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How do Ritz Vectors React to the Imposed Load?

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Abstract
All real structures behave dynamically when loaded or displaced. Additional inertial forces are equal to the force in acceleration using Newton's second law. If the forces or the change of places are applied very slowly, the inertial forces can be ignored and a static analysis can be done. Therefore, it can be said that dynamic analysis is a simple extension of static analysis. In addition, all potential real structures have unlimited degrees of freedom. Therefore, the most critical part of structural analysis is creating a model with a limited number of degrees of freedom that has several almost massless members and several nodes, which can estimate the behavior of the structure appropriately. The mass of the structure can be concentrated in the nodes. Also, for a linear elastic system, the stiffness characteristics of the members can be estimated very accurately - according to the experimental data - although the estimation of dynamic loading, energy loss, and boundary conditions can be very difficult. Therefore, the nonlinear dynamic analysis method is usually used in theoretical studies and is not used in routine designs. So, it is a worthwhile task to propose a simple and computer method for seismic estimation of structures. For this purpose, POA has been introduced. In many cases, we can get more valuable information from POA than nonlinear dynamic analysis, this method is simple and economical and has attracted more attention nowadays.

Keywords: Dynamic Analysis, Load, Earthquake, POA, Ritz Vectors

1. Introduction

In 1975, Freeman and his colleagues obtained a quick evaluation method, which is similar to today's capacitance spectrum method (Aghazadeh, M. *et al*., 2017) [2]. In 1981, Saiid & Sozen proposed a nonlinear dynamic analysis method on a one-degree-of-freedom system equivalent to SDOF. Based on this idea, Fajfar & Fischinger obtained the first version of the N2 method in the middle of 1980. (N means non-linear analysis and number 2 mathematical models of one degree of freedom FSDO and several degrees of freedom MDOF). However, the earthquake engineering community did not pay much attention to simple nonlinear methods until the mid-1990s (Naghibi Iravani, S., *et al*., 2024a) [26].

There are many differences in simplified nonlinear methods under the title of capacity spectrum method in ATC 40 and Triservices regulations, Japanese building standard, nonlinear static analysis method applied in FEMA273 and FEMA356, N2 method implemented in Eurocode 8 draft, and modal pushover analysis (Sohrabi, S., 2024b) [38]. Also, several non-linear methods have been implemented in the SEAOC blue book. All existing pushover analysis methods combine a multi-degree-of-freedom model with the response spectrum analysis of a one-degree-of-freedom system equivalent to SDOF (Sadigh Sarabi, M., *et al*., 2024c).

Elastic and non-elastic spectrums are applied based on an equivalent damping and period. Which achieves the inelastic spectrum of the yield point (Dehghan S., 2024a) [7]. The methods provided in the FEMA regulations are formulated based on the acceleration-displacement (A-D) format. In this form, the capacity of the structure is directly compared with the demand of ground motion in an earthquake on a structure. The force-displacement curve obtained by non-linear static analysis shows the capacity of the structure (Zakerhaghighi *et al*., 2015) [41].

The shearing forces of the foundation and the change of the roof locations are returned to the acceleration spectra and the displacement spectra of a one-degree-of-freedom system equivalent to SDOF, and the values of these spectra define the capacity diagram. Seismic demand spectrum values have different contents among different methods (Zaker Haghighi *et al*., 2014) [40]. In all cases, the intersection of the capacity curve and the demand spectrum gives an estimate of the inelastic acceleration (resistance) and the change in demand location. Therefore, many simplified methods of design based on shape change control (which is called design based on space change) have been developed and are being developed (Gheitarani, N., *et al*., 2024b; Karimimansoob, V. *et al*., 2024c).

All methods are limited based on planar structures, which have recently been hypothesized to develop and apply existing methods for asymmetric structures using 3D analysis. Non-linear static analysis methods have been developed to determine and estimate the seismic response of asymmetric structures. The proposed methods include torsional responses resulting from the effects of three-dimensional analyses of these asymmetric buildings. In the new form, these methods are used for non-linear static analysis by using dynamic and elastic analyses of buildings to determine the change of target locations and the form of lateral load distribution (Gheitarani, N., *et al*., 2013a).

