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Traveling around the Sun: A Project to Study Planetary Motion in an Attractive Central Force Field with the help of MATHEMATICA

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Abstract
The present work focuses on the creation of an interactive environment of active learning of an exploratory type, by exploiting the capabilities of the MATHEMATICA software, for the study of the planetary motion around the central star. Through the software environment, students are given the opportunity to experiment by implementing interdisciplinary projects in the fields of Physics, Mathematics and Informatics. The results of the pilot research point to the understanding of the cognitive subject by the learners, as well as to the strengthening of skills, such as communication, collaboration, problem solving, critical thinking, reflection and creativity, showing the direction of the main research that follows.

Keywords: Teaching Physics, Planetary Motion, Simulation of Orbits, MATHEMATICA

1. Introduction

Physics teaching in University Departments follows an integrated approach, combining lectures with the use of appropriate teaching tools, demonstration experiments, aiming at basic principles and laws of Physics, tutorial sessions in theoretical topics and exercises, as well as laboratory exercises with a focus on the measurement experiment. Using visual aids during lectures can help visualize the concepts being taught and make lessons more accessible and interesting. Demonstration experiments focusing on principles and laws of physics provide students with a first contact with the real application of theoretical science concepts. Tutorial sessions and practice strengthen theoretical understanding, while laboratory measurement exercises play an essential role in familiarizing learners with experimental techniques and, more generally, with the scientific method of examining the natural world. Teaching success is assessed through the extent to which an understanding of the behavior of physical systems is achieved, the learning of methodologies used to solve physical problems, and students' ability to use experimental and computational techniques. This fact is particularly important, as it should enable them to apply the concepts, models and techniques (both experimental and computational) they have learned, to situations and problems of daily experience, that is, to real life situations. Such an educational approach constitutes a complete educational experience as it combines theoretical knowledge, experimental practice and application of knowledge to practical problems of everyday life.

A key issue in Science Education concerns the determination of the optimal teaching approach. This approach should encourage active participation in the learning process and facilitate a deep understanding of the subject matter. To address today's challenges in teaching, some researchers propose a teaching approach that integrates multiple representations of specific cognitive fields, the use of which contributes to the construction of cognitive schemas, which can be applied to the analysis of phenomena and the solving of related problems.

Research efforts that began many years ago (Merrill, 1980, Thornton, 1987, Mc Dermott, 1993) [21, 31, 20] indicate that the use of Information and Communication Technologies (ICT) is particularly effective in creating representations of natural systems that are taught in all the levels of education (Tsihouridis & Vavougios, 2023) [33]. The use of appropriate software and all kinds of digital tools allows and facilitates the simulation and visualization of phenomena, the supervisory presentation of principles and laws of Physics and of course the collection, processing and presentation of "small" but also "large volume" data. Moreover, it supports personalized teaching and learning, a crucial aspect in today's Information Society (Martin, Squadron and Young, 1991, Laws, 1991, Neves *et al*., 2011) [19, 16, 23] leading researchers to support the introduction of interactive teaching methodologies, a fact that results in the reform of the following teaching practices.

But which specialized software can improve the situation by providing simulations with exploratory and/or experimental procedures compatible with the learning objectives required? What computational tools can be used to produce attractive simulations minimizing the distance between the simulation of the physical system behavior and experimental performance?

2. Study of the Physical world & Modelling

When studying the natural world, the following methodological scheme is generally followed (Williamson, 1996, Vavougios & Thanos, 2005, Vavougios & Karakasidis, 2008) [38, 36, 35]: In every natural system under study, a model of it is constructed in the laboratory. This model constitutes the physical model to which, through a mathematical description (such as a set of algebraic, differential, integral differential equations, difference equations, etc.) we match the mathematical model. The solutions of this mathematical model approximate the behavior of the physical system by predicting its behavior for various initial conditions, which can then be compared with the experiment, thus, enabling the improvement of both the physical and the corresponding mathematical model. The repetition of this process contributes to the gradual improvement of the understanding of natural phenomena as well as to the understanding of the relevant laws that govern their behavior. The role of the simulation of the natural system under study, that allows “computational experimentation” with it and an initial understanding of its behavior, is considered important (Baumann, 1996 [4], Acheson, 1997, Leader, 2004 [17], Gould *et al*., 2006 [10], Steinhauser, 2012 [29], Hehenberger *et al*., 2016 [13], Feynman, 2018 [8], Downey, 2021 [7]).

