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An Interdisciplinary Teaching Proposal for the Introduction of Fractals to Junior High School

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Abstract
The aim of this paper is to integrate Fractals into the conventional High School curriculum through an interdisciplinary approach between Mathematics and Biology. The proposed activities aim to assist educators in delivering a comprehensive and captivating learning journey for students, encouraging deeper comprehension, creativity, and interconnections across various scientific disciplines. Fractals are intricate geometric shapes characterized by self-similarity, meaning they display similar patterns at multiple scales. They are formed through iterative processes, distinguishing them from traditional geometric figures and offer an enjoyable and stimulating method to introduce numerous mathematical and biological concepts. By connecting fractals to various mathematical concepts and real-world applications, junior High School students can enhance their problem-solving skills and gain a greater appreciation for the beauty and complexity of Mathematics.

Keywords: Fractals, Fractal Dimension, Interdisciplinary Education, Science, Mathematics, Biology, Junior High School Education

1. Introduction

At the start of every school year, educators are tasked with engaging students in the learning process. They must select an instructional approach they believe will be effective. This task is challenging due to various issues, such as the lack of student motivation [1]. Implementing interdisciplinary lesson plans that include Fractals offers many benefits. Studying Fractals promotes creativity and enhances problem-solving skills. Additionally, Fractals encourage cross-disciplinary connections, allowing students to explore historical, ecological, and scientific contexts. Overall, interdisciplinary lessons involving Fractals boost student engagement, motivation, and critical thinking, making learning more enjoyable and meaningful. By incorporating these interdisciplinary lessons, educators can provide a comprehensive and engaging educational experience that fosters deeper understanding, creativity, and connections across different subjects. This interdisciplinary approach offers numerous advantages, cultivating essential lifelong learning skills that are crucial for students' future education [2].

Introducing Fractal patterns in nature and Biology to Junior High School students can be done in a visually appealing way, helping them see the connection between Fractals and biological structures. Presenting Fractal patterns and Biology in an engaging and relatable manner allows students to appreciate the beauty and functionality of natural systems and understand the basic links between Fractals and Biology. Students are aware of specific patterns in Mathematics, especially in geometry. Through their exposure to Fractal sets, they will discover that similar repeating patterns exist in nature, which they might have previously overlooked. In Biology, many natural structures, such as trees, display branching patterns that resemble Fractals. The branching of blood vessels, bronchial tubes in the lungs, nerve cells, and even the plant circulatory system exhibit Fractal characteristics. This resemblance is due to the evolutionary optimization of biological systems for efficiency and resource distribution. The goal of this teaching proposal is to introduce Junior High School students to a new approach to science lessons. It will provide them with opportunities to work in groups, collaboratively experimenting, imagining, investigating, and ultimately understanding complex mathematical concepts [3]. They will also recognize the connection between Mathematics and nature [4].

This work continues our research into the integration of new technologies in teaching and learning, particularly their application in High Schools [5]. This paper presents an interdisciplinary educational plan specifically designed for Junior High

School students, highlighting the importance of developing interdisciplinary skills at this educational level. The proposal aims to introduce Junior High School students to a novel approach to learning Mathematics. It will enable them to work cooperatively in groups, experimenting, imagining, investigating, and understanding complex mathematical concepts, particularly the geometry of Fractals, through real-world biological examples. We believe it is crucial for students to gain experience with interdisciplinary teaching proposals, helping them develop essential skills, strategies, and critical thinking abilities that will benefit their entire educational journey. This study is organized into two sections: the first section offers a concise explanation and introduction to fractals, while the second section details an educational plan centered on fractals and biology for Junior High School students.

2. Fractals and Fractal Dimension

In traditional geometry classes, we often focus on familiar shapes like lines, circles, squares, cubes, cylinders, and spheres. However, when we look at the natural world, we encounter a vast array of shapes that differ significantly from these simple geometric objects. Nature presents us with intricate and complex forms such as clouds, lightning bolts, ice crystals, sponges, and coastlines. These shapes challenge our understanding of geometry and Highlight the richness and diversity of the world around us, offering a fascinating contrast to the shapes we typically study in "classical" geometry.