2. Theoretical

Basic principles of dynamic analysis: Considering the above, it is necessary to reduce the existing errors. Multiple dynamic analyses using different dynamic models, loading, and boundary conditions should be used. It may be necessary to perform even 20 computer analyses to design a new structure or estimate an existing structure (Norouzian M. M., *et al*., 2024) [30]. Due to the large number of computer analyses required for a dynamic sample analysis. Appropriate numerical methods for calculations should be used in computers (Sadigh Sarabi, M., *et al*., 2023b).

Dynamic balance: The force balance for a multi-degree-of-freedom system with concentrated mass can be written as a function of time as follows:

$$F(t)I + F(t)D + F(t)S = F(t)$$

F(t)I: Vector of inertial forces acting on the mass
F(t)D: Vector of viscous damping force, or energy loss.
F(t)S: Vector of internal forces endured by the structure
F(t): Vector of applied loads.

It is based on physical laws and is valid for both linear and non-linear systems. For many structural systems, the estimation of the linear behavior of the structure is done and converted into a group of linear second-order differential equations (Sadigh Sarabi, M., *et al*., 2024d) [32].

$$\mathbf{M}\ddot{\mathbf{u}}(\mathbf{t})_a + \mathbf{C}\dot{\mathbf{u}}(\mathbf{t})_a + \mathbf{K}\mathbf{u}(\mathbf{t})_a = \mathbf{F}(\mathbf{t})$$

Where M is the mass matrix, C is the damping matrix, and K is the stiffness matrix. The time-dependent vectors are the absolute values of change of location, speed, and acceleration. For earthquake loading F(t) external force is equal to zero (Norouzian, M. M., & Sarabi, 2023) [31]. The basic seismic movement is three components u(t) which are considered at a point under the foundation of the building. Therefore, according to, which are relative quantities (compared to earthquake components). Therefore, the absolute values of change of location, speed, and acceleration can be removed (Sadigh Sarabi, M., *et al*., 2023a).

$$u(t)a = u(t) + \{rx\}\ u(t)xg + \{ry\}\ u(t)yg + \{rz\}\ u(t)zg$$

$$\dot{u}(t)a = \dot{u}(t) + \{rx\}\ \dot{u}(t)xg + \{ry\}\ \dot{u}(t)yg + \{rz\}\ \dot{u}(t)zg$$

$$\ddot{u}(t)a = \ddot{u}(t) + \{rx\}\ \ddot{u}(t)xg + \{ry\}\ \ddot{u}(t)yg + \{rz\}\ \ddot{u}(t)zg$$

Where {ri} is a vector whose degrees of freedom are 1 and the rest of its elements are zero (Gheitarani, N., *et al*., 2024c).

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = -Mx\ \ddot{u}(t)xg - My\ \ddot{u}(t)yg - Mz\ \ddot{u}(t)zg$$

$$Mi = M\{ri\}$$

There are various classic methods to solve the equation, each of which has its own merits and demerits, which we will summarize (Khanian, M., *et al*., 2013) [21].

Step-by-step solution method: The most general method of dynamic analysis is the incremental method where the equilibrium equations are solved at times when there are a large number of such incremental methods for solving (Khanian, M., *et al*., 2019) [22]. In general, these methods include solving a complete group of equilibrium equations in each time increment (Dehghan S. *et al*., 2024). In the case of non-linear analysis, it may be necessary to re-form the stiffness matrix of the structure (Norouzian & Sarabi, 2023) [31]. Also, it is possible that we need to repeat each time step to reach the balance (Naghibi Iravani, S., *et al*., 2024b) [27]. From the computational point of view, it may take a lot of time to solve a system with several hundreds of degrees of freedom (Gheitarany, N., *et al*., 2013b). In addition, we may need to add numerical or virtual damping to many of these incremental solutions to obtain a stable solution (Sohrabi, S., 2024a) [39]. For several non-linear structures that are affected by ground motion, incremental numerical solution methods are necessary (Kahvand, M., *et al*., 2015) [18]. For very large structural systems, a combination of Modal superposition and incremental methods can be very effective (for systems with a small number of nonlinear elements).

Modi superposition method: The most common and effective approach for seismic analysis of linear structures is the superposition method. After a group of orthogonal vectors were estimated. This method converts the large set of balance equations into a relatively smaller number of second-order differential equations, which significantly reduces the calculation time. It has been shown that the seismic movements of the earth excite only the low frequencies of the structure. Normally, earthquake movements are recorded at intervals of 200 points per second (Norouzian, M. M., 2024) [29]. Therefore, the basic loading data does not include information above 50 rounds per second. According to this article, regardless of modes and higher frequencies, they usually do not cause errors.

