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Common Fixed Point Theorems for Eight Occasionally Weakly Compatible Maps in Interval-Valued Fuzzy Metric Space for Rational Expression

[1] **Kavita Shrivastava,** [2] **Heera Ahirwar**
[1, 2] Department of Mathematics and Statistics, Dr. Harisingh Gour Vishwavidyalaya Sagar, Madhya Pradesh, India
(kavita.rohit@rediff.com, heera.maths15@gmail.com)

Corresponding Author: **Heera Ahirwar**

Abstract
In this paper, we use interval valued fuzzy metric space (IVFMS) for rational expression and occasionally weakly compatible mapping to establish certain common fixed-point outcomes. Additionally, we create a novel class of function ψ that can aid researchers and hasten the sustainable development of common fixed-point existence and uniqueness wherever a pair of mappings satisfies occasionally weakly compatible mapping.

Keywords: Interval-valued Fuzzy Metric Space, Common Fixed Point, Occasionally Weakly Compatible Mapping

1. Introduction

By 1975, Kromosil and Michalek [1] proposed the idea of fuzzy metric space. Abu Osman [7] employed fuzzy metric space membership functions in 1983 to determine the fuzzy metric between two fuzzy sets; this approach differed from that suggested by [1]. Fuzzy metric space was reformulated in 1994 when Veeramani and George [16] presented the revised idea of Continuous t-norm. In response to this letter, numerous authors and researchers have thoroughly and extensively examined a variety of issues pertaining to this space from many angles, including compatible mapping, R-weakly computing mapping, weak compatible mapping, CLR-Property, E.A. property, etc. and have produced new findings on fuzzy metric space. Sushil Sharma [19, 20] first presented the novel idea of fuzzy 2-metric space in 2002 and provides several. Recently in 2023, Vaishali deshmukh, *et al*. [13] proved some common fixed-point results using pairwise compatible mappings and owc mappings for interval valued fuzzy metric space.

In this paper, we use the notion of interval-valued t-norm to prove certain popular fixed point theorems for new rational expression eight mapping through occasionally weakly compatible mapping on an interval-valued fuzzy metric space.

2. Preliminaries

Definition 2.1 [9] In a non empty set X, a mapping $A_m: X \to [I]$ is called an interval-valued fuzzy set on X. Collection of all interval-valued fuzzy sets on X is denoted by $IVF(X)$.

If $A_m \in IVF(X)$, let $A_m(x) = [A_m^-, A_m^+]$, $A_m^-(x) \le A_m^+(x)$ for all $x \in X$, then the set $A_m^-: X \to I$ and $A_m^+: X \to I$ are called lower fuzzy set and upper fuzzy set of A_m and if $A_m^-(x) = A_m^+(x)$ then is called degenerate fuzzy set for all $x \in X$.

Definition 2.2 [9] A binary operation of the form is an interval valued t_{norm} is $*_I: [I] \times [I] \to [I]$ on $[I]$ such that for all $\overline{u}, \overline{v}, \overline{w} \in [I]$ if satisfying following four conditions:

(1) **Commutativity:** $\overline{u} *_I \overline{v} = \overline{v} *_I \overline{u}$,

(2) **Associativity:** $[\overline{u} *_I \overline{v}] *_I \overline{w} = \overline{u} *_I [\overline{v} *_I \overline{w}]$,

(3) **Monotonicity:** $\overline{u} *_I \overline{v} \le \overline{u} *_I \overline{w}$ *whenever* $\overline{v} *_I \overline{w}$,

(4) **Boundary condition:** $\overline{u} *_I \overline{1} = \overline{u}$, $\overline{u} *_I \overline{0} = [u^-, u^+] *_I [0,1] = [0, u^+]$.

Note: Each interval valued t_{norm} satisfies some additional boundary conditions for all $\overline{u} \in [I]$:

$\overline{u} *_I \overline{0} = \overline{0} *_I \overline{u} = \overline{0}$,
$\overline{I} *_I \overline{u} = [0,1] *_I [u^-, u^+] = \overline{0}$,
$\overline{1} *_I \overline{u} = \overline{1}$.

Example 2.3: (a) $\overline{u} *_I \overline{v} = [u^- . v^-, u^+ . v^+]$; (b) $\overline{u} *_I \overline{v} = [u^- \wedge v^-, u^+ \wedge v^+]$;

Definition 2.4 [13]: Let $\{\overline{u}_i\} = \{[u_i^-, u_i^+]\}, i \in \mathbb{N}^+$ be a sequence of interval numbers in *[I]*, $\bar{u} = [u^-, u^+] \in [I]$, if $\lim_{i\to\infty} u_i^- = u^-$ and $\lim_{i\to\infty} u_i^+ = u^+$ then the sequence $\{\overline{u}_i\}$ is convergent to $\overline{u}$ and denoted by $\lim_{i\to\infty} \overline{u}_i = \overline{u}$.

