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Method for Determining the Objective Function in the Problem of Optimal Control of Robot Manipulator

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Abstract

An industrial robot is a device that controls multiple axes simultaneously. The robot kinematics problem is studied in two main aspects: kinematic synthesis and kinematic analysis. The solution of the inverse kinematics problem is one of the important information to control the robot, in which attention need to paid to the speed of forming the

solution and the accuracy of the solution to the inverse dynamic problem. These factors determine control quality as well as real-time control capabilities. In this paper, a method to determine the objective function Q in the optimal control problem of a 3-link planar robot manipulator.

Keywords: Optimal Control, Optimization Method, Robot Manipulator

1. Introduction

Optimal control [1-9] is a basic specialty in automatic control, it has the role of determining and creating control laws for the system in order to achieve the defined objectives in the form of the objective function Q . In solving the inverse kinematics problems, it is necessary to have a common algorithm for different types of robots, the purpose of which is to apply computers in preparing robot control data. Furthermore, the algorithm must be finite and have a short processing time to meet real-time control requirements.

The inverse kinematics problem is paid much attention because its solution is the basis for building a program to control the robot motion tracking a given trajectory. The task of the working part is set in the working space, while the control action is placed on the joint, so the joint variable is the direct control object. Therefore, the inverse kinematics problem must always be solved after a finite number of iterations. If the problem does not converge, a warning must be given. When applying optimization theory in robot control in the worst case, the optimization algorithm still ends with a clear conclusion after a finite, predictable period of time.

2. The Optimal Problem of the Accuracy of Position and Direction of the Actuator

The goal of kinematic control is to achieve the accuracy of position and direction of the actuator. Thus, it is only necessary to determine the values of the joint variables to ensure the smallest position and direction errors while satisfying the structural constraints.

Goì $q = \{q_1, q_2, \dots, q_n\}$: is a vector of joint variables.

$Q = f(q)$: Descriptive function of the position and direction error of the actuator.

The problem of determining the value of joint variables is written as follows:

$$Q = f(q_1, q_2, \dots, q_n) \rightarrow \min \tag{1}$$

Where:

$$q_i \in D; \quad i = 1 \div n \tag{2}$$

This is an optimal problem. The solution of (2) is also the solution of the equations describing the relationship of the joint variables.

$$\begin{cases} s_x - a_{12} = 0 \\ a_x - a_{13} = 0 \\ a_y - a_{23} = 0 \\ p_x - a_{14} = 0 \\ p_y - a_{24} = 0 \\ p_z - a_{34} = 0 \end{cases} \tag{3}$$

Square both sides of this system of equations and add each side to get:

$$(s_x - a_{12})^2 + (a_x - a_{13})^2 + (a_y - a_{23})^2 + (p_x - a_{14})^2 + (p_y - a_{24})^2 + (p_z - a_{34})^2 = 0$$

Obviously, the left side is not negative, so the smallest value of the left side is zero, equivalent to (3) being satisfied. Let Q be the function on the left side:

$$Q = (s_x - a_{12})^2 + (a_x - a_{13})^2 + (a_y - a_{23})^2 + (p_x - a_{14})^2 + (p_y - a_{24})^2 + (p_z - a_{34})^2 \tag{4}$$

This form of function has its own name, the Rosenbrock-Banana function [10], therefore a suitable algorithm must be determined to solve the problem. The problem of optimal control of the robot manipulator is determining the shortest time for the robot manipulator to move to the required position by finding the optimal solution of the objective function that make the objective function (4) minimum such that $Q \rightarrow \text{Min}$

3. Determine the Objective Function used in Control for a 3-Degree-of-Freedom Robot

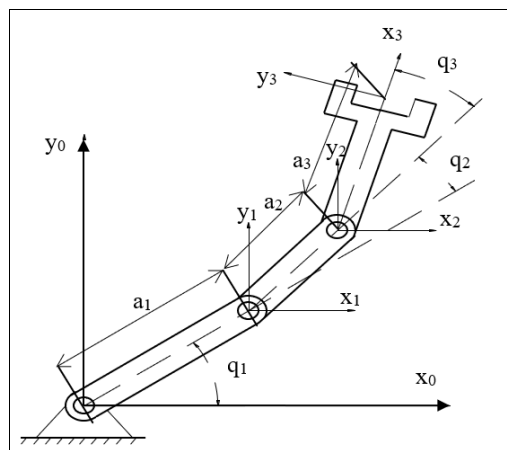


Fig 1: Kinematic diagram of 3-link planar mechanism (3 rotating joints)

Kinematic equations:

$$\begin{pmatrix} p_x \\ p_y \\ p_z \\ 1 \end{pmatrix} = {}^0A_3 = \begin{pmatrix} a_1 \cos(q_1) + a_2 \cos(q_1 + q_2) + a_3 \cos(q_1 + q_2 + q_3) \\ a_1 \sin(q_1) + a_2 \sin(q_1 + q_2) + a_3 \sin(q_1 + q_2 + q_3) \\ 0 \\ 1 \end{pmatrix} \tag{5}$$

Orientation of the clamp:

$$\begin{vmatrix} n_x & s_x & a_x \\ n_y & s_y & a_y \\ n_z & s_z & a_z \end{vmatrix} = \begin{vmatrix} \cos(q_1 + q_2 + q_3) & -\sin(q_1 + q_2 + q_3) & 0 \\ \sin(q_1 + q_2 + q_3) & \cos(q_1 + q_2 + q_3) & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad (6)$$

Because of the planar mechanism, it is capable of satisfying simultaneous positioning and orientating in the plane with coordinates $z = const$

Suppose the orientation description of the clamp axis is selected through the parameter:

$$s_y = \cos \text{in}(y_3; y_0) = a_{22} \quad (7)$$

So, the general form of the objective function is:

$$Q = (s_y - a_{22})^2 + (p_x - a_{14})^2 + (p_y - a_{24})^2 \rightarrow \text{Min} \quad (8)$$

The objective function for a 3-link planar robot (3 rotating joints) has the form:

$$Q = (\cos(q_1 + q_2 + q_3) - a_{22})^2 + (a_1 \cos q_1 + a_2 \cos(q_1 + q_2) + a_3 \cos(q_1 + q_2 + q_3) - a_{14})^2 \\ + (a_1 \sin q_1 + a_2 \sin(q_1 + q_2) + a_3 \sin(q_1 + q_2 + q_3) - a_{24})^2 \rightarrow \text{Min} \quad (9)$$

Constraint Condition

The conditions for selecting the solution according to the operating limit of the joint variable are determined as follows:

$$-3,14(\text{rad}) \leq q_i \leq 3,14(\text{rad}) \quad \text{with } i = 1 \rightarrow 3 \quad (10)$$

4. Conclusion

The goal of kinematic control is to achieve the accuracy of position and direction of the actuator. Thus, it is only necessary to determine the values of the joint variables to ensure the smallest position and direction errors while satisfying the structural constraints. The problem of optimal control of the robot manipulator is determining the shortest time for the robot manipulator to move to the required position by finding the optimal solution of the objective function that make the objective function minimum.

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6. References

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