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Centered Triangular Sum Labeling of *m, n*−Snowflake Graph

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Abstract
Graph labeling is an active field of study in graph theory. A graph's labeling is the assigning of values to its vertices and edges under specified conditions. Harary [3] introduced the concept of sum labeling in 1990. Hedge and Shankaran [2] introduced a labeling of graph with q edges to admit triangular sum labeling if it is a one-to-one function that vertex of G is mapping to the natural numbers that induces a bijection $f^*: E(G) \to \{T1, T2, ..., Tq\}$ of the edges of the graph G defined by $f^*(uv) = f(u) + f(v)$, $\forall e = uv \in E(G)$ are the first q triangular numbers. From the triangular number they derived the centered triangular number. In this paper we proved that snowflake graph and Bi-snowflake graph admits the triangular and centered triangular sum labeling.

Keywords: Graph Labeling, Sum Labeling, Centered Triangular Sum Labeling, Snowflake Graph, Bi-snowflake Graph

1. Introduction

Graph labeling is a current field of research in graph theory. A labeling of a graph is an assignment of values to the vertices and edges of graphs subject to certain conditions. Sum labeling is a fascinating and actively investigated topic in graph theory. The sum labeling issue entails assigning numeric labels to vertices and edges of a graph in such a way that specific requirements or constraints are met. In 2008, Hedge and Shankaran [2] introduced the TSL (triangular sum labeling) of a graph with a vertex and edges to admit the triangular sum labeling. Centered triangular sum labeling requires finding label arrangements that satisfy both the sum constraint and the centered condition. Then we are going to show that the snowflake graph and Bi-snowflake graph under the triangular and CTSL (centered triangular sum labeling).

2. Prelims

Definition 1: The generalized $m, n-$Snowflake graph $G = K(1,m) \oplus K(1,n)$, where $m \geq 3$, $n \geq 2$ is a graph with $in + (i + 1)$ vertices and $i(n + 1)$ edges where $m \geq 3$ & $n \geq 1$ where the vertices of the star graph $K(1,m)$ adjacent to $u1, u2, ..., ui$ respectively.

Definition 2: A Bi snowflake graph is derived by joining the apex vertices of two copies of $m, n-$Snowflake graph it is denoted by $B(h1, h2, m, n)$. $h1, h2-$its represents the half of the snowflake graph.

3. Main Result

In this section we prove the newly defined newly defined $m, n-$Snowflake graph for $m \geq 3$ is a centered triangular sum graph.

Theorem 1
The m, n−snowflake graph admits the CTSL.

Proof: Let G denote the $m, n-$Snowflake graph $K(1,m) \oplus K(1,n)$ for $m \geq 3$ and $n \geq 2$.
Let $f(V)$ and $f^*(E)$ denote the number of vertices and edges respectively.
Let u be the middle vertex.
Let $u1, u2, ..., ui$ be the vertices adjacent to the central vertex u and are labeled in a clockwise direction.

Let $v^{I} j$, $v^{II} j$, ..., $v^{m} j$ respectively.
Vertex set is given by,

$$f(V) = \{u, ui, vI\,j, vII\,j, vIII, vIV\,j, vV\,j, vV\,I\,j, ..., vm\,j: 1 \leq i \leq m,\ 1 \leq j \leq n\}$$

Edges set is given by,

$$f^{*}(E) = \{uui, uivI\,j, uivII\,j, uivIII, uivIV\,j, uivV\,j, uivV\,I\,j, ..., uivm\,j: 1 \leq i \leq m,\ 1 \leq j \leq n\}$$

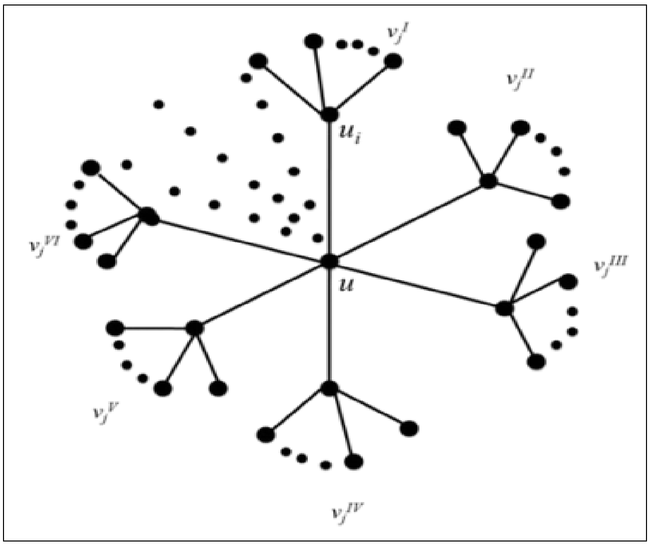

Fig 1: *m, n*−snowflake graph

Clearly G has $V = in + (i + 1)$ and $E = i(n + 1)$ for $m \leq i \leq n$, $n \geq 2$ and $1 \leq j \leq n$, define $Cn = 1/\ 2(3n2 - 3n + 2)$.