Analysis of the response spectrum: The primary mode superposition analysis method, which is limited only to linear elastic structures, obtains the complete response of the time history of the changes in the shapes of the nodes and member forces due to specific ground motion. Using this method has two disadvantages:

This method creates a high output volume, which increases the design process, especially when we want to use the results for design control. The analysis should be repeated for several more earthquakes to ensure that all modes are excited. There are significant computational advantages in using response spectrum analysis to predict the changes in member locations and forces in structural systems (Karimimansoob *et al*., 2024b) [20]. This method only includes the calculation of the maximum amount of changes in the places and forces of the members using a smoothed spectrum which is the average of several earthquakes. Then, it is necessary to use the CQC, SRSS, or CQC3 methods to obtain the most tolerant peak value of position or force change.

Solution in the frequency domain: The basic approach used in solving dynamic equilibrium equations in the frequency domain is the expansion of external forces F(t) in the form of Fourier series expressions or Fourier integrals. The solution consists of complex expressions that cover the time range. Therefore, it is a very effective method for repetitive loads such as mechanical, acoustic, sea waves, and wind vibrations (Gheitarani, N., *et al*., 2024a). However, the use of solving in the frequency domain for the analysis of structures that are affected by earthquakes has several disadvantages. Understanding the mathematics used is very difficult for many structural engineers. Therefore, it is very difficult to be sure of the correctness of the solution.

For the type of seismic loading, this method is not numerically efficient. Transferring the results from the frequency domain to the time domain, even using FFT methods, requires a significant amount of numerical calculations. The method is limited to linear building systems. The method for solving the approximate nonlinearity of soil/structure action and response in construction has been used without sufficient theoretical justification. For example, this method is used as an iterative behavior to construct linear equations, the linear damping sentences are changed after each iteration to estimate the energy consumption in the soil. Therefore, the dynamic balance in the soil is not satisfied (MM Norouzian & N Gheitarani, 2023) [25].

Solving linear equations: The step-by-step solution of dynamic equations, solving in the frequency domain, and estimating eigenvectors and Ritz vectors all require solving linear equations, which are described below (Sadigh Sarabi, M., *et al*., 2024b).

AX=B (1-7-1)

Where A is a symmetric N×N matrix that has a large number of zero terms. B and X matrices, which are "N × M", indicate that more than one loading mode can be solved at a time and that there are several methods to reduce the memory consumed by A and solve the device at the same time (Karimimansoob *et al*., 2024a) [19]. (Goose's elimination method, skyline solution, and other very diverse methods that are used to invert matrices, including the methods of subtraction, triangulation, matrix reduction, Jordan method, etc.)

3. Methodology

Two-stage separation method in the analysis of structures: The first step in the analysis of structures using finite elements is to isolate the structure to obtain the characteristics of stiffness, mass, and damping of the structure for use in the equations of dynamic balance (motion). Then, a new separation can be done using the combination of general and linear independent shape functions, which are obtained from the previous modeling, to determine the response of the structure (Dizaji, A. *et al*., 2023) [9]. The second reduction method is not interesting for linear static analysis. Because only one step is required for this analysis (Dehghan S., 2024b) [6]. However, this second reduction is suitable for static nonlinear analysis as well as linear and nonlinear dynamic analysis where several steps must be performed and in each step, a system of linear and nonlinear equations must be solved (Sadigh Sarabi, M., *et al*., 2024a) [33].

Separation of dynamic linear problems using direct vector superposition: Studying the deformation characteristics due to static loads and the numerical response time history of a complex structure reveals that the large number of degrees of freedom remaining in the analysis is often dictated by the topology of the building rather than by the complexity of the expected behavior. Usually, the geometry of the structure does not allow separation into a small number of elements, but the behavior can be determined using a small number of degrees of freedom. This research is generally true for structural dynamic issues such as earthquake analysis - in which modal analysis studies on the frequency content and spatial distribution of excitation have shown, that the response is controlled by a relatively small number of low-frequency modes.

In the case of vibrational excitation analysis, only a small number of intermediate frequencies may be excited. However, in the case of Multi-Shock excited systems, the modes of action related to medium and high frequencies may maintain their importance during the time frame under investigation. Changing the origin from the original coordinate system to the generalized modal coordinate system, which is required in the traditional formulation of solving large eigenvalue problems, is interesting when the number of participating modes is small compared to the original degrees of freedom.