Definition 2.5 [13]: An interval valued $t_{norm} *_I$ is continuous iff it is continuous in its first component, i.e for each $\overline{v} \in [I]$, if $\lim_{i\to\infty} \overline{u}_i = \overline{u}$, then $\lim_{i\to\infty}(\overline{u}_i *_I \overline{v}) = (\lim_{i\to\infty} \overline{u}_i *_I \overline{v}) = \overline{u} *_I \overline{v}$, Where $\{\overline{u}_i\} \subseteq [I], \overline{u} \in [I]$.

Definition 2.6 [9]: A 3-tuple $(\Xi, \mathfrak{M}_{IVFMS}, *_I)$ is called interval valued fuzzy metric space *(IVFMS)* if Ξ is an arbitrary set, $*_I$ is a continuous interval valued t_{norm} on *[I]* and $\mathfrak{M}_{IVFMS}$ is a fuzzy set on $\Xi^2 \times (0, \infty)$ satisfying the following conditions:

(1) $\mathfrak{M}_{IVFMS}(x, y, t_{norm}) > \bar{0}$;
(2) $\mathfrak{M}_{IVFMS}(x, y, t_{norm}) = \bar{1}$ $for\ all\ t > 0$ iff $x = y$;
(3) $\mathfrak{M}_{IVFMS}(x, y, t_{norm}) = \mathfrak{M}_{IVFMS}(y, x, t_{norm})$;
(4) $\mathfrak{M}_{IVFMS}(x, y, t_1) *_I \mathfrak{M}_{IVFMS}(y, z, t_2) \leq \mathfrak{M}_{IVFMS}(x, z, t_1 + t_2)$; $\forall x, y, z \in \Xi$ and $t_1, t_2, > 0$
(5) $\mathfrak{M}_{IVFMS}(x, y, *_I) : [0, \infty] \to (I)$ is continuous;
(6) $\lim_{t\to\infty} \mathfrak{M}_{IVFMS}(x, y, t_{norm}) = \bar{1}$; $\forall x, y, z \in \Xi, t_{norm} > 0$.

Definition 2.7 [13]: Let $(\Xi, \mathfrak{M}_{IVFMS}, *_I)$ is an IVFMS,

(a) If $\beta > t_{norm} > 0$ then $\mathfrak{M}_{IVFMS}(x, y, t_{norm}) \leq \mathfrak{M}_{IVFMS}(x, y, \beta)$ for $x, y \in \Xi$.
(b) A sequence $\{x_n\}$ in Ξ is referred to as a Cauchy sequence if for all $\bar{\epsilon} > \bar{0}$ and $t_{norm} > 0$ then there exists a $n_0 \in N\ \bar{\epsilon}\ \mathfrak{M}_{IVFMS}(x, y, t_{norm}) > 1 - \bar{\epsilon}$, for all $x, y \geq n_0$.
(c) Every Cauchy sequence is convergent and called complete *IVFMS.*

Definition 2.8 [13]: Two self mapping A and B on *IVFMS* $(\Xi, \mathfrak{M}_{IVFMS}, *_I)$ are said to be compatible if $\lim_{n\to\infty} \mathfrak{M}_{IVFMS}(ABx_n, BAx_n, t_{norm}) = \bar{1}\ for\ all\ t_{norm} > 0$ whenever $\{x_n\}$ is a sequence in Ξ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = u$, $for\ all\ u \in \Xi$.

Definition 2.9 [10]: Two self mapping A and B on *IVFMS* $(\Xi, \mathfrak{M}_{IVFMS}, *_I)$ are said to be a weakly compatible if they commute at their coincidence point that is for $\forall\ u \in \Xi$, $Au = Bu$ $implies\ that$ $ABu = BAu$ $for\ all\ t_{norm} > 0$.

Definition 2.10 [10]: Two self mapping A and B on *IVFMS* $(\Xi, \mathfrak{M}_{IVFMS}, *_I)$ are said to be satisfy E.A property if their exist a sequence $\{x_n\}$ in Ξ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = u$, $for\ all\ u \in \Xi$.

Lemma 2.11 [13]: Let S and T be a self mapping of a *IVFMS* $(\Xi, \mathfrak{M}_{IVFMS}, *_I)$. If S and T have unique point of coincidence $w = Sx = Tx$, then w is a unique common fixed point of S and T.

Lemma 2.12 [9]: $\mathfrak{M}_{IVFMS}(x, y, *_I)$ is non decreasing function $for\ all\ x, y \in \Xi$.

Lemma 2.13 [13]: Let *IVFMS* $(\Xi, \mathfrak{M}_{IVFMS}, *_I)$ if their exist $k \in (0,1)$ $such\ that$ $\mathfrak{M}_{IVFMS}(x, y, kt_{norm}) \geq \mathfrak{M}_{IVFMS}(x, y, t_{norm})$ then $x = y$ and , $k \in (0,1), t_{norm} > 0$ for all $x, y \in \Xi$.