3. Software used for the study of natural systems

The use of software is believed to contribute to the development of multiple representations in the teaching and learning of Physics, allowing the creation of simulations and models that facilitate interactive teaching and allow students to approach cognitively demanding areas with difficult mathematical formalism, thus facing realistic problems. Particularly effective and popular in applied sciences is the use of the mathematical packages MATHEMATICA and MATLAB included in their teaching program. In the present research and for the configuration of the Project, the use of MATHEMATICA was chosen, due to its ease of use and its individual special characteristics (Blachman, 1991, Gray & Glynn, 1992, Wolfram, 1992, 1996, 2003, 2020, Tam, 1997, Zimmerman *et al*., 2002, Shifrin, 2008, Abel & Braselton, 2009, Ruskeepää, 2009, Tsihouridis *et al*., 2012, Wellin 2013, Grozin, 2014, Napolitano, 2018) [5, 11, 39, 40, 41, 42, 30, 43, 28, 2, 25, 32, 37, 12, 22].

These are:

- The simplified expression of mathematical symbols and relationships.
- The easy creation of various graphs depicting functions, curves and surfaces.
- Animation capabilities that depict the motion of systems in both standard space and phase space.
- The ability of the software to solve almost any mathematical problem with a known calculation method (analytical or numerical).

The activities chosen to be implemented by the students, in the context of the Project, utilizing their knowledge from the courses of Theoretical Engineering, Numerical Analysis and Programming are as follows:

- To formulate a model corresponding to the rotational motion of a material point of mass m around a gravitational center of mass M.
- To graphically present the trajectory of the material point which rotates around the center of gravity located at the origin of the axis system of a coordinate system.
- To create an animation of the movement by representing the vector quantities (speed of rotation, attractive force).
- To use the model in exploratory approaches (formulating hypotheses, changing motion parameters, creating measurement tables, experimenting, formulating conclusions compatible with natural laws).
- To be able to create and study their own programs/models with MATHEMATICA® for other natural systems of interest.

The proposed model aims to study the planetary motions around the central star with the simplest possible theoretical approach at a teaching level, aiming at the understanding of the phenomenon by Senior High School learners and first year University students.

4. Planetary motion

4.1 The theoretical model

The force of gravity causes objects near the surface of the earth to fall, allows the moon to rotate around the earth, and keeps the planets in orbit around the sun. Two objects of mass m_A and mass m_B separated by a distance r receive a force F of magnitude

$$F = G\frac{m_A m_B}{r^2}$$

Where $G = 6{,}67 * 10^{-11} Nt.\frac{m^2}{Kg^2}$ and are fixed (Newton's Law of Universal Gravitation). The motion is easier to visualize with a coordinate system where mass m_A is located at the beginning of the axes. The other mass m_B is located at another point with coordinates (x, y), as shown in the diagram below:

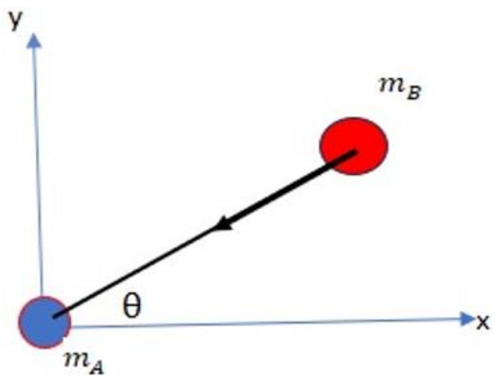

Fig 1: The movement of the earth around the sun

Let us consider that m_A is the mass of the sun and m_B is the mass of the earth. The direction of the gravitational force of the mass of the earth (due to the mass of the sun) is represented by the vector, as shown in the diagram above. The components F_X και F_y of the force of the gravity with respect to the xx' and yy' axes respectively are:

$$F_x = -G\frac{m_A m_B}{r^2}cos\theta$$

$$F_y = -G\frac{m_A m_B}{r^2}sin\theta$$

The distance between the two masses can be written as:

$$r = \sqrt{x^2 + y^2}$$

Where for the trigonometric numbers cosθ and sinθ it applies:

$$cos\theta = \frac{x}{r} \qquad sin\theta = \frac{y}{r}$$

The use of Newton's 2nd Law, in combination with a numerical approach, gives for the components of the acceleration:

$$a_x[t] = \frac{F_x[t]}{m_B} = \frac{Gm_A x[t]}{r^3}$$

$$a_y[t] = \frac{F_y[t]}{m_B} = \frac{Gm_A y[t]}{r^3}$$

From which, the equations for the components of the velocity of the moving object result:

$$v_x[t + \Delta t] = v_x[t] + \frac{F_x[t]}{m_B}\Delta t$$

$$v_y[t + \Delta t] = v_y[t] + \frac{F_y[t]}{m_B}\Delta t$$

These equations are independent of the mass m_B. The quantity Gm_A, for the implementation of the motion simulation, is considered constant and we depict the position of the mass m_B, in relation to time t, through the realized changes of its position vector, $\overrightarrow{r(t)}$, in conjunction with time t, considering and calculating the change in the components of x[t] and y[t].

To simulate the motion of the earth, which we will consider as mass m_B, in relation to the Sun, of mass m_A, we will use the equations that give the components of the position vector, in relation to the two axes, with respect to time t, as follows:

$$x[t + \Delta t] = x[t] + v_x[t + \Delta t]\Delta t$$

$$y[t + \Delta t] = y[t] + v_y[t + \Delta t]\Delta t$$

4.2 The simulation code

Using MATHEMATICA II, or more advanced versions, we can solve tables of values and create various dynamic visualizations. The use of the "Manipulate" command gives us the possibility to create dynamic visualizations and insert interactive controls and dynamically change the parameters of the depiction/visualization. It can also be used to investigate the effect of different inputs on a visual representation, such as adjusting the parameters of a mathematical function whose animation is requested.

The researchers believed that the students, with their knowledge of the theoretical framework and its numerical approach, would be able, using the commands "Manipulate" and "Module", to define the variables, the functions, as well as the initial conditions of the system under study, which will then be used in the calculation of the trajectory of the rotating body of mass m_B. Using the command "Do" students would be able to perform iterative calculations, so that, at each performance step, the components of positions, velocities and accelerations are calculated. At the end, using the capabilities of the "Manipulate" command to create the simulation of the natural system, emphasizing the sequence:

Initial data → assumption-observation → calculations-verification → extraction of results to be able to experiment with it for various changes of the initial conditions by studying the resulting trajectories / forms of movement.

In detail, for the creation of the code, we have an algorithm, consisting of the following steps:

1. **Definition of Initial Conditions and Constants:** In the first part of the "Module", the initial conditions and constants used to calculate the trajectory and the velocity vector are defined. Initially, the start time t_i, the completion time t_f, the number of steps S, the step d, as well as the initial values of position and velocity as well as the acceleration of the movement are defined.
2. **Trajectory Calculation:** We use an iterative loop to calculate the motion trajectory using the equations of motion given by the numerical study. Using the "Do" command, the position, velocity and acceleration of the moving body are calculated for the requested points in the selected time duration.
3. **Creation of the Trajectory and Velocity Vector:** Using the calculated points, the trajectory is created, using the "ListLinePlot" command. Also, the velocity vector is entered, that depicts it at the point defined by the slider.
4. **Display Graphs:** The trajectory points, velocity and gravity vectors are displayed on the graph via the "Show" command.
5. **The "Manipulate" command:** Allows the observation of the evolution of the trajectory in real time.

Then, the program which was implemented is presented, following the steps. The program constitutes a fusion of ideas from the proposal of Varley (1996) [34], in combination with a similar project proposed on the WOLFRAM website by Árpád Kósa (2011) [3].

The specific code is relatively simple and feasible by the students of the Physics or Mathematics Department as well as students of the 3rd grade of Senior High School

(Lyceum) with a particular interest in physics, mathematics and programming as our pilot research shows and is as follows:

4.3 The program

The program is as follows:

```
Manipulate [
Module [{n, r, ax, ay, Vx, Vy, x, y, d1, trajectory, velocity Vector, gravityForce},
ti = 0.;
tf = 3.4;
S = 500.;
d = tf/S;
x[0] = 1.0;
Vx[0] = 0.;
y[0] = 0.0;
Vy[0] = 0.7;
a = 1.0;
Do[r[n] = Sqrt[(x[n]^2) + (y[n]^2)];
ax[n] = -a*x[n]/(r[n]^3);
ay[n] = -a*y[n]/(r[n]^3);
Vx[n + 1] = Vx[n] + ax[n]*d;
Vy[n + 1] = Vy[n] + ay[n]*d;
x[n + 1] = x[n] + Vx[n + 1]*d;
y[n + 1] = y[n] + Vy[n + 1]*d, {n, 0, S}];
d1 = Table[{x[n], y[n]}, {n, 0, S}];
trajectory = Table[{x[u], y[u]}, {u, 0, p}];
velocityVector =
Graphics[{Red,
Arrow[{{x[Round[p]],
y[Round[p]]}, {x[Round[p]] + 0.2 Vx[Round[p]],
y[Round[p]] + 0.2 Vy[Round[p]]}}],
Text[Style["u", Red], {x[Round[p]] + 0.2 Vx[Round[p]]/2,
y[Round[p]] + 0.2 Vy[Round[p]]/2}, {0, -0.5}]}];
gravityForce =
Graphics[{Blue,
Arrow[{{x[Round[p]],
y[Round[p]]}, {x[Round[p]] + 0.2 ax[Round[p]],
y[Round[p]] + 0.2 ay[Round[p]]}}],
Text[Style["F", Blue], {x[Round[p]] + 0.2 ax[Round[p]]/2,
y[Round[p]] + 0.2 ay[Round[p]]/2}, {0, -0.5}]}];
Show[ListPlot[d1, AxesLabel -> {"x", "y"}, AspectRatio -> 1.0,
PlotRange -> {{-1.2, 1.2}, {-1.2, 1.2}}],
ListLinePlot[trajectory, PlotStyle -> Red],
Graphics[{PointSize[0.05], Point[{x[Round[p]], y[Round[p]]}]}],
velocityVector, gravityForce]], {{p, 0, "Time"}, 0, S, 1, Animator,
AnimationRunning -> False, AnimationRate -> 1}]
```

By running the program, for the given initial conditions, we result in that, not only is the trajectory obtained, but rather an interactive working environment, which gives the elliptic trajectory but also allows experimentation for various changes of the parameters and initial conditions of the form of Fig 2:

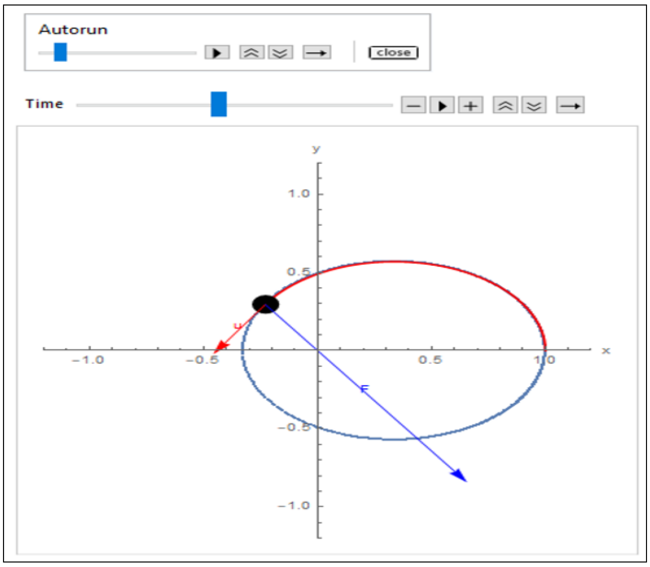

Fig 2: A snapshot of the program performance for the initial values entered in it

This environment was used by the researchers as a teaching intervention tool which was attempted in the context of a pilot study, which we refer to below.

5. The pilot study

5.1 The sample

The sample consisted of a total of 27 male and female students of the Physics department of the University of Thessaly who, in the context of their undergraduate studies, had been taught motion of a material point in a central force field, elements of the mathematical package MATLAB and programming in Python language in previous years of study, without however, knowing the math package MATHEMATICA. The male and female students worked in groups using laptops in the Laboratory of Science Education of the Physics Department of the University of Thessaly.