One of the foremost mathematical challenges is deciphering the formation of these irregular shapes, such as the seemingly random patterns in clouds or the intricate structure of snowflakes. Fractal geometry was developed specifically to describe and analyze the complex shapes and patterns observed in nature, providing a framework to understand and characterize their unique and often chaotic forms. This field was pioneered by Benoit Mandelbrot, who coined the term "Fractal" from the Latin word "fractus," meaning fragmented or broken [6]. In Mathematics, Physics, and other sciences, a Fractal is a geometric shape that repeats itself identically at every scale of magnification. This unique property of Fractals is known as self-similarity [7].

Our world is replete with fractals, as numerous natural objects exhibit striking self-similarity. From the intricate patterns of snowflakes to the branching of trees and the structure of coastlines, fractals are prevalent in nature, revealing a remarkable repetition of patterns at various scales [8]. Common examples of fractal structures include broccoli, fern leaves, and trees. These patterns of self-similarity extend to the human body as well, with fractal-like organization observable in the branching of lungs, the intricate network of neurons, and the complex arrangement of blood vessels. These natural and biological examples illustrate the pervasive presence of fractals in both the environment and within ourselves [9]. However, it is important to note that the Fractals observed in nature do not exhibit infinite detail upon magnification, unlike those derived from mathematical formulas.

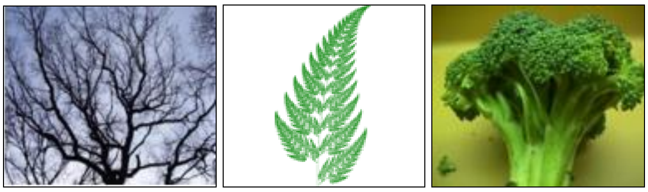


Fig 1: Examples of Fractals

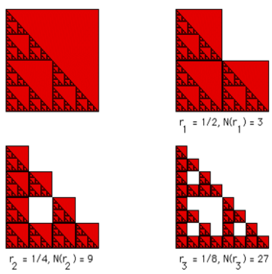
Unlike classical geometry, Fractals are irregular and may have non-integer dimensions. Fractal geometry has been shown to effectively quantify irregular patterns, such as winding lines, crumpled surfaces, and intricate shapes, and to estimate the complexity of systems [10]. A commonly utilized technique for determining the dimension of a fractal is the box-counting method. This approach is simple, computationally feasible, and suitable for analyzing patterns, whether they exhibit self-similarity or not [11, 12]. In the box-counting procedure, an image is covered with a sequence of grids of decreasing sizes, and two values are recorded for each grid. In the box-counting procedure, an image is overlaid with a series of grids of progressively smaller sizes, and two values are noted for each grid.: the number of squares that intersect with the image (N(s)) and the side length of the squares (s).

The Fractal dimension is then determined by the regression slope D ($1 \leq D \leq 2$) of the line formed by plotting log(N(s)) against log(1/s) [10]. An image with a Fractal dimension of 1 is considered smooth, while one with a Fractal dimension of 2 is deemed very rough and irregular. The linear regression equation used to estimate the Fractal dimension is:

$$\log(N(s))=\log(K)+D\log(1/s),$$

where K is a constant and N(s) is proportional to (1/s)D. This box-counting method can be easily performed using free software suitable for calculating Fractal dimensions, and it can enhance interdisciplinary learning, as Fractals and their dimensions are relevant to many subjects in the basic education curriculum [13]. Fractal dimension is a valuable characteristic used to describe roughness and self-similarity in an image, and it has applications in texture segmentation, shape classification, and graphical analysis in various fields [14].

An example of box counting in Sierpinski triangle: By covering a triangle with progressively smaller boxes, we observe the pattern depicted in the table on the right..



$N(1) = 1$
$N(1/2) = 3$
$N(1/4) = N((1/2)^2) = 9=3^2$
$N(1/8) = N((1/2)^3) = 27 = 3^3$
and in general
$N((1/2)^n) = 3^n$.