Definition 2.14: Two self mapping A and B on *IVFMS* $(\Xi, \mathfrak{M}_{IVFMS}, *_I)$ are said to be occasionally weakly compatible (*OWC)* iff their exist a coincidence point $u \in \Xi$ of A and B such that A and B commutes at the point.

3. Main Result

Theorem 3.1: Let P, M, N, T, A, F, R and D be a self mapping of IVFMS $(\Xi, Ð_{IVFMS}, *_I)$ satisfying the following conditions:

(i) $(RP, NT) and (DM, AF)$ are occasionally weakly compatible.
(ii) $(R, P), (D, M), (A, F), (N, T), (DM, F)$ and (T, RP) are commuting pairs and $Px = P^2x, Mx = M^2x$ $for\ all\ x \in \Xi$.
(iii) For $\Theta \in \varphi$ then there exist $k \in (0,1)$.

$$Đ_{IVFMS}(RPx, DMy, kt_{norm}) \geq \varphi\left[\begin{array}{c} Đ_{IVFMS}(NTx, AFy, t_{norm}), Đ_{IVFMS}(NTx, RPx, t_{norm}), Đ_{IVFMS}(NTx, DMy, t_{norm}), \\ \frac{Đ_{IVFMS}(NTx, RPx, t_{norm}) + Đ_{IVFMS}(NTx, AFy, t_{norm})}{\bar{1} + Đ_{IVFMS}(RPx, AFy, t_{norm})} . Đ_{IVFMS}(RPx, AFy, t_{norm}), \\ \frac{aĐ_{IVFMS}(NTx, RPx, t_{norm}) + bĐ_{IVFMS}(DMy, AFy, t_{norm}) + cĐ_{IVFMS}(RPx, AFy, t_{norm})}{a + b + c\, Đ_{IVFMS}(DMy, NTx, t_{norm})}, \\ \frac{aĐ_{IVFMS}(RPx, DMy, t_{norm}) + bĐ_{IVFMS}(DMy, NTx, t_{norm}) + cĐ_{IVFMS}(RPx, AFy, t_{norm})}{a + b + c} \end{array}\right]$$

Such that $\forall\, x, y \in \Xi, a, b, c, d \geq 0$ with a and b cannot be simultaneously zero. $\varphi: [I]^6 \to [I]$, such that $\varphi(t_{norm}, \bar{1}, t_{norm},$ $\varphi: [I]^6 \to [I]$, such that $\varphi(t_{norm}, \bar{1}, t_{norm}, t_{norm}, 1, t_{norm}) \geq t_{norm}$ for $t_{norm} > 0$.
Then P, M, N, T, A, F, R and D have a unique common fixed point.

Proof: Pair (RP, NT) and (DM, AF) are occasionally weakly compatible then there are points $\sigma, \upsilon \in \Xi$ such that $RP\sigma = NT\sigma$ and $DM\upsilon = AF\upsilon$.

To claim, $= DM\upsilon$.

If not then by inequality (iii)

$$Đ_{IVFMS}(RP\sigma, DM\upsilon, kt_{norm}) \geq \varphi\left[\begin{array}{c} Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), Đ_{IVFMS}(RP\sigma, RP\sigma, t_{norm}), Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \\ \frac{Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}) + Đ_{IVFMS}(RP\sigma, RP\sigma, t_{norm})}{\bar{1} + Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})} . Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \\ \frac{aĐ_{IVFMS}(RP\sigma, RP\sigma, t_{norm}) + bĐ_{IVFMS}(DM\upsilon, DM\upsilon, t_{norm}) + cĐ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})}{a + b + c\, Đ_{IVFMS}(DM\upsilon, RP\sigma, t_{norm})}, \\ \frac{aĐ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}) + bĐ_{IVFMS}(DM\upsilon, RP\sigma, t_{norm}) + cĐ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})}{a + b + c} \end{array}\right]$$

$$Đ_{IVFMS}(RP\sigma, DM\upsilon, kt_{norm}) \geq \varphi\left[\begin{array}{c} Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \bar{1}, Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \\ \frac{Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}) + \bar{1}}{\bar{1} + Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})} . Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \\ \frac{a.\bar{1} + b.\bar{1} + cĐ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})}{a + b + c\, Đ_{IVFMS}(DM\upsilon, RP\sigma, t_{norm})}, \\ \frac{aĐ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}) + bĐ_{IVFMS}(DM\upsilon, RP\sigma, t_{norm}) + cĐ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})}{a + b + c} \end{array}\right]$$

$$Đ_{IVFMS}(RP\sigma, DM\upsilon, kt_{norm}) \geq \varphi\left[\begin{array}{c} Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \bar{1}, Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \\ Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \frac{a.\bar{1} + b.\bar{1} + cĐ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})}{a + b + c\, Đ_{IVFMS}(DM\upsilon, RP\sigma, t_{norm})}, \\ \frac{(a + b + c)Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})}{a + b + c} \end{array}\right]$$

$$Đ_{IVFMS}(RP\sigma, DM\upsilon, kt_{norm}) \geq \varphi\left[\begin{array}{c} Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \bar{1}, Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \\ Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}),\ \bar{1}, \\ Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}) \end{array}\right]$$

$$Đ_{IVFMS}(RP\sigma, DM\upsilon, kt_{norm}) \geq Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})$$

