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A New Method to Stabilize and Swing up of Inverted Pendulum by Utilizing Different Approaches

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Abstract
This paper introduces a simplified methodology for tackling the control problem that arises in the context of the rotating inverted pendulum. The rotating inverted pendulum (RIP) is a complex system that has attracted considerable interest in several academic disciplines due to its complicated structure and nonlinear properties. The control approach may be decomposed into two distinct subproblems. The first subproblem pertains to the management of the pendulum's swing-up, which is achieved by the use of a PD cascade approach. The second subproblem concerns the implementation of a fuzzy-PD regulator for the purpose of balancing and stabilizing the whole system. The transition between these subproblems is also considered. The simulation and analysis of the plant, swing-up control, and stabilizing system are conducted inside the computational framework of Matlab/Simulink.

Keywords: Rotating Inverted Pendulum, Fuzzy-PD, Nonlinear Properties, PD Cascade Approach

1. Introduction

The inverted pendulum, regardless of its rotating nature, has been extensively examined in several domains including both linear and nonlinear control. The study examines a difficult underactuated mechanical system known as the rotating inverted pendulum. The study of this problem in the field of control theory involves looking at it from a number of different angles, such as the analysis of multivariable systems, non-minimum phase processes or unstable systems, complex non-linear dynamics, modeling uncertainty, and resiliency, to name a few [1, 2]. Applications of the RIP, or recursive identification and parameter estimation, are not limited to the aforementioned fields; position control, robotics, and aerospace vehicle control are just a few more. In order to stabilize systems and implement swing-up control, several different control algorithms have been proposed. In scientific literature, the two subproblems are often addressed independently via the use of traditional PID controllers, resilient and adaptable techniques, or intelligent systems. Several writers integrate disparate methodologies into a hybrid control system.

In this study, a traditional proportional-derivative (PD) algorithm is proposed for the control of the rotating inverted pendulum during the swing-up phase. Furthermore, enhancements are made to optimize the algorithm's performance by reducing the swing-up time. The inclusion of an impulsive control action in the PD algorithm leads to a significant reduction in swing-up time, as shown in the aforementioned articles. Furthermore, the researchers used robust and nonlinear techniques in order to achieve the swing-up of the robotic inverted pendulum (RIP). Another approach, referred to as energy-based control, fails to account for the response torques exerted by the pendulum on the arm [3, 4]. Additionally, a comprehensive study is performed to examine the stability characteristics of the energy-based control approach. The use of intelligent methodologies in the domain of pendulum dynamics has shown the inherent advantages of attaining an upright stance [5, 6].

One often used approach for addressing the stability issue of an inverted pendulum involves simplifying the system by linearizing it around the desired equilibrium point [7]. The use of the linearized model may be employed in combination with either the pole placement strategy or the linear quadratic regulator (LQR) to attain stability. Considering the intrinsic attributes of the botanical organism, including the presence of uncertainties and nonlinearities, it might be more beneficial to investigate the use of modern systems. One instance illustrating this concept is the development of a fuzzy controller, which incorporates five distinct membership functions for both input variables: the angle of the pendulum in relation to its upright position, as well as the angular velocity of the pendulum. The present study introduces a unique version of the fuzzy controller, which integrates a straightforward fuzzy logic controller (FLC) with a control rule table that has the skew-symmetric quality [8, 9, 10]. This suggested modification maintains the equivalent performance levels as the previous version. This paper presents evolutionary methodologies for the design of a rotating inverted pendulum controller. Genetic algorithms (GA), particle swarm optimization

(PSO), and ant colony optimization (ACO) represent a subset of the approaches examined within this particular framework.

The discourse also encompasses the examination of the switching control approach. The use of energy is regarded as the appropriate condition for transitioning from the swing up mode to the stabilization phase [11].

This paper presents a compelling proposition that offers a comprehensive solution to address the dual challenges of control in a genuine inverted pendulum system via the use of a single fuzzy rule foundation. By combining two different fuzzy controllers into a single, the authors were able to effectively minimize the number of tuning parameters needed. Altering the mode of operation from swing up to stability is possible by altering the functional signals and modifying the gains.