In general, the finite element analysis method estimates the least accurate frequencies very well. While low accuracy or lack of accuracy is expected to approximate the shape of higher modes and higher frequencies. This is because the higher modes are very volatile. It is difficult to present them by the size of practical meshing done for engineering calculations. Therefore, there is little justification for using the dynamic response of high-frequency mode shapes in the analysis. Ideally, finite element meshes should be chosen in such a way that the mode shapes related to important vibration frequencies are best estimated (Farrokhirad & Gheitarani, 2024) [10]. And then the solution can be obtained by considering the response of these modes.

This research can be done by analyzing the superposition, according to the important modes of the finite element system. Estimating natural frequencies of mode shapes for large structural systems requires a significant amount of numerical operations. However, as pointed out by Wilson *et al*. (29), the direct importance of this information in engineering may be of limited value. The frequency values indicate possible states of intensification and mode shapes related to low frequencies indicate which parts of the structure are the most flexible. In most cases, approximate values can provide this information (Aghazadeh, M. *et al*., 2018) [3].

In performing most of the analyses, the only reason to estimate complete and accurate eigenvectors is because of their replacement to reduce the size of the system in a superposition analysis:

- The use of Ritz vectors in the dynamics of structures.
- Rayleigh method for single degree of freedom systems.

The basic idea in Rayleigh's method, which is used to approximate the vibration frequency of a single degree of freedom system, is the principle of energy stability. Energy in a freely vibrating system should remain constant if there is no damping force to absorb it. Therefore, the maximum strain energy in the elastic structure must be equal to the maximum kinetic energy of the mass. This method can be applied to any multi-degree-of-freedom system that can be expressed as a single-degree-of-freedom system by using Ritz {X} hypothetical spatial change forms.

Therefore, eigenvectors are approximated. It can be shown that the reduced eigenvalue problem leads to the approximate frequency r and their corresponding mode shapes. r eigenvalues resulting from the Rayleigh-Ritz approximation are the upper limit of the eigenvalues resulting from the exact solution. Static compression process, mode component composition, subspace iteration, and various other methods can be understood as Ritz analysis. The techniques differ only in the selection of basic Ritz vectors that are assumed in the analysis.

The Ritz process can be used in finite element formulation to reduce dynamic balance. The dynamic equilibrium equations for the finite element model and considering {u} which is the nodal position change vector are written as follows.

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F(s,t)\}$$

Where [M], [C], and [K] are n×n square matrices for mass, damping, and stiffness, and {F(s,t)} is the dynamic loading vector imposed on the structure, which is a function of space and time. The dot symbol indicates the derivative concerning time.

4. Findings

The nonlinear static process of NSP or POA, as stated in FEMA273 and FEMA356, is widely used among structural engineers today. During the past years, much research has been done on the assumptions and limitations of this method. For example, matching loading that tries to model the time-dependent distribution of inertial forces and considering modes other than the main oscillation mode have been among the solutions. Paying attention to the dynamic principles of the structure, the MPA method is proposed, which allows the user to consider all the modes that have a significant effect on the response. This method maintains the same simplicity of the standard nonlinear static incremental analysis processes, and at the same time, the load distribution is also independent of time. Since 1997, several researchers have developed the POA method for asymmetric buildings. An approximate method was developed by applying height-wise distribution of lateral force and performing plane POA on the center of mass of floors (Norouzian & Gheitarani, 2023) [25].

But the authors stated that "this method does not seem to be very accurate." Another method that included (i) a three-dimensional analysis of the elastic response spectrum to obtain the change of roof location and height force distribution for the resistant elements and (ii) a planar analysis for each position element was also presented by Several studies were also conducted on the effect of frame-wall interaction in frame-wall structures (Dstefano, Rutenberg). Researchers were also presented about the application of lateral force in various places of the plan. Relatively few comparisons between POA and RHA (Response History Analysis) showed limited success. Therefore, providing a method that provides relatively acceptable approximate answers is a need for engineers (Ghadarjani *et al*., 2013a).

Here, our main goal has been to develop MPA to estimate the seismic demand of symmetrical and asymmetrical buildings in plan. To express MPA, it is necessary to develop a new concept called Uncoupled Modal Response History Analysis (UMRHA), which is shown. For linear elastic systems, it is equivalent (RHA) and for nonlinear systems, it is an approximate method. Then MPA is presented and it is shown that it is equivalent to response spectrum analysis (RSA) for elastic systems and its assumptions are investigated for nonlinear systems.