By lemma 2.13, we get $RP\sigma = DM\upsilon$
Therefore, $RP\sigma = NT\sigma = DM\upsilon = AF\upsilon$. suppose that μ is another point in Ξ such that $RP\mu = NT\mu$ By inequality (iii) we have $RP\mu = NT\mu \ = DM\upsilon = AF\upsilon$. so $RP\sigma = RP\mu$ and $RP\sigma = AF\sigma = \tau$ is the unique of coincidence of RP and AF. then by lemma 2.11, τ is a common fixed point of RP and AF.
In the same manner, there is a unique fixed point λ such that $DM\lambda = AF\lambda = \lambda$.
Assume that $\mu \neq \lambda$, we have.

by inequality (iii)

$$Đ_{IVFMS}(\mu, \lambda, kt_{norm}) = Đ_{IVFMS}(RP\mu, DM\lambda, kt_{norm})$$

$$\geq \varphi\left[\begin{array}{c} Đ_{IVFMS}(NT\mu, AF\lambda, t_{norm}), Đ_{IVFMS}(NT\mu, RP\mu, t_{norm}), Đ_{IVFMS}(NT\mu, DM\lambda, t_{norm}), \\ \frac{Đ_{IVFMS}(NT\mu, AF\lambda, t_{norm}) + Đ_{IVFMS}(NT\mu, RP\mu, t_{norm})}{\bar{1} + Đ_{IVFMS}(RP\mu, AF\lambda, t_{norm})} . Đ_{IVFMS}(RP\mu, AF\lambda, t_{norm}), \\ \frac{aĐ_{IVFMS}(NT\mu, RP\mu, t_{norm}) + bĐ_{IVFMS}(DM\lambda, AF\lambda, t_{norm}) + cĐ_{IVFMS}(RP\mu, AF\lambda, t_{norm})}{a + b + c\, Đ_{IVFMS}(DM\upsilon, RP\sigma, t_{norm})}, \\ \frac{aĐ_{IVFMS}(RP\mu, DM\lambda, t_{norm}) + bĐ_{IVFMS}(DM\lambda, NT\mu, t_{norm}) + cĐ_{IVFMS}(RP\mu, DM\lambda, t_{norm})}{a + b + c} \end{array}\right]$$

$$Ð_{IVFMS}(\mu,\lambda,kt_{norm}) \geq \varphi\begin{bmatrix} Ð_{IVFMS}(\mu,\lambda,t_{norm}),Ð_{IVFMS}(\mu,\mu,t_{norm}),Ð_{IVFMS}(\mu,\lambda,t_{norm}), \\ \frac{Ð_{IVFMS}(\mu,\lambda,t_{norm})+Ð_{IVFMS}(\mu,\mu,t_{norm})}{\bar{1}+Ð_{IVFMS}(\mu,\lambda,t_{norm})}.Ð_{IVFMS}(\mu,\lambda,t_{norm}), \\ \frac{aÐ_{IVFMS}(\mu,\mu,t_{norm})+bÐ_{IVFMS}(\lambda,\lambda,t_{norm})+cÐ_{IVFMS}(\mu,\lambda,t_{norm})}{a+b+c\,Ð_{IVFMS}(\mu,\lambda,t_{norm})}, \\ \frac{aÐ_{IVFMS}(\mu,\lambda,t_{norm})+bÐ_{IVFMS}(\lambda,\mu,t_{norm})+cÐ_{IVFMS}(\mu,\lambda,t_{norm})}{a+b+c} \end{bmatrix}$$

$$Ð_{IVFMS}(\mu,\lambda,kt_{norm}) \geq \varphi\begin{bmatrix} Ð_{IVFMS}(\mu,\lambda,t_{norm}),\bar{1},Ð_{IVFMS}(\mu,\lambda,t_{norm}), \\ \frac{Ð_{IVFMS}(\mu,\lambda,t_{norm})+\bar{1}}{\bar{1}+Ð_{IVFMS}(\mu,\lambda,t_{norm})}.Ð_{IVFMS}(\mu,\lambda,t_{norm}), \\ \frac{a.\bar{1}+b.\bar{1}+cÐ_{IVFMS}(\mu,\lambda,t_{norm})}{a+b+c\,Ð_{IVFMS}(\mu,\lambda,t_{norm})}, \\ \frac{aÐ_{IVFMS}(\mu,\lambda,t_{norm})+bÐ_{IVFMS}(\lambda,\mu,t_{norm})+cÐ_{IVFMS}(\mu,\lambda,t_{norm})}{a+b+c} \end{bmatrix}$$