The central vertex is labeled as $f(u) = 0$ and

$$f(ui) = Ci, \tag{1}$$

The end vertices of the first star label:

$$f(vI\,j) = C(m+j) - f(u1), \tag{2}$$

The end vertices of the second star label:

$$f(vII\,j) = C(m+n+j) - f(u2), \tag{3}$$

The end vertices of the third star label:

$$f(vIII_j) = C(m+2n+j) - f(u3), \tag{4}$$

The end vertices of the fourth star label:

$$f(vIV\,j) = C(m+3n+j) - f(u4), \tag{5}$$

The end vertices of the fifth star label:

$$f(v^{V} j) = C(m+4n+j) - f(u5), \tag{6}$$

The end vertices of the m^{th} star label:

$$f(vm\,j) = C(m+kn+j) - f(um), \tag{7}$$

Thus, the edge labels and the vertices u, ui, vI j, vII j, vIII j, vIV j, vV j, vV I j, ..., vm j all are distinct.

Hence, the snowflake graph G admits the CTSL.
An illustration of 6, 5− snowflake graph is shown in Fig 2

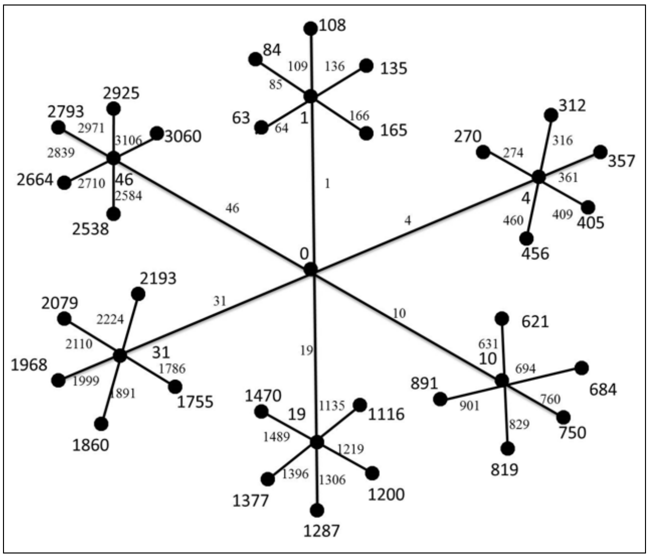

Fig 2: 6, 5−snowflake graph

Theorem 2

The Bi-snowflake graph admits the CTSL.

Proof: Let G denote the m, n−Snowflake graph for $m \geq 3$ and $n \geq 2$.

Then the Bi-snowflake graph is denoted as $B(h1, h2, m, n)$.
Let $f(V)$ and $f*(E)$ denote the number of vertices and edges respectively.
The vertex set is given by,

$$f(V) = \{u, ui, v, vi, uI\, j, uII\, j, ..., um\, j, vI\, j, vII\, j, ..., vm\, j: 1 \leq i \leq k, 1 \leq j \leq n\}$$

The edges set is given by,

$$f*(E) = \{uv, uui, vvi, uium\, j, vivm\, j: 1 \leq i \leq k, 1 \leq j \leq n, 1 \leq m \leq n\}$$

Clearly G has $V = i(n + 1) - 1$ and $E = in(i + 1)$ For $m \leq i \leq n$, $n \geq 2$ and $1 \leq j \leq n$, define

$$Cn = 1/\, 2(3n^2 - 3n + 2)$$

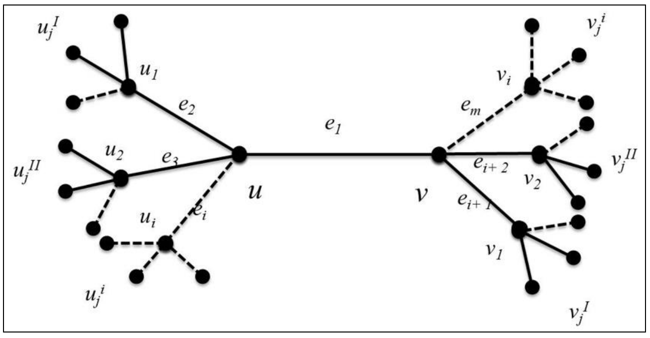

Fig 3: Bi snowflake graph $B(h1, h2, m, n)$

The vertex is labeled as $u = 0$ and $v = 1$

$$f(ui) = Ci, \quad (8)$$

$$f(vi) = Ci{+}1 \quad (9)$$

The end vertices of the first star in $h1$ label:

$$f(uI\, j) = C(m{+}j{+}1) - f(u1), \quad (10)$$

The end vertices of the second star in $h1$ label:

$$f(uII\, j) = C(m{+}j{+}n{+}1) - f(u2), \quad (11)$$

The end vertices of the $u_i$star in $h1$ label:

$$f(u_{ij}) = C(m+j+kn+1)-f(u_i), \quad (12)$$

The end vertices of the first star in $h2$ label:

$$f(v_{Ij}) = C(m+j+(k+1)n+2)-f(v_1), \quad (13)$$

The end vertices of the second star in $h2$ label:

$$f(v_{IIj}) = C(m+j+(k+2)n+1)-f(v_2) \quad (14)$$

The end vertices of the $v_i$star in $h2$ label:

$$f(v_{ij}) = C(m+j+sn+1)-f(v_i), \quad (15)$$

Thus, the edge and vertex $u, v, u_i, v_i, u_{Ij}, u_{IIj}, u_{III}, ..., u_{mj}, v_{Ij}, v_{IIj}, v_{III_j}, ..., v_{mj}$ all are distinct.

