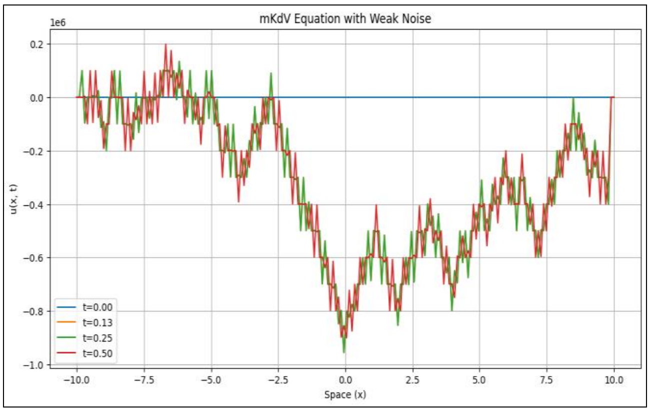

Fig 3: The behaviour of wave solution of mKdV in case of weak noise

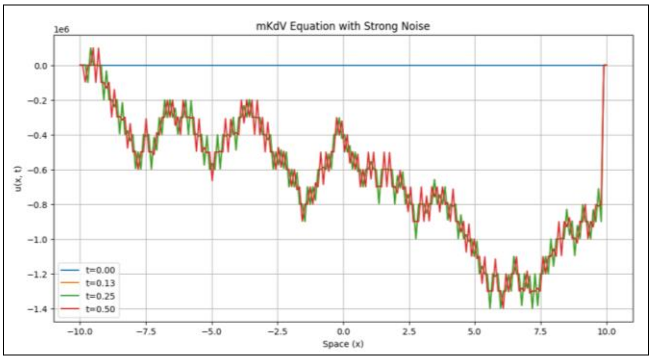

Fig 4: The behaviour of wave solution of mKdV in case of strong noise

5. Results and discussion

Many complex phenomena are explained by applying stochastic processes, such as Brownian motion and Markov processes to a variety of nonlinear partial differential equations, including those found in fluid dynamics, optical fiber communications, plasma physics, and quantum mechanics. These processes allow for a deeper understanding of how randomness influences the behavior of waves and solutions in nonlinear systems. The stochastic approach provides a practical framework for modeling random events in real-world systems, offering insights into how noise and uncertainty impact wave propagation, energy, and stability in these models.

In particular, the nonlinear Schrödinger equation (NLSE) under Brownian motion demonstrates notable phenomena of wave propagation influenced by noise. The Brownian process is a very effective technique for coping with a wide range of real-world random occurrences. When stochastic noise is added to the NLSE, the system's behavior changes, showing more rapid wave collapse and destabilization as the noise intensity increases. This behavior is crucial in fields such as optical communications and deep-water waves, where noise can significantly affect the performance and stability of the system. Through the application of Brownian motion, the NLSE transforms into a system where randomness plays a dominant role, with the resulting stochastic solutions providing a comprehensive understanding of the dynamics of wave collapse, energy dissipation, and wave steepening.

Likewise, the modified KdV and nonlinear wave equations, which describes shallow water waves, exhibits altered wave behavior when influenced by both Brownian motion and Markov processes. Noise can either enhance wave stability by damping perturbations or lead to energy dissipation and wave collapse. In particular, noise induced by the Markov process, which depends on the system's current state, has shown to contribute to both short-term perturbations and long-term instability. These effects are crucial for understanding wave formation and dissipation in coastal and fluid dynamics systems, where noise plays a pivotal role in shaping the behavior of waves over time. The stochastic solutions derived under these processes provide insights into how randomness, especially under Brownian motion, can lead to dynamic shifts in wave propagation, causing either increased steepening or dissipation of energy over time. This is particularly relevant in systems involving long-distance wave propagation, where stochastic effects can cause significant deviations from deterministic predictions.

In all cases, whether applying Brownian motion or Markov processes, the stochastic solutions provide valuable insights into the role of randomness in nonlinear wave propagation and energy dynamics. The simulations conducted, using stochastic methods, have shown that both short-term and long-term behaviors of these systems are highly sensitive to noise intensity and type. Figures generated from the simulations depict the dynamical behavior of these solutions under various noise intensities, showing how increasing the stochastic coefficient impacts the system's stability, phase shifts, and energy distribution.

In summary, the application of stochastic processes to NPDES significantly enhances our understanding of how randomness impacts the stability and dynamics of wave systems. While noise can lead to wave collapse and energy dissipation in some scenarios, it can also enhance the stability of solutions under different conditions. These results are critical for practical applications in fields such as fluid dynamics, optical communication, and quantum mechanics, where understanding the influence of stochasticity is essential for optimizing system performance and predicting system behavior under random influences.

6. Conclusions

In applied science, nonlinear partial differential equations (NPDEs) offer a suitable setting for the development of new applied mathematics and physical scientific research. It is crucial to create more efficient computer and simulation methods for analyzing these equations. By solving these equations, researchers may approximately identify themselves from the process described, which helps them learn about some truths that are hard to explain from everyday observation. Understanding real-world phenomena in fields such as fluid mechanics, mechanical engineering, optical fiber communications, quantum mechanics, and plasma physics has been made easier by mathematical and computer modeling. A stochastic process is one in which random variables are generated from observations at specific times. Brownian motion is a stochastic process that is both a Martingale process and a Markov process.

Mathematicians, like no one else, feel most comfortable applying SPDE and stochastic processes to natural models We investigated the influence of different probability types on the behavior of the proposed solution. It turns out that random effects change the intensity of energy waves or disruptions caused by turbulence in the model environment. Using Matlab software, various profile images are provided that show the dynamic behavior of the proposed solution.

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