Using for example the third line of the above table to calculate the fractal dimension of the Sierpinski triangle: $9=(1/(1/4))^D \Rightarrow 9=4^D \Rightarrow \log 9=\log 4^D \Rightarrow \log 9=D\log 4 = D=\log 9/\log 4 \Rightarrow D\approx 1.585$

3. Pedagogical utilization of Fractals

Teaching fractals in the classroom can be extraordinarily captivating, especially for Junior High School students. Integrating fractals into the curriculum not only ignites a fascination with the beauty and intricacy of mathematics but also significantly enhances students' critical thinking and problem-solving skills. As students explore and create fractal patterns, they develop a deeper understanding of mathematical concepts and gain valuable insights into complex, dynamic systems. This approach makes learning both engaging and intellectually stimulating, bridging abstract mathematical theory with tangible, visual experiences.

Integrating fractals into a junior High school math curriculum offers significant benefits across various subjects. In geometry, fractals present a fascinating study of shapes that demonstrate self-similarity—where patterns repeat at progressively smaller scales—enhancing students' comprehension of symmetry and similarity. In algebra, the distinctive attributes of fractals can be analyzed through ratios and proportions, providing insights into how their length, area, and perimeter vary with different scales. Moreover, in biology, visually compelling lessons can reveal the prevalence of fractal patterns in nature and biological structures, bridging mathematical concepts with the natural world and sparking students' interest in both disciplines. This interdisciplinary approach not only enriches their understanding but also highlights the interconnectedness of mathematical and natural phenomena.

By presenting Fractals and Biology in a relatable manner, students can appreciate the beauty and functionality of natural systems and grasp the basic link between Fractals and Biology. This approach helps students understand complex concepts in a more accessible and engaging way.

Establishing a connection between Biology and Mathematics is crucial for evolving the paradigms of both scientific disciplines. However, merely incorporating some mathematical content into Biology or vice versa is insufficient; appropriate pedagogical models must also be developed [15]. Over the past two decades, various authors [16, 17] have highlighted the need for new teaching methods that better align with contemporary challenges, which require reflection and reconsideration of our educational approaches Introducing fractals in schools helps develop teaching methods that focus on placing students at the center of the educational process. A student-centered teaching model seeks to address student diversity, promote academic responsibility, encourage active participation, and nurture self-directed learning skills [17].

4. State of the Art

This teaching approach is ideally conducted in the IT laboratory, where students can utilize computers and the interactive whiteboard commonly found in School laboratories. The important of this method lies in its accessibility; students don't need advanced mathematical skills or prior knowledge. The objective is to actively engage as many students as possible, inspiring them to explore and experiment with new concepts while fostering a culture of discovery and creativity. To facilitate this, the classroom will be organized into small, dynamic groups, where students can collaborate closely, exchange ideas, communicate effectively, reflect on their learning experiences, and express mathematical concepts with clarity and confidence. This setup aims to create a vibrant learning environment that promotes active participation and deepens students' understanding of the subject matter. This collaborative environment fosters active participation and encourages students to share their thoughts, ask questions, and explore concepts together. This encourages students to cultivate scientific dialogue practices and enhances their overall learning experience. A high level description of the proposed pedagogical approach is given below:

Teaching Approach: Teaching Approach: Enriched lectures, collaborative learning, hands-on activities and exploratory instruction.

Goals:

1. *Recognize fractal patterns.*
2. *Comprehend the self-similarity of fractals.*
3. *Adjust parameters in a function or process technique.*
4. *Calculate the fractal dimension.*

Teaching Tools: Whiteboard, computer, projector, paper, and pencils.