Hence, the Bi snowflake graph G admits the CTSL.
An illustration of Bi snowflake graph $B(3, 2, 5, 3)$ is shown in Fig 4

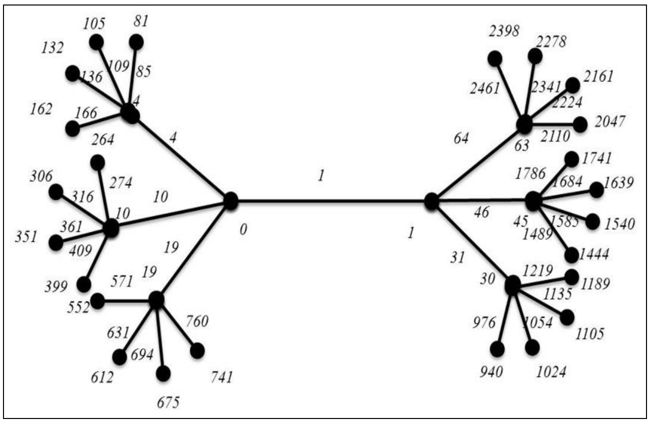

Fig 4: Bi snowflake graph $B(3, 2, 5, 3)$.

Theorem 3
The Bi-snowflake graph admits the TSL.
Proof: Let G denote the $m, n-$Snowflake graph for $m \geq 3$and $n \geq 2$.
Then the Bi-snowflake graph is denoted as $B(h1, h2, m, n)$.

Vertex set is given by

$$f(V) = \{u, u_i, v, v_i, u_{Ij}, u_{IIj}, ..., u_{mj}, v_{Ij}, v_{IIj}, ..., v_{mj}: 1 \leq i \leq k, 1 \leq j \leq n\}$$

The set of edges is given by

$$f*(E) = \{uv, uu_i, vv_i, u_iu_{mj}, v_iv_{mj}: 1 \leq i \leq k, 1 \leq j \leq n, 1 \leq m \leq n\}$$

Clearly G has $V = i(n + 1) -1$ and $E = in(i + 1)$ for $m \leq i \leq n$, $n \geq 2$ and $1 \leq j \leq n$, define $Tn = 1\ 2n(n + 1)$.

The vertex is labeled as $f(u) = 0$ and $f(v) = 1$

$$f(u_i) = T_i, \quad (16)$$

$$f(v_i) = T_{i+1}, \quad (17)$$

The end vertices of the first star in $h1$ label:

$$f(u_{Ij}) = T(m+j+1)-f(u_1), \quad (18)$$

The end vertices of the second star in $h1$ label:

$f(uII\ j) = T(m+j+n+1)-f(u2),$ (19)

The end vertices of the *ui*star in *h*1 label:

$f(ui\ j) = T(m+j+kn+1)-f(ui),$ (20)

The end vertices of the first star in *h*2 label:

$f(vI\ j) = T(m+j+(k+1)n+2)-f(v1),$ (21)

The end vertices of the second star in *h*2 label:

$f(vII\ j) = T(m+j+(k+2)n+1)-f(v2)$ (22)

The end vertices of the *vi*star in *h*2 label:

$f(vi\ j) = T(m+j+sn+1)-f(vi),$ (23)

Where $1 \leq k \leq s \leq n$ Thus the edge labels and the vertices all are distinct.
Hence, the Bi snowflake graph *G* admits the TSL.
An illustration of Bi snowflake graph $B(h1, h2, m, n)$ is shown in Fig 5.

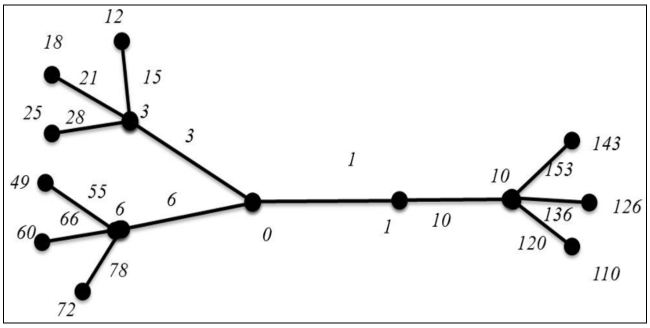

Fig 5: Bi snowflake graph $B(h2, h2, 4, 3)$

4. Conclusion

In this paper, we looked into the CTSL of the snowflake and the bi-snowflake graph. This study adds the new findings and the ideas of graph labeling. The CTSL can be proven for many graphs. Also, more-CTSL can be studied.

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