$$Ð_{IVFMS}(\mu,\lambda,kt_{norm}) \geq \varphi\begin{bmatrix} Ð_{IVFMS}(\mu,\lambda,t_{norm}),\bar{1},Ð_{IVFMS}(\mu,\lambda,t_{norm}), \\ Ð_{IVFMS}(\mu,\lambda,t_{norm}), \\ \frac{a.\bar{1}+b.\bar{1}+cÐ_{IVFMS}(\mu,\lambda,t_{norm})}{a+b+c\,Ð_{IVFMS}(\mu,\lambda,t_{norm})} \\ \frac{(a+b+c)Ð_{IVFMS}(\mu,\lambda,t_{norm})}{a+b+c} \end{bmatrix}$$

$$Ð_{IVFMS}(\mu,\lambda,kt_{norm}) \geq \varphi\begin{bmatrix} Ð_{IVFMS}(\mu,\lambda,t_{norm}),\bar{1},Ð_{IVFMS}(\mu,\lambda,t_{norm}), \\ Ð_{IVFMS}(\mu,\lambda,t_{norm}),\bar{1}, \\ Ð_{IVFMS}(\mu,\lambda,t_{norm}) \end{bmatrix}$$

$$Ð_{IVFMS}(\mu,\lambda,kt_{norm}) \geq Ð_{IVFMS}(\mu,\lambda,t_{norm})$$

By lemma 2.13, we get $\mu = \lambda$.
Therefore μ is a common fixed point of $RP, NT, DM, and\ AF$.
Uniqueness: Let ϱ is another common fixed point of $RP, NT, DM, and\ AF$. Then

By inequality (iii)

$$Ð_{IVFMS}(\mu,\varrho,kt_{norm}) = Ð_{IVFMS}(RP\mu,DM\varrho,kt_{norm})$$
$$\geq \varphi\begin{bmatrix} Ð_{IVFMS}(NT\mu,AF\varrho,t_{norm}),Ð_{IVFMS}(NT\mu,RP\mu,t_{norm}),Ð_{IVFMS}(NT\mu,DM\varrho,t_{norm}), \\ \frac{Ð_{IVFMS}(NT\mu,AF\varrho,t_{norm})+Ð_{IVFMS}(NT\mu,RP\mu,t_{norm})}{\bar{1}+Ð_{IVFMS}(RP\mu,AF\varrho,t_{norm})}.Ð_{IVFMS}(RP\mu,AF\varrho,t_{norm}), \\ \frac{aÐ_{IVFMS}(NT\mu,RP\mu,t_{norm})+bÐ_{IVFMS}(DM\varrho,AF\varrho,t_{norm})+cÐ_{IVFMS}(RP\mu,AF\varrho,t_{norm})}{a+b+c\,Ð_{IVFMS}(DMv,RP\sigma,t_{norm})}, \\ \frac{aÐ_{IVFMS}(RP\mu,DM\varrho,t_{norm})+bÐ_{IVFMS}(DM\varrho,NT\mu,t_{norm})+cÐ_{IVFMS}(RP\mu,DM\varrho,t_{norm})}{a+b+c} \end{bmatrix}$$

$$Ð_{IVFMS}(\mu,\varrho,kt_{norm}) \geq \varphi\begin{bmatrix} Ð_{IVFMS}(\mu,\varrho,t_{norm}),Ð_{IVFMS}(\mu,\mu,t_{norm}),Ð_{IVFMS}(\mu,\varrho,t_{norm}), \\ \frac{Ð_{IVFMS}(\mu,\varrho,t_{norm})+Ð_{IVFMS}(\mu,\mu,t_{norm})}{\bar{1}+Ð_{IVFMS}(\mu,\varrho,t_{norm})}.Ð_{IVFMS}(\mu,\varrho,t_{norm}), \\ \frac{aÐ_{IVFMS}(\mu,\mu,t_{norm})+bÐ_{IVFMS}(\varrho,\varrho,t_{norm})+cÐ_{IVFMS}(\mu,\varrho,t_{norm})}{a+b+c\,Ð_{IVFMS}(\mu,\lambda,t_{norm})}, \\ \frac{aÐ_{IVFMS}(\mu,\varrho,t_{norm})+bÐ_{IVFMS}(\varrho,\mu,t_{norm})+cÐ_{IVFMS}(\mu,\varrho,t_{norm})}{a+b+c} \end{bmatrix}$$

$$Ð_{IVFMS}(\mu,\varrho,kt_{norm}) \geq \varphi\begin{bmatrix} Ð_{IVFMS}(\mu,\varrho,t_{norm}),\bar{1},Ð_{IVFMS}(\mu,\varrho,t_{norm}), \\ \frac{Ð_{IVFMS}(\mu,\varrho,t_{norm})+\bar{1}}{\bar{1}+Ð_{IVFMS}(\mu,\varrho,t_{norm})}.Ð_{IVFMS}(\mu,\varrho,t_{norm}), \\ \frac{a.\bar{1}+b.\bar{1}+cÐ_{IVFMS}(\mu,\varrho,t_{norm})}{a+b+c\,Ð_{IVFMS}(\mu,\varrho,t_{norm})}, \\ \frac{aÐ_{IVFMS}(\mu,\varrho,t_{norm})+bÐ_{IVFMS}(\varrho,\mu,t_{norm})+cÐ_{IVFMS}(\mu,\varrho,t_{norm})}{a+b+c} \end{bmatrix}$$