Equations of motion: Consider a combination of frames resistant to lateral force. This means that the distribution of mass or hardness is not symmetrical. Assuming a rigid floor, we can define three degrees of freedom in the center of mass CM.

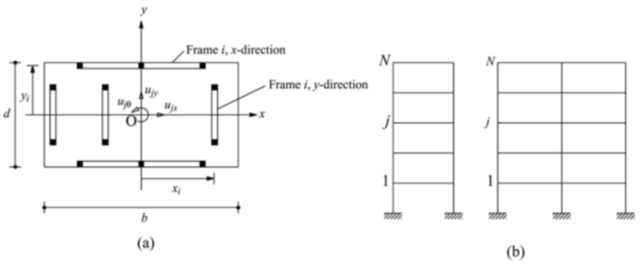

Fig 1: (a) plan (b) frames equation of motion

Basic Idea: Two methods for the approximate analysis of inelastic buildings will be investigated. Uninterrupted time history analysis (UMRHA) and MPA. In the UMRHA method, the time history of a structure's response to Peff, n(t), the nth mode of vibration is obtained by a nonlinear RHA on a one-degree-of-freedom structure, and the summation of these modal responses gives the total response. In the MPA method, the maximum response to Peff, n(t) is calculated by a non-linear static analysis. Maximum modal responses are combined by modal combination rules to obtain the final response (Maleki, M., *et al*., 2024) [23].

Inelastic systems: Although modal analysis is not valid for a non-linear system, its answer can be discussed and investigated in the form of modal coordinates of an elastic system. Each structural element in the corresponding elastic system is defined in such a way that it has a stiffness equal to the initial stiffness of the same member in the inelastic system (Naghibi Iravani, S., *et al*., 2024c) [28]. Because we consider the properties of mass and damping to be the same, the natural modes and periodicities of these two systems are assumed to be the same. The response of a nonlinear system to Peff,n(t) cannot be expressed in this part (Ghadarjani *et al*., 2013b).

5. Results

As stated before, one of the best reduction methods known for linear dynamic problems is the "mode superposition technique." This includes the selection of r modes of free vibration without damping which is the result of solving the eigenvalue problem as base vectors. With this special choice, it can be easily shown that the reduced matrices [C]*, [M]*, and [K]* are theoretically obtained with the assumption of damping as a fraction of the critical damping. The reduced system is obtained as r-independent equations, each of which can be integrated alone. 6. However, this is not a necessary condition for the non-combination of finite differential equations in a reduction method.

The lack of generality in the codes based on the Reilly-Ritz method is due to the difficulty in selecting general functions that lead to answers with the expected degree of accuracy in a computer analysis. This situation has significantly increased the popularity of using precise eigenvectors for fashion superposition. However, Wilson and colleagues have developed a simple numerical algorithm to create a special class of Ritz vectors, which are called here as (WYD Ritz vectors) or load-dependent Ritz vectors. Which provides answers with more accuracy and less computer time than the traditional special vector approach for a wide range of studied problems.

It creates a group of Mody coordinates [z] that can be used to diagonalize the system. The exact eigenvalues for the reduced system and the squared values are the approximate frequencies for the complete system. The eigenvectors [z] can be used to create a final set of load-dependent and orthogonal Ritz vectors.

[▫X] =[X][Z]

The category vectors [▫X] are orthogonal to both stiffness and mass matrices in the complete system. Some of these vectors can be a good approximation of the exact mode shapes of the structure. In case of arbitrary damping, a solution of the complex eigenvalue problem is necessary if the Mody coordinates are to be non-twin. It should be noted that the numerical effort required to solve the reduced system of degree r is usually very small in comparison with the complete original system of degree n. Since the Ritz vectors depending on the load of the automatic form are generated in a fraction of the numerical effort required to calculate the characteristic vectors of the main system. It is an effective solution to reduce three-dimensional structural systems such as soil/structure, dam/reservoir, and sea platforms, which require a lot of numerical effort and are expensive to solve through the classical eigenvalue problem (Norouzian & Gheitarani, 2024) [30].