5.2 The research tools

Semi-structured interview

Beginning of the intervention: During the pilot phase of the research and the beginning of the teaching intervention, the participants answered the questions of a semi-structured interview which consisted of 17 axes, whose purpose was to diagnose their initial ideas about:

- The motion of a planet around the Sun,
- Kepler's Laws,
- The gravitational force that determines the movement,
- The solution of the differential equation of motion and the derivation of the trajectory equation analytically,
- The finding of the period of motion of the planet,

At the same time, the axes of the interview aimed to investigate:

The possibility of finding a simple solution to the problem numerically, the creation of an algorithm and the implementation of a relevant program to solve the problem and the type of computing tools (programming language, MATHEMATICA, MATLAB, other computing tools) they would like to use to implement the algorithm.

The interview also aimed to diagnose the sources which the participants would use to look for implemented corresponding programs (books, internet search engines, AI programs such as Chat GPT, or other tools) and in case they decided to use artificial intelligence tools, how they would use them (finding a program, understanding it, help in creating it, etc.). Additionally, participants were encouraged to experiment with the programs they would have chosen by changing parameters and initial values of positions and speeds.

Finally, the diagnosis of their ability to formulate conclusions, regarding the issues that follow, was regarded as particularly important in this research effort. These issues were:

- The theoretical model they chose,
- Its numerical processing,
- The corresponding program,
- The tools that had been used in it and
- The results from its use.

After the completion of the intervention: After the completion of the intervention the students participated in a semi-structured interview with the same axes/areas of discussion. The only difference was that now they had created their own program and therefore their conclusions referred to it.

Usability questionnaire / System Usability Scale (SUS)

The System Usability Scale (SUS) is a computational tool which is used to assess the usability of various systems and applications. According to literature, it is characterized of simplicity, it is quick in its operation and easy for users to handle. Apart from the aforementioned benefits, SUS is also characterized by its possibility to trace problems that occur between users and system. The specific tool involves a set of ten questions the answers of which are given on a Likert Scale with replies that range between strongly agree to strongly disagree. In the case of this research, students will be considered as 'users' and the whole process with the program simulating movement, with which learners can experiment, will be considered as 'system' and they competed also the SUS questionnaire after the completion of the intervention

The didactic intervention

A group-based intervention with inquiry elements followed, in which students worked in teams. Each learner worked with a laptop with MATHEMATICA 14 installed in them, for which the University of Thessaly is a legal user. The purpose of the intervention was to create an environment, to simulate the movement of a planet around the Sun, and to experiment with its use.

The teaching intervention followed the following phases:

1. In the first phase, lasting 3 hours, the participants had the opportunity to familiarize themselves with MATHEMATICA and the commands they would use in order to implement a program for the movement of a planet around the Sun, simulating the specific movement with the help of the researchers.
2. In the second phase, lasting 2 hours, they should produce the equations of motion of a planet around the sun with the help of the knowledge they had acquired from the teaching of the Theoretical Mechanics course and the unit of motion in a central force field.
3. In the third phase, lasting 2 hours, the task was to create a simple algorithm and with the use of MATHEMATICA to implement a program to move a planet around the Sun by simulating its movement.
4. In the fourth phase, lasting 2 hours, the focus was on the students' experimentation with the program. The students experimented by varying both the initial conditions and the parameters of the problem, creating closed and open trajectories and seeing whether their predictions were verified or disproved in practice.

The researchers recorded their observations and comments during the implementation of the phases. Upon completion, the usability questionnaire was filled in and the participants were re-interviewed with semi-structured questions that revolved around the same axes.

The students worked with the help of the researchers designing and implementing the program and then experimented with it.

The results

The following observations emerged from the initial participant interviews:

Regarding the planetary motion around the star (Sun), the majority of students choose the elliptical orbit (13/27 of the learners elliptical, 8/27 circular, 6/27 don't know).

Concerning Kepler's laws, 12/27 learners state that they know them, while 6/27 of the students know that they derive from Newton's law for the gravitational pull of the Sun - Planet.

It is noteworthy that, although they have been taught movement in a central field of forces and intuitively perceive the connection of an attractive force of central nature with elliptical or circular trajectories, they hesitate to declare it. When asked to formulate the equation of the trajectory of a particle moving under a central force in the general case, very few students (6/27) formulate the equation in its final form, while, when asked for the trajectory form for the case of the inverse square attractive force, they do not attempt to give an answer related to the previous question but they return to the answers they originally gave for elliptical (13/27) or circular orbits (8/27).