$$Ð_{IVFMS}(\mu,\varrho,kt_{norm}) \geq \varphi\begin{bmatrix} Ð_{IVFMS}(\mu,\varrho,t_{norm}),\bar{1},Ð_{IVFMS}(\mu,\varrho,t_{norm}), \\ Ð_{IVFMS}(\mu,\varrho,t_{norm}), \\ \frac{a.\bar{1}+b.\bar{1}+cÐ_{IVFMS}(\mu,\varrho,t_{norm})}{a+b+c\,Ð_{IVFMS}(\mu,\varrho,t_{norm})} \\ \frac{(a+b+c)Ð_{IVFMS}(\mu,\varrho,t_{norm})}{a+b+c} \end{bmatrix}$$

$$Ð_{IVFMS}(\mu,\varrho,kt_{norm}) \geq \varphi\begin{bmatrix} Ð_{IVFMS}(\mu,\varrho,t_{norm}),\bar{1},Ð_{IVFMS}(\mu,\varrho,t_{norm}), \\ Ð_{IVFMS}(\mu,\varrho,t_{norm}),\bar{1}, \\ Ð_{IVFMS}(\mu,\varrho,t_{norm}) \end{bmatrix}$$

$$Ð_{IVFMS}(\mu,\varrho,kt_{norm}) \geq Ð_{IVFMS}(\mu,\varrho,t_{norm})$$

By lemma 2.13, we get $\mu = \varrho$.
Therefore ϱ is a common fixed point of $RP, NT, DM, and\ AF$.
Since $(R,P),(D,M),(A,F),(N,T),(DM,F)\ and\ (T,RP)$ are commuting pairs and $Px = P^2x, Mx = M^2x\ for\ all\ x \in \Xi.$ so that we can easily show that ϱ is a common fixed point of $P,M,N,T,A,F,R\ and\ D$.

Remark * *Now we define new class of* ϱ *as follows.*

Let ϱ *be the class of all mapping* $\varrho:[I]\to[I]$ *such that*

(a) ϱ *is non-decreasing and* $\lim_{n\to\infty}\varrho^n(Q)=\bar{1},\forall Q\in(I)$;
(b) $\varrho(Q)>Q,\forall Q\in(I)$;
(c) $\varrho(\bar{1})=\bar{1}$

Example: *Define* $\varrho:[I]\to[I]$ *by* $\varrho(Q)=\frac{2Q}{Q+1},\forall Q\in[I],$

$\varrho^2(Q)=\frac{4Q}{3Q+1}, \varrho^3(Q)=\frac{8Q}{7Q+1}, \ldots\ldots, \varrho^n(Q)=\frac{2^nQ}{(2^n-1)Q+1}, \forall Q\in[I].$

$\lim_{n\to\infty}\varrho^n(Q)=\frac{2^nQ}{(2^n-1)p+1}=\bar{1}, \forall Q\in[I]$

Clearly, $\varrho(Q)>Q\ and\ \varrho(\bar{1})=\bar{1}, \forall Q\in[I]$

Theorem 3.2: Let $P,M,N,T,A,F,R\ and\ D$ be a self mapping of IVFMS $(\Xi,Ð_{IVFMS},*_I)$ satisfying the following conditions:
(i) $(RP,NT) and\ (DM,AF)$ are occasionally weakly compatible.
(ii) $(R,P),(D,M),(A,F),(N,T),(DM,F)\ and\ (T,RP)$ are commuting pairs and $Px = P^2x, Mx = M^2x\ for\ all\ x\in\ \Xi.$
(iii) For $\Theta\in\ \varphi$ then there exist $k\in(0,1)$.

$$Ð_{IVFMS}(RPx,DMy,kt_{norm}) \geq \varphi\left\{min\begin{bmatrix} Ð_{IVFMS}(NTx,AFy,t_{norm}),Ð_{IVFMS}(NTx,RPx,t_{norm}),Ð_{IVFMS}(NTx,DMy,t_{norm}), \\ \frac{Ð_{IVFMS}(NTx,RPx,t_{norm})+Ð_{IVFMS}(NTx,AFy,t_{norm})}{1+Ð_{IVFMS}(RPx,AFy,t_{norm})}.Ð_{IVFMS}(RPx,AFy,t_{norm}) \\ \frac{aÐ_{IVFMS}(NTx,RPx,t_{norm})+bÐ_{IVFMS}(DMy,AFy,t_{norm})+cÐ_{IVFMS}(RPx,AFy,t_{norm})}{a+b+c\,Ð_{IVFMS}(DMy,NTx,t_{norm})}, \\ \frac{aÐ_{IVFMS}(RPx,DMy,t_{norm})+bÐ_{IVFMS}(DMy,NTx,t_{norm})+cÐ_{IVFMS}(RPx,AFy,t_{norm})}{a+b+c} \end{bmatrix}\right\}$$

