Int. j. adv. multidisc. res. stud. 2023; 3(6):949-954

Received: 19-10-2023
Accepted: 29-11-2023

International Journal of Advanced Multidisciplinary Research and Studies

ISSN: 2583-049X

Characteristics of stochastic soliton structures for the nonlinear Schrodinger equation

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Abstract
This article discusses the travelling wave solutions for the nonlinear Schrodinger equation (NLSE) through Itô sense in optical fiber. He's semi-inverse approach is used to generate some innovative travelling wave solutions. The Ritz approach is used to obtain these solutions. We also show the effect of the multiplicative noise on the solutions. Additionally, some graphs are shown to illustrate the dynamical behaviour of solutions using the Matlab packet programme. The He's semi-inverse method actually shows accuracy and efficiency in solving a variety of nonlinear systems arising in applied sciences.

Keywords: Schrodinger Equation, He's Variational Principle, Brownian Process, Solitary Wave Solutions, Optical Fiber

1. Introduction

Comprehending the dynamic wave patterns associated with nonlinear partial differential equations (NPDEs) is imperative in comprehending the fundamental mechanisms of intricate phenomena [1–3]. In actuality, the NPDEs have been the subject of the most in-depth study across a variety of applied science domains [4, 5]. Scientists have recently focused their attention on nonlinear stochastic partial differential equations (NSPDEs) [6–8]. Numerous nonlinear stochastic scientific phenomena that have applications in various fields were produced by the waves, such as fluid mechanics, biomathematics, quantum mechanics and many others [9, 10]. Particles moving stochastically in random potentials are essential to many processes. The influence of randomness on the dispersion of soliton solutions has been given more and more attention lately. This effect is critical in describing many complex phenomena.

Stochastic calculus is a very vital branch of mathematics that studies stochastic processes and allows for the simulation and modelling of random systems [11, 12]. The Brownian motion is a classic stochastic process that is a martingale and a Markov process [13]. Brownian motion is a common stochastic process in dispersive systems. The physical mechanism of asset pricing is utilised by the Brownian motion process. Indeed, stochastic processes and partial differential equations are inextricably linked. Numerous processes rely on particles moving stochastically in random potentials. A stochastic differential equation, which, despite its name, is essentially an integral equation, is used to describe this process [12, 14].

The nonlinear Schrodinger equation (NLSE) essentially describes the dynamics of optical soliton propagation in nano-fibers, microelectronics, bimolecular dynamical modes, superfluid, coastal water motions and many others. A dynamical balance between the wave's linear dispersive spreading and nonlinear self-interaction is included in this equation. Because of the possible applications of NLSE, soliton solutions have been studied from several viewpoints [15–17]. Moreover, a wide range of deterministic and stochastic NLSE frameworks can be used to describe the numerous nonlinear wave events in applied research [18–21]. We take into account the NLSE induced by multiplicative noise in Itô sense, which given as follows [22]:

$$iU_t + U_{xx} + \beta \,|\, U \,|^2 \, U + \sigma U \, \Pi_t = 0, \qquad (1.1)$$

$\beta \in \mathbb{R}$- {0} is the nonlinear coefficient, σ is the noise strength. The term U_{xx} represents the dissipation term & $|U|^2 U$ represents the nonlinearity term. The noise Π_t is the time derivative of the Brownian motion Π(t). Here, we implement He's variations technique [23–25] to introduce vital stochastic solution. We also show the noise influence on solitary propagations for this equation.

The rest of the article is structured as follows. Sec. 2 introduces the description of He's variational principle. Sec. 3 offers some new stochastic solutions for NLSE induced by multiplicative noise in Itô sense. Sec. 4 discuss the results obtained and introduced some graphs to illustrate the behaviour of stochastic solution. Sec. 5 demonstrates how the nonlinear parameter affects how solutions behave. Finally, in Sec. 6, final remarks are reported.