In the question for the determination of the period of the planet, although it was expected that, those who proposed at least circular orbits would be able to calculate the period of rotation, this did not happen, creating particular concern for the researchers. It is worth mentioning the case of a student who stated that, observing the movement of the planet with a telescope is required.

The students were taught introductory numerical analysis, programming in C and Python, and had experience in the computational mathematics package MATLAB. When they were asked to give an algorithm for solving the problem where numerical analysis is used and proposing a program that corresponds to it, they did not manage to formulate an algorithm and/or a program and a had quite a difficulty to make the connection between the knowledge of the corresponding courses with the specific application that was requested.

In the next question, the researchers, despite the fact that the students did not appear to be ready to formulate a program, tried to understand what possible tools they would use to design and implement such a program. The participants responded that they would use programming languages (8/27 learners), MATLAB (14/27 students) while, when prompted to use one of the tools they suggested in order to write a program to solve the differential equation of the problem, none of the participants responded.

In the next question about the sources to search for a ready-made program to use, 8/27 students suggested Physics books, 18/27 online sources, while 3/27 learners suggested that the teacher should suggest it.

When asked if they would use Artificial Intelligence tools and how, 12/27 of the participants answered that they would use them to understand commands and programs, while 12/27 of them answered that they would ask AI to create the appropriate program for them.

Finally, they were particularly reluctant to come to conclusions about how they would work to design and implement such a program.

A different picture now emerges from the interviews of the participants after the completion of the teaching intervention and the creation of the program. All 27 participants state that the trajectories are an ellipse. A number of 24/27 of them stated that Kepler's three empirical laws apply to the specific motion of the Planet around the Sun, and only 3/27 of the learners still do not know them. Another 24/27 students argued that the three laws derive from and are explained with the help of Newton's laws, while 3/27 of them stated that they do not know from which law these derive. Finally, 26/27 participants associated the planet's elliptical orbit with an inverse-square gravitational force of the form F(r) exerted by the Sun on it, while one failed.

Additionally, starting from Newton's second Law, with the form of the force for gravitational attraction being the one proposed by Newton, they arrived at the equation for the trajectory. Although they have been taught this in theory, they failed to calculate the period of the planet's motion when the orbit was elliptical, while asking them to consider it circular and determine the period 6/27 of them succeeded.

Based on the results, 27/27 learners knew, in theory, how to solve a second order differential equation numerically but the algorithm was created with a lot of help from the researchers starting from Newton second law and using the numerical solution method proposed by Varley, which leads to a particularly simple and working algorithm for the problem. Having learned the basic commands of MATHEMATICA for creating a program, when asked what they would use to implement a program, 24/26 learners chose MATHEMATICA, while 3/27 of them chose the Python programming language.

To the question about where they would look for such a program with the experience they had now acquired, they answered (having the possibility of multiple choice): In Engineering books 18/27 learners, on the internet 18/27, in ChatGPT 6/27 students.

Regarding how to use AI tools, the answers (of multiple-choice type) were the following: 18/27 learners to find programs, 12/27 to understand them, 9/27 to find a ready-made program 15/17 to have help in creating their own program and end 6/27 of them for anything else. The participants worked with the created program by changing parameters and initial conditions of position and speed, initially creating trajectories similar to the original lack of the program and then continued their experimentation by producing closed and open trajectories, obtaining what they described as an unprecedented and pleasant experience.

The students described the theoretical model used as comprehensible. They found the numerical solution easy, of moderate difficulty to create a program with MATHEMATICA to calculate the trajectory, without additional elements (velocity and force vectors, etc.). However, they found the creation of the complete program more difficult and quite demanding in relation to time needed to do it. Having already experienced MATLAB, they considered MATHEMATICA to be a particularly functional and very powerful tool, but ChatGPT was equally powerful, from which they obtained information about the commands of MATHEMATICA, numerical solutions of differential equations and finally from WOLFRAM Demonstrations Project/Physics ideas for the creation of program.

5.3 In relation to the usability of the teaching environment

In order to assess the ease of use of the teaching environment, the System Usability Scale (SUS) questionnaire was administered to the students. The Cronbach coefficient of the questionnaire is 0,810. The analysis of the questionnaire data, regarding the usability of the teaching environment, is depicted on Fig 3. After calculating the SUS score, by using the formula SUS score = (X+Y) *2.5, where X equals the Sum of the points for all odd numbered minus 5 (X=Sum (odd number question points)-5) and Y equals 25 minus the sum of the points for all even-numbered questions (Y=25 - Sum (even number question points)) there was an acceptability score of 73,61 (with 100 the highest score possible). According to the Usability Scale the teaching environment can be characterized as close to Excellent.