***The effect of finite element formulation on the creation of load-dependent Ritz vectors*:** The stiffness mass matrices are normally symmetric and positive definite, although the following two exceptions may occur**:**

- If the structure can move freely as a rigid body (such as an airplane or a ship), in this case, the stiffness matrix is positive and semi-definite and of n-b rank. Where b is the number of independent movements of the rigid body.
- If no mass is assigned to some nodal displacements, zero rows and columns will be created in the mass matrix and the mass matrix will be single.
- To deal with the difficulty matrix problem with defective rank (n-b), a definite positive matrix is shifted as follows:

$$([K]-\omega_0^2[M])$$

Can be used instead of the original [K] matrix. The load-dependent Ritz vector method will theoretically generate the same vectors, albeit in a different order, for any arbitrary permuted matrix. The load-dependent Ritz vectors will be the eigenvalues of the reduced system matrices and their corresponding eigenvectors estimate the roots of the physical model closer to the specified point of interest from the eigenspectrum (Gheitarani, N., *et al.*, 2020) [14].

The total number of independent load-dependent vectors that can be generated, including any existing rigid body mode, is equal to the rank of the mass matrix, s. Therefore, the size of the reduced problem, r, cannot be greater than s. In the end, it should be noted that for large systems or a special class of problems, coordinate reduction methods such as static compaction and substructuring techniques can precede the application of load-dependent Ritz vectors algorithm to obtain system matrices ([M],[K],{f}) are smaller used in the vector calculation process. The advantages of such solution processes should be carefully evaluated so as not to increase the number of operations required for the solution.

6. Conclusion

Reduction methods are not reliable if error estimates are not used effectively. In classical methods of direct vector superposition for linear systems, only an approximation of the smaller modes of the structure is used. Even if the interest is in frequencies close to a certain value to investigate the state of intensification resulting from oscillating forces, the specified frequencies should be at the lower end of the spectrum. Because the approximation made in physical idealization by finite elements causes higher frequencies to be wrong in the mathematical model.

Therefore, the higher modes are excluded from the analysis and it should be checked that the remaining vectors correctly present the spatial distribution and effectively and possibly cover the applied loading frequency limit. In this section, error estimates are shown to ensure that the specified loading is correctly determined by the load-dependent Ritz foundation. Also to measure the relationship between the load-dependent Ritz solution in the reduced system and the exact characteristic solution in the main system is presented. An analogy between the load-dependent Ritz reduction method and the well-known methods of static correction and Sheta b Mody will be expanded to improve Mody summation when working on reduced vector groups. Finally, some attention to frequency in the process of error estimation will be discussed.

Spatial error estimates for loading presentation: An important aspect of direct vector superposition for solving dynamic equilibrium equations is related to the number of vectors that should remain in the analysis. As the number of vectors that must remain in the solution for a satisfactory solution increases, the cost of analysis increases rapidly. The easiest way to find out how many vectors, or general coordinates, should remain in finite element analysis is to use an iterative solution. To add new vectors to the solution until convergence. This method is not economical even for small systems**.** A better approach is to develop an error criterion at the level of the vector generation algorithm, to specify the time to stop generating a new vector, and to ensure convergence for desired values of the response.

***Presentation of loading by Ritz basis vectors depending on the load*:** The cases of inaccuracy in mode shortening are caused by removing load components perpendicular to the existing modes in the solution. The main idea for measuring a part of the external force vector that is not included in the vector superposition is to expand the load vector in the form of shortened base vectors and define the error criteria that are dependent on the residuals. A whole set of Ritz vectors orthogonal to the mass forms a basis for the n-dimensional linear space, so an arbitrary vector such as the spatial distribution of loading {f(s)} can be written as follows.

$$\{f(s)\} = [M]\{X_1\}p_1 + [M]\{X_2\}p_2 + \ldots + [M]\{X_n\}p_n$$

$$\{f(s)\} = [M][X]\{p\}$$

In using exact eigenvectors or load-dependent Ritz vectors in a direct superposition analysis, the participation coefficient is a direct indicator of whether {Xj} will interfere in the dynamic solution or not. The coefficient of participation, pj, therefore, may be seen as the coordinates of load vectors described in vector form. Therefore, the error in presenting the load using the reduced number of residual vectors, r, may be defined as follows. An extension of the presentation of loading is necessary to perform more general 3D analyses. In this case, the loading can be written as follows:

$$\{F(s,t)\} = \sum_{j=1}^{3}\{f_j(s)\}g_j(t)$$

Where the spatial distribution in the j direction of y, x) or z) and gj(t) is the corresponding time function. The first analytical approach may involve applying the sum sign to the level of response calculations.