2. Description of the method

Consider the NPDEs:

$$G(U, U_t, U_x, U_{tt}, U_{xx}, ...) = 0, \tag{2.1}$$

G is a polynomial in $U(x,t)$ and its partial derivatives. Using the wave transformation

$$U(x,t) = u(\zeta), \qquad \zeta = x - vt, \tag{2.2}$$

Converts Eq.(2.1) into the following ODE:

$$H(u, u', u'', u''', ...) = 0, \tag{2.3}$$

H is a polynomial in u(ζ) and its total derivatives, while. According to He's semi-inverse technique [23–25], integrate equation (2.3) term by term, gives constant(s) of integration that can be chosen zero for simplicity. We develop the following trial-function using He's semi-inverse approach.

$$J(u) = \int L \, d\zeta, \tag{2.4}$$

L dependent on u and its derivatives, is the Lagrangian function of the problem as given by Eq. (2.3).

We can find various sorts of solitary wave solutions using the Ritz approach, including $u(\zeta) = A\, sech(B\,\zeta)$, $u(\zeta) = A\, sech(B\zeta)$, $u(\zeta) = A\, tanh(B\,\zeta)$ and $u(\zeta) = A\, coth(B\,\zeta)$. In this article we search about the solutions in the form:

$$u(\zeta) = A\, sech(B\zeta), \tag{2.5}$$

A and *B* are constants to be determined. Substituting from equation (2.5) into equation (2.4) and making *J* stationary with respect to *A* and *B* give

$$\frac{\partial J}{\partial A} = 0, \tag{2.6}$$

$$\frac{\partial J}{\partial B} = 0. \tag{2.7}$$

Solving simultaneously the equations (2.6) and (2.7) we obtain the values of *A* and *B*. Consequently, the solitary wave solution given by equation (2.5) will be determined.

3. The stochastic solutions

Using the transformation [22]:

$$U(x,t) = u(\zeta)\, e^{i(kx - ct + \sigma \Pi(t))}, \qquad \zeta = x - vt, \tag{3.1}$$

k, *c*, *v* are constants, gives

$$U'' + \beta\, u^3 - \left(c - k^2\right) u = 0, \tag{3.2}$$

from real part and v = 2k from imaginary part. Using $c - k^2 = \lambda$, then Eq (3.2) becomes

$$U'' + \beta\, u^3 - \lambda\, u = 0. \tag{3.3}$$

According to the He's semi-inverse method stated in [23–25], one constructs the following variational formulation from Eq. (3.3) as:

$$J(u) = \int_0^\infty \{\frac{1}{2}(u')^2 - \frac{\beta}{2}u^4 + \frac{\lambda}{2}u^2\} d\zeta. \tag{3.4}$$

We use the Ritz approach to look for a solitary wave solution in the form

$$u(\zeta) = A \sec h(B\,\zeta), \tag{3.5}$$

Where *A*, *B* are an unknown constant. Substituting Eq. (3.5) into Eq. (3.4), gives

$$\begin{aligned} J &= \int_0^{\infty} \left[\frac{1}{2}A^2B^2\, sech^2(\zeta)\tanh^2(\zeta) - \frac{1}{4}\alpha A^4\, sech^4(\zeta) + \frac{\lambda}{2}\, sech^2(\zeta)\right] d\zeta \\ &= \frac{A^2B}{6} - \frac{\beta A^4}{6B} + \frac{\lambda\, A^2}{2B}. \end{aligned} \tag{3.6}$$

Differentiating *J* with respect to *A*, *B* and putting $\frac{\partial J}{\partial A} = 0$ and $\frac{\partial J}{\partial B} = 0$
yields

$$\frac{\partial J}{\partial A} = \frac{AB}{3} - \frac{2\beta A^3}{3B} + \frac{\lambda\, A}{B}. \tag{3.7}$$

$$\frac{\partial J}{\partial B} = \frac{A^2}{6} + \frac{\beta A^4}{6B^2} - \frac{\lambda\, A^2}{2B^2}. \tag{3.8}$$