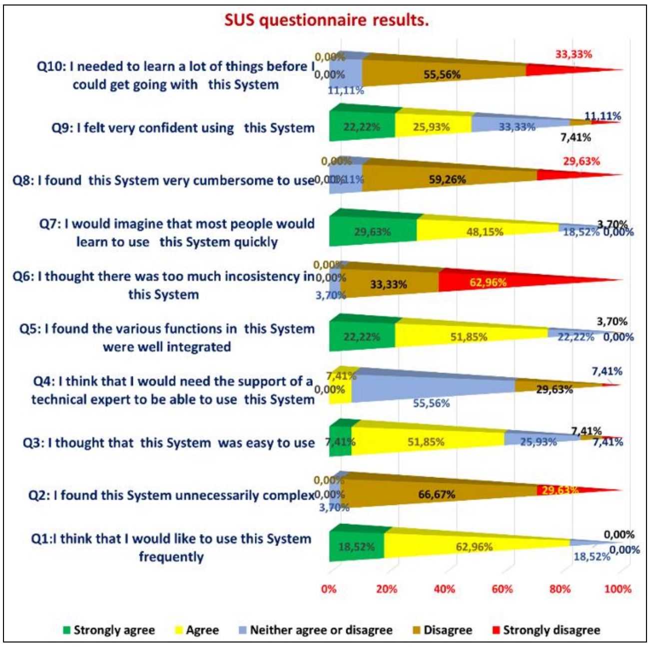

Fig 3: SUS questionnaire results

6. Conclusions

The aim of the research was to utilize the possibilities of MATHEMATICA in order to create an interactive environment of active learning of an investigative type, with the help of which students could experiment. The focus was on the implementation of a program, the testing and the connection of the knowledge they had acquired from Physics, Mathematics and Programming (with an emphasis on the use of mathematical packages) for the study of the problem of planetary motion and in general body motion in a central force field.

A particularly simple model and method was chosen for this pilot research. It should be noted that in future main research the focus will be the possibility of extending the Project in order to determine whether the students of the Second year will use equations of motion under a central attractive force, producing the trajectory equation of the moving object (Marion & Thornton, 1995, Acheson, 1998, Fishbane *et al*., 2004) [18, 1, 9] and then to establish their ability to use methods of numerical analysis to approach the problem (Kumar *et al*., 2014, Chen, 2019, Kalantzis *et al*., 2021, Sari *et al*., 2021, Sharma *et al*., 2023, Putranta *et al*., 2023) [15, 6, 14, 26, 27, 24].

The pilot study revealed that, after 5 hours of teaching-interaction, the students were able to create similar codes to the one used as a model by the researchers and then experiment with them about what can happen in the simulated system, by actively interacting among them and working in a collaborative environment.

A special feature of their effort was the use of ChatGPT in order to find similar codes, but also to request information about the commands used, in order to understand their meaning, in addition to the teaching they had received from the research team, but also then to use MATHEMATICA to check if the programs they found were "running". It should also be noted that in the ones they chose, they experimented with changing parameters and initial values and most of the time they seemed to be "surprised" by the results. It seems that such an effort, belonging to the area of Computational Physics, makes the participants understand more deeply the importance of what they were theoretically taught. This is because the simulation gives the feeling of observing the planetary motion during its evolution and can potentially confirm or disprove ideas that arise when manipulating the mathematical/theoretical model of planetary motion in terms of changing parameters or initial conditions.

Because at this stage the research is pilot, we will not further expand on issues related to the in-depth understanding and building of knowledge about the concepts of the physical quantities involved and their interdependence by the students. The main research is expected to follow but we believe that the pilot results provide indications that the project is expected to encourage the active participation of students in the exploratory learning process and contribute to the essential understanding of the studied natural system. Concluding, the challenge to simulate the movement of a material point around a center of gravity seems to provide students with a high degree of motivation, encouraging them to develop skills such as, the development of critical-analytical thinking, problem solving and the development of the interdisciplinary connection of cognitive fields of the subject they study and finally creativity.

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