Such that $\forall\ x,y\in\Xi, t_{norm}>0,\ a,b,c,d\geq 0$ with a and b cannot be simultaneously zero.
Then $P,M,N,T,A,F,R\ and\ D\ \ have\ a\ unique\ common\ fixed\ point.$

Proof: Pair $(RP,NT) and\ (DM,AF)$ are occasionally weakly compatible then there are points $\sigma,\upsilon\in\ \Xi$ such that $RP\sigma=NT\sigma$ and $DM\upsilon=AF\upsilon.$

To claim, $=DM\upsilon$.
If not then by inequality (iii)

$$Ð_{IVFMS}(RP\sigma,DM\upsilon,kt_{norm}) \geq \varphi\left\{min\begin{bmatrix} Ð_{IVFMS}(RP\sigma,DM\upsilon,t_{norm}),Ð_{IVFMS}(RP\sigma,RP\sigma,t_{norm}),Ð_{IVFMS}(RP\sigma,DM\upsilon,t_{norm}), \\ \frac{Ð_{IVFMS}(RP\sigma,DM\upsilon,t_{norm})+Ð_{IVFMS}(RP\sigma,RP\sigma,t_{norm})}{\bar{1}+Ð_{IVFMS}(RP\sigma,DM\upsilon,t_{norm})}.Ð_{IVFMS}(RP\sigma,DM\upsilon,t_{norm}), \\ \frac{aÐ_{IVFMS}(RP\sigma,RP\sigma,t_{norm})+bÐ_{IVFMS}(DM\upsilon,DM\upsilon,t_{norm})+cÐ_{IVFMS}(RP\sigma,DM\upsilon,t_{norm})}{a+b+c\,Ð_{IVFMS}(DM\upsilon,RP\sigma,t_{norm})}, \\ \frac{aÐ_{IVFMS}(RP\sigma,DM\upsilon,t_{norm})+bÐ_{IVFMS}(DM\upsilon,RP\sigma,t_{norm})+cÐ_{IVFMS}(RP\sigma,DM\upsilon,t_{norm})}{a+b+c} \end{bmatrix}\right\}$$

$$Đ_{IVFMS}(RP\sigma, DM\upsilon, kt_{norm}) \geq \varphi\left\{min\left[\begin{array}{c} Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \bar{1}, Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \\ \frac{Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}) + \bar{1}}{\bar{1} + Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})} . Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \\ \frac{a.\bar{1} + b.\bar{1} + cĐ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})}{a + b + c\, Đ_{IVFMS}(DM\upsilon, RP\sigma, t_{norm})}, \\ \frac{aĐ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}) + bĐ_{IVFMS}(DM\upsilon, RP\sigma, t_{norm}) + cĐ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})}{a + b + c} \end{array}\right]\right\}$$

$$Đ_{IVFMS}(RP\sigma, DM\upsilon, kt_{norm}) \geq \varphi\left\{min\left[\begin{array}{c} Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \bar{1}, Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \\ Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \frac{a.\bar{1} + b.\bar{1} + cĐ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})}{a + b + c\, Đ_{IVFMS}(DM\upsilon, RP\sigma, t_{norm})}, \\ \frac{(a + b + c)Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})}{a + b + c} \end{array}\right]\right\}$$

$$Đ_{IVFMS}(RP\sigma, DM\upsilon, kt_{norm}) \geq \varphi\left\{min\left[\begin{array}{c} Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \bar{1}, Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}), \\ Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}),\ \bar{1}, \\ Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm}) \end{array}\right]\right\}$$

$$Đ_{IVFMS}(RP\sigma, DM\upsilon, kt_{norm}) \geq \varphi(Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})$$
$$Đ_{IVFMS}(RP\sigma, DM\upsilon, kt_{norm}) \geq Đ_{IVFMS}(RP\sigma, DM\upsilon, t_{norm})$$

By lemma 2.13, we get $RP\sigma = DM\upsilon$

Therefore, $RP\sigma = NT\sigma = DM\upsilon = AF\upsilon.$ suppose that μ is another point in Ξ such that $RP\mu = NT\mu$ By inequality (iii) we have $RP\mu = NT\mu\ = DM\upsilon = AF\upsilon.$ so $RP\sigma = RP\mu$ and $RP\sigma = AF\sigma = \tau$ is the unique of coincidence of RP and $AF.$ then by lemma 2.11, τ is a common fixed point of RP and $AF.$

In the same manner, there is a unique fixed point λ such that $DM\lambda = AF\lambda = \lambda$.

Assume that $\mu \neq \lambda$, we have.