Proposed Teaching Method Stages:

1. *Provide a brief historical overview.*
2. *Search for fractals in the environment and nature.*
3. *Investigate and study fractals and their properties.*
4. *Examine real-world problems involving fractals.*

Emphasis will be placed on the concept of dimension, starting with Euclidean geometry and progressing Fractal Dimension. The teacher will present mathematical Fractal images to captivate students' interest, prompting questions and reflections on the properties of these unique objects. Students will engage in iterative procedures and calculations to determine the perimeter and area of Fractal shapes, fostering both enjoyment and the development of higher-level math and reasoning skills not typically encountered in traditional High School math. The intricate designs and vibrant colors of these images are sure to leave a lasting impression on students.

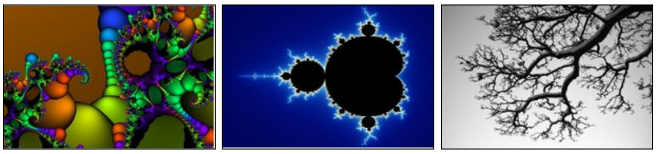


Fig 2: Fractals

Additionally, a brief yet comprehensive historical overview is essential. Fractals, a cornerstone of modern Mathematics, have experienced significant growth in recent years. A good first case to study is Sierpinski's triangle. Students will draw Sierpinski's triangle in which they will in fact establish self-similarity as well as some of its basic properties such as infinite length and zero area. A first acquaintance with Sierpinski's triangle can be made by using special worksheets that will give them specific instructions for the construction of this strange triangle. The purpose is for each group of children to draw and paint with vivid colors their own triangle, then they could all be grouped together, i.e. set them up in such a way that the children's triangles create and compose a new and large triangle after many repetitions.

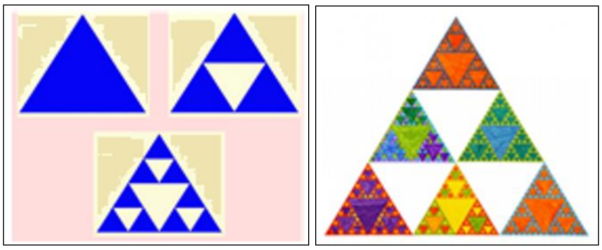


Fig 3: Students create the Sierpinski triangle

Then, after the students had a fun and creative time drawing Sierpinski's triangle, it was time to look at it from a more formal mathematical point of view. For this reason we will hand out to the children specially designed worksheets that will explain precisely the iterative process for designing this particular triangle. The process in the form of an algorithm would be as follows.

[1]. Draw an equilateral triangle.
[2]. Find the midpoints of its sides and join them.
[3]. Remove the middle triangle.
[4]. Repeat from step 2.

Upon completion of the above process, students should observe this strange triangle they constructed and try to answer questions regarding its perimeter and area. The interest in this part of the teaching is that the students will be confronted with their existing knowledge since they will come to the conclusion that Sierpinski's triangle has zero area and infinite perimeter. So this shape is something different from what they have already learned to deal with, study and negotiate in Euclidean Geometry. Obviously they will not be able to calculate its dimension since it requires mathematical knowledge that they do not yet possess, such as logarithm. But they can deal with the dimension of this triangle by drawing logical conclusions.

This is where proper guidance from the teacher is critical. Students must observe and finally conclude that this strange triangle cannot have dimension 2, since it has zero area. On the other hand it obviously cannot have dimension 1 since it is not a simple line. So we conclude that the dimension of this shape is between 1 and 2 and thus the students are slowly introduced to the concept of fractional dimension. Then, we will also explain to the students that they will apply the new scientific ideas they acquired to a real problem from the science of Biology. It is important to emphasize that students do not need advanced mathematical skills or special prerequisite knowledge. The primary goal has always been to engage as many students as possible, encouraging them to observe and experiment with new concepts, and generate new knowledge through their discoveries and creations.

Fields like Biology and Mathematics have long been interconnected. Through interdisciplinary teaching, we aim for our students to develop more adaptable knowledge, allowing them to apply it in diverse contexts and transfer it effectively [18]. Engaging in interdisciplinary projects is crucial in modern education. Thus, as educators, we must devise strategies to equip our students with tools that will enhance their future professional endeavors and critical thinking abilities. This project will utilize interdisciplinary approaches, fostering a critical debate among students that integrates knowledge across various subjects to promote interdisciplinary learning [19]. This approach creates new opportunities for educational institutions to explore alternative pathways for meaningful academic and ongoing educational programs, where formal and informal learning can coexist [17].