Solving these equations gives:

$$A = \left(\frac{2\lambda}{\beta}\right)^{\frac{1}{2}}, \qquad B = \sqrt{\lambda}. \tag{3.9}$$

Hence the solutions (3.5) takes the form

$$u(\zeta) = \left(\frac{2\lambda}{\beta}\right)^{\frac{1}{2}} sech(\sqrt{\lambda}\zeta). \tag{3.10}$$

Hence the stochastic solution of (1.1) is

$$U(x,t) = \left(\frac{2\lambda}{\beta}\right)^{\frac{1}{2}} e^{i(kx-ct+\sigma\Pi(t))} sech(\sqrt{\lambda}(x-vt)) \tag{3.11}$$

4. Results and Discussion

Many intriguing complicated phenomena are explained by the solitary wave for the NLSE via the Brownian motion process. These phenomena are of great importance in optical fiber communications, deep water, plasma physics, quantum mechanics, superfluid, condensed matter physics and many others. The Brownian process is a highly successfull strategy for dealing with a wide range of real-world random events. Brownian motion is a fundamental building block of stochastic calculus and the key to modelling stochastic systems. The stochastic NLSE equation is transform to nonlinear ordinary differential equations through the $\Pi(t)$ function. Specifically, we use the Brownian motion approach to analyse the NLSE model.

Most standard articles examined the proposed NLSE model in the deterministic scenario. As opposed to our approach, we study this equation in the stochastic scenario, that is, when they are induced by multiplicative noise through Itô sense. We have been applying the He’s semi-inverse approach to the NLSE model with multiplicative noise in the Itô sense in order to obtain vital hyperbolic secant stochastic solutions. According to Weisstein [26], this sort of solution secant solution occurs in the profile of a laminar jet. The He’s semi-inverse approach was used to find some innovative and concise random solutions for the NLSE model with multiplicative random parameter. This method’s main advantages over others are that it can solve a wider range of physical models and eliminates expensive and time-consuming computations.

Because of its crucial uses, the impact of a noise parameter on the propagation of soliton solutions has received more attention in recent decades. In this sense, the stochastic solutions are important for understanding how NLSE waves propagate when they emerge in different physical perspective areas. We also drawn some corresponding profile pictures in order to show the dynamical behaviour of these solutions. Fig 1 depicts the dynamical behaviour of solution (3.11) in the deterministic ($\sigma = 0$) and stochastic ($\sigma = 2$) cases. This graphic shows that the efficiency of randomization and the capacity to achieve rapid wave collapse increase with increasing noise term σ. The impact of the intense randomness coefficient on phase shift, band width, structure and amplitude is illustrated in Figs. 2, 3. Fig 4 illustrates the envelope waves for the presented solution.

5.The influence of β

One of the primary objectives of this article is to illustrate the impact of β on the properties of the wave modes. The wave images of solution (3.11) for a range of β values are displayed in Fig 5. It is discovered that increasing β decreases the amplitude of the optical solution (3.11) with no change in direction or space. Moreover, there is no variation or reversal in the amplitude.

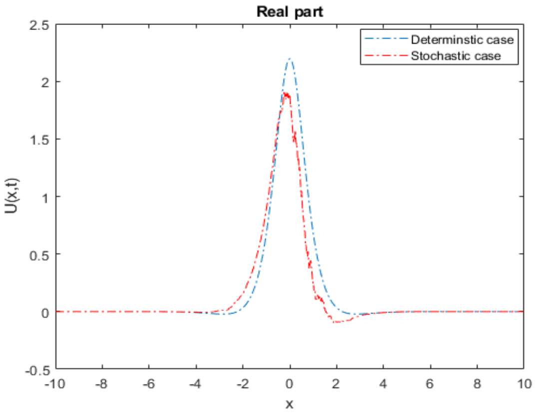

Fig 1: Effect of stochastic refinement on solitary wave solution (3.11)

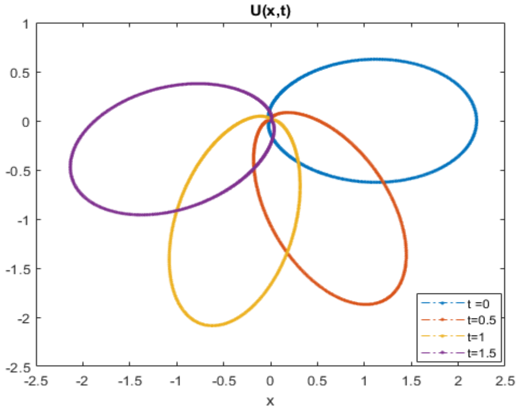

Fig 2: Trajectory of solution (3.11) with different values of t

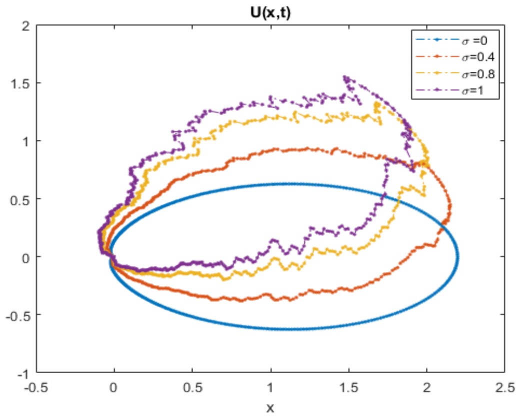

Fig 3: Trajectory of solution (3.11) with different values of σ

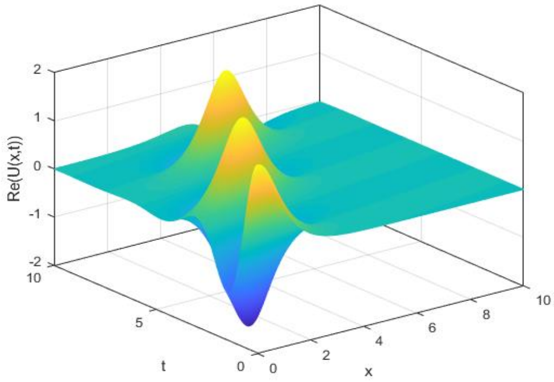

Fig 4: 3D plot for solution (3.11)

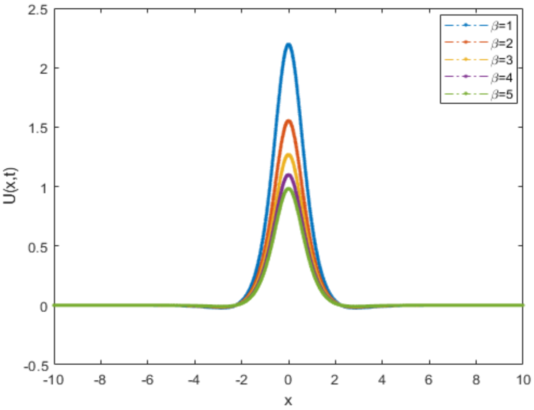

Fig 5: Variations of solution (3.11) with β = 1, 2, 3, 4, 5

6. Conclusions

We have investigated the nonlinear Schrodinger's equation induced by multiplicative noise through Itô sense, utilizing He's semi-inverse technique. We employ this approach to generate some novel travelling wave solutions. The key merits of this technique over others are that it averts time-consuming and expensive computations and has a wider range of applications for solving diverse sciences difficulties. We depict the effect of the multiplicative noise on dynamical behaviour of the solutions. We also depicts effect of the nonlinear coefficient on the behaviour of the solution. Finally, the He's semi-inverse approach is applicable to other complex models, therefore it will be used in future studies.

7. References

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