By inequality (iii)

$$Đ_{IVFMS}(\mu, \lambda, kt_{norm}) = Đ_{IVFMS}(RP\mu, DM\lambda, kt_{norm})$$
$$\geq \varphi\left\{min\left[\begin{array}{c} Đ_{IVFMS}(NT\mu, AF\lambda, t_{norm}), Đ_{IVFMS}(NT\mu, RP\mu, t_{norm}), Đ_{IVFMS}(NT\mu, DM\lambda, t_{norm}), \\ \frac{Đ_{IVFMS}(NT\mu, AF\lambda, t_{norm}) + Đ_{IVFMS}(NT\mu, RP\mu, t_{norm})}{\bar{1} + Đ_{IVFMS}(RP\mu, AF\lambda, t_{norm})} . Đ_{IVFMS}(RP\mu, AF\lambda, t_{norm}), \\ \frac{aĐ_{IVFMS}(NT\mu, RP\mu, t_{norm}) + bĐ_{IVFMS}(DM\lambda, AF\lambda, t_{norm}) + cĐ_{IVFMS}(RP\mu, AF\lambda, t_{norm})}{a + b + c\, Đ_{IVFMS}(DM\upsilon, RP\sigma, t_{norm})}, \\ \frac{aĐ_{IVFMS}(RP\mu, DM\lambda, t_{norm}) + bĐ_{IVFMS}(DM\lambda, NT\mu, t_{norm}) + cĐ_{IVFMS}(RP\mu, DM\lambda, t_{norm})}{a + b + c} \end{array}\right]\right\}$$

$$Đ_{IVFMS}(\mu, \lambda, kt_{norm}) \geq \varphi\left\{min\left[\begin{array}{c} Đ_{IVFMS}(\mu, \lambda, t_{norm}), \bar{1}, Đ_{IVFMS}(\mu, \lambda, t_{norm}), \\ \frac{Đ_{IVFMS}(\mu, \lambda, t_{norm}) + \bar{1}}{\bar{1} + Đ_{IVFMS}(\mu, \lambda, t_{norm})} . Đ_{IVFMS}(\mu, \lambda, t_{norm}), \\ \frac{a.\bar{1} + b.\bar{1} + cĐ_{IVFMS}(\mu, \lambda, t_{norm})}{a + b + c\, Đ_{IVFMS}(\mu, \lambda, t_{norm})}, \\ \frac{aĐ_{IVFMS}(\mu, \lambda, t_{norm}) + bĐ_{IVFMS}(\lambda, \mu, t_{norm}) + cĐ_{IVFMS}(\mu, \lambda, t_{norm})}{a + b + c} \end{array}\right]\right\}$$

$$Đ_{IVFMS}(\mu, \lambda, kt_{norm}) \geq \varphi\left\{min\left[\begin{array}{c} Đ_{IVFMS}(\mu, \lambda, t_{norm}), \bar{1}, Đ_{IVFMS}(\mu, \lambda, t_{norm}), \\ Đ_{IVFMS}(\mu, \lambda, t_{norm}), \\ \frac{a.\bar{1} + b.\bar{1} + cĐ_{IVFMS}(\mu, \lambda, t_{norm})}{a + b + c\, Đ_{IVFMS}(\mu, \lambda, t_{norm})} \\ \frac{(a + b + c)Đ_{IVFMS}(\mu, \lambda, t_{norm})}{a + b + c} \end{array}\right]\right\}$$

$$Đ_{IVFMS}(\mu, \lambda, kt_{norm}) \geq \varphi\left\{min\left[\begin{array}{c} Đ_{IVFMS}(\mu, \lambda, t_{norm}), \bar{1}, Đ_{IVFMS}(\mu, \lambda, t_{norm}), \\ Đ_{IVFMS}(\mu, \lambda, t_{norm}), \bar{1}, \\ Đ_{IVFMS}(\mu, \lambda, t_{norm}) \end{array}\right]\right\}$$

$$Đ_{IVFMS}(\mu, \lambda, kt_{norm}) \geq \varphi\{Đ_{IVFMS}(\mu, \lambda, t_{norm})\}$$

Since $\varrho(\mathcal{Q}) \geq \mathcal{Q}$ $(by\, remark\, *\ (b)\&(c))$ for all $\mathcal{Q} \in (I]$, it is only possible when $Đ_{IVFMS}(\mu, \lambda, t_{norm}) = \bar{1}$
$Đ_{IVFMS}(\mu, \lambda, kt_{norm}) \geq Đ_{IVFMS}(\mu, \lambda, t_{norm})$ $Đ_{IVFMS}(\mu, \lambda, kt_{norm}) = \bar{1}$.

We get $\mu = \lambda$, (by def. 2.6 (2)).

Therefore μ is a common fixed point of $RP, NT, DM, and\ AF$.

4. Conclusion

In this paper, we shown common fixed point result which has been remove continuity of mapping and containment of ranges by using common E.A like property and weakly compatible mapping on interval valued fuzzy metric space *(IVFMS)* for rational expression. These results will very helpful for the researchers and accelerate sustainable development for the existence and uniqueness of common fixed point in theoretical mathematics.

5. References

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