Integrating these disciplines early in the educational process is essential for preparing students to blend both fields at the graduate and postgraduate levels. While many biological processes can be modeled using traditional High School mathematics, some extend beyond the standard curriculum, as illustrated by fractal structures in biology. Despite the well-documented fractal nature of physical objects in nature and the fact that numerous biological structures exhibit fractal patterns, this knowledge is frequently neglected by curriculum developers in both mathematics and science [15].

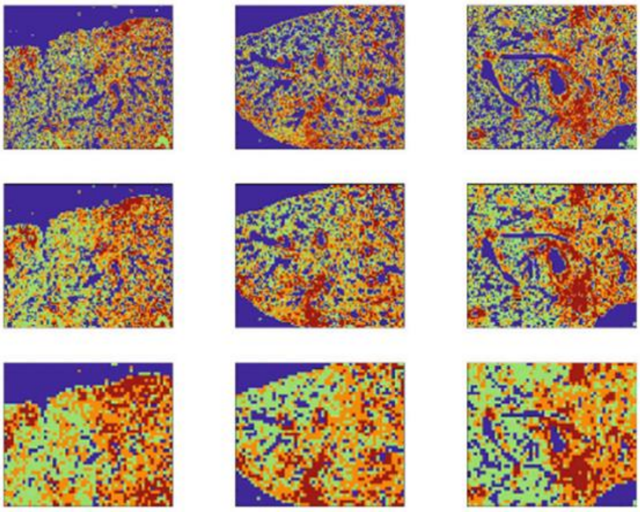


Fig 4: The Fractal dimension in the images of the lungs has been calculated with the Box counting method

In previous work, we analyzed images of lungs from mice suffering from cystic fibrosis. We used Fractal dimension to find the affected area with great success [14]. In this work students will apply the concept of Fractal dimension to analyze images of mouse lungs. Acting as budding scientists, students will carefully examine the lung images to identify areas of concern. They will understand that an increase in Fractal dimension, indicated by the color red, signifies potential issues within the lung. Armed with this knowledge, students will be able to pinpoint problematic areas that require specialized attention from a medical specialist. This hands-on approach not only reinforces their understanding of Fractals but also empowers students to make meaningful observations and contributions in the field of Biology.

5. Conclusions

Introducing fractals in an engaging and interactive way allows students to gain a greater appreciation for the beauty and complexity of mathematics while sharpening their critical thinking and problem-solving skills. Studying fractals provides a versatile, interdisciplinary learning experience that deepens students' understanding across different mathematical areas. By connecting fractals to various mathematical concepts and real-world applications, junior High School students can improve their problem-solving abilities and develop a greater appreciation for the intricacies of mathematics.

Moreover, the interdisciplinary nature of studying Fractals goes beyond Mathematics, enhancing students' understanding across multiple subjects. Integrating graphics into teaching opens new possibilities for delivering lessons, as it is scientifically recognized that visuals help maintain student attention and make the lesson more comprehensible [20].

Many courses lack sufficient engagement and fail to spark curiosity among students. Inspiring and motivating students is crucial, as it ignites a passion for learning and opens doors to further exploration. This emphasis on inspiration and motivation is fundamental for long-term academic success, surpassing the mere delivery of information. Therefore, fostering an environment of joy, excitement, and a love for learning is imperative. In future studies, we aim to implement these principles in the classroom and report our observations and outcomes.

Many courses suffer from an overload of content, a lack of critical thinking, and insufficient enjoyment. It is crucial to inspire and motivate students, as once they are, they can access numerous resources to deepen their understanding of a subject. Fostering inspiration and motivation is far more crucial for long-term success than just imparting information. Therefore, we should focus on cultivating joy, excitement, and a genuine passion for learning [21]. In future research, we will apply these principles in the classroom and disseminate our findings.

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