The primary aims of this work are to identify an accurate model for the RIP (Resonant Inductive Power transfer) and to develop, simulate, and analyze the control mechanisms associated with the RIP. The control approach involves the use of a swinging-up strategy to elevate the inverted pendulum and subsequently achieve stability in the upright position. In the first phase, the primary objective of the swing-up control issue is to enable the pendulum to transition from a downward orientation to an upright configuration. The process of providing the DC motor, which is responsible for propelling the mechanical system, with an appropriate voltage of sufficient magnitude and polarity is how this is accomplished. During the second stage, the balance/stabilizing control system is introduced when the pendulum reaches an angular displacement of 20° from its final vertical position. Despite the presence of several prospective resolutions, it is crucial to tackle these tasks in a fundamental manner to provide a basis for eventual practical implementation [12, 13].

The ensuing portion of the work is organized in the following fashion. This paper presents a comprehensive explanation of the mathematical model of the RIP system and its related open loop responses in Section II. In this study, Section III provides a comprehensive exposition of the swing-up control technique used for the spinning inverted pendulum. The control strategy used in this technique involves the implementation of a cascade control system, which relies on conventional proportional-derivative (PD) regulators. This section also covers the balance/stabilizing control system that employs a fuzzy-PD methodology. The results of the simulation are outlined in Section V, while the research finishes with a comprehensive analysis of the ramifications [14].

2. The Rotating Inverted Pendulum Model

Inverted Pendulum's Rotation Explained

The Furuta pendulum, sometimes referred to as the inverted rotating pendulum, is equipped with a servo motor system which enables its rotational movement. This system is responsible for driving a separate output gear, which operates independently. The experimental setup consists of a rotating pendulum arm, as seen in Fig 1(a), with a radius denoted as r and *a* mass denoted as m_r. This arm is attached to the gear's output, and its pivot point, *P*, is connected to a pendulum of length 2l and mass *m*. The servo motor is responsible for enabling the pivot arm to rotate within the horizontal plane XY. The pendulum is deliberately manipulated to undergo oscillations inside an XZ plane, which is consistently maintained in a perpendicular alignment with the rotating arm. The primary goal is to successfully sustain the pendulum in a vertical position.

The inverted pendulum is shifted from its upright position in the simplified schematic design seen in Fig 1(a). Displacement is denoted by α, while the angle θ of the pivoting arm is indicated by. In addition, the angular velocities of the inverted pendulum and the pivot arm are represented as "α dot" and "θ dot," correspondingly. The dynamics of the inverted pendulum system are characterized by the parameters "α" and "θ", which are generalized coordinates.

The physical rotating inverted pendulum system, as seen in Fig 2(b), is constructed using aluminum material with certain dimensions and mass properties. The length (l) of the pendulum is 0.14 units, while the radius (*r*) measures 0.115 units. The mass (*m*) of the pendulum is 0.08 units, and the combined mass of the pendulum and the rotational axis (m_r) is 0.35 units.

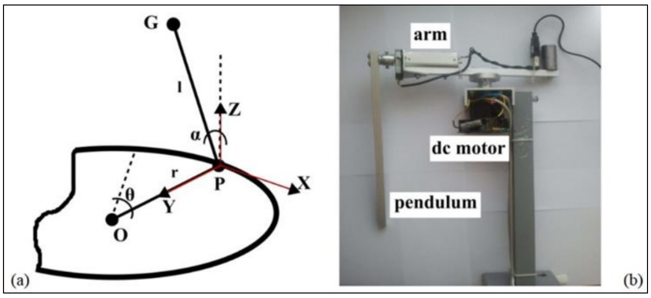

Fig 1: Two representations of the RIP system: (a) a schematic showing the setup (pendulum in an upright position); and (b) a physical depiction of the system (pendulum in a downward position). The user supplies screenshots

The RIP as a Dynamic Model

The dynamic model of the inverted