Therefore, the load-dependent Ritz vector algorithm for each axis is implemented using {fx(s)},{fy(s)},{fz(s)} as the initial load distribution to obtain the three-dimensional vector basis, which is Their superposition gives the overall answer (Aghazadeh, M. *et al*., 2019) [1]. This method is most efficient when the loading vectors correspond to the main vectors of the structure so that when the structure is subjected to independent stimuli, we do not have simultaneity in the response (Sohrabi, S., 2024a) [39]. Actual numerical experiments on the seismic response of a three-dimensional structure have shown that the sum sign can be effectively applied to the level of the initial load distribution to create a load-dependent Ritz vector foundation constructed from the following initial vector.

{f(s)}={fx(s)}+{fy(s)}+{fz(s)}

It should be noted that although a basis vector can be used in all calculations, it is very important. To take into account the stability of load directionality error calculations, an independent evaluation should be done to provide the dynamic loads obtained by the shortened base for the "X", "Y" and "Z" directions. For example, if directivity is not considered in the three-dimensional analysis, the coefficient of participation, pj, obtained from {Xj}T{f(s)}, which {f(s)} is obtained will be equal to:

pj=px,j+py,j+pz,j

$$f_r(s)\} = \sum_{j=1}^{r}(p_{x,j} + p_{y,j} + p_{z,j})[M]\{X_j\}$$

This equation gives unstable answers according to the specific loading direction. Because the participation coefficients applied to each component of the vector [M]{Xj} in the directions x, y, and z must be the corresponding participation coefficients of the same direction and not the sum of the participation coefficients in all directions. The vector {Er(s)} which is defined by the equation (5-6) is the spatial error in the presentation of loading due to the shortening of the base of the vector. Because {Er} is a vector with single components, it is difficult to develop a direct understanding method for the value of the force vector that is omitted in the calculations. Two different proposals, which consider loading directionality, will be presented to measure the error in loading representation due to base vector truncation.

***Error estimates using the sum of loads provided*:** For earthquake analysis, an effective mass corresponds to a part of the total mass responding to the earthquake in each mode. It is usually used as a good indicator of the relative participation of a specific mode in the whole response of the structure. An example expansion in the "X" direction when using exact eigenvectors orthonormal to the mass as basis vectors is shown below. The load {fx(s)}, acting in the X direction, is defined as follows. {fx(s)}=[M]{rx}, where [M] is the mass matrix and {rx} is the effect vector corresponding to the change of position obtained in each degree of freedom of the structure due to a single position change at the base of the building in the "X" direction. The mode participation coefficient for mode j is defined as follows.

$$*p_{x,j} = \{\phi_j\}^T[M]\{r_x\}$$

The total mass in the "X" direction is given by the following equation.

mxx={rx} T [M]{rx}

It is done. The value corresponding to a proportion of the total mass, in the x direction, which is provided by the modi contribution of the vector, and the full expansion is found in coil and penizine. Therefore, the total value is the spatial distribution of loading in the X direction, which is provided by Madi Group. The percentage of total mass, which should remain to achieve satisfactory convergence, is somewhat debatable. But for example, the 2800 guideline suggests that at least 90% should be used. It is natural to generalize this criterion for the spatial distribution of arbitrary loads. By using a load-dependent truncated vector handle [Xr] to write an expression similar to and in full form and summation in X direction by premultiplication in {rx}T.

$$f_{r,xx} = \{r_x\}^T[M][X_r][X_r]^T\{f_x(s)\}$$

It is important to pay attention to this point that when using the load-dependent Ritz vector method, it should be used because the coefficients of participation px,j and □px,j, which are orthonormal vectors concerning [M], i.e. {Xj}, or orthogonal vectors to [M] and [K], i.e. {▫Xj} are obtained, they are different and only the final sum is the same. Therefore, if the sum of the squared participation coefficients to calculate the share of the total load provided by the base, is used, different answers are obtained if single bases [▫Xr] are used. This criterion can also be expanded to consider the moment imposed on some degrees of freedom.

Numerical experiments on small building structures have shown that for earthquake loading, the error criterion based on the sum of forces presented shows a type of convergence with increasing unity. The value of ez, ey, and ex-changes between 0 and 100 and shows the relative percentage of the total load presented in the solution by the vector group. For the more general type of dynamic loads, the error criterion based on the sum of the presented forces will not necessarily show the convergence of unity, and negative values as well as average values greater than 100 are also possible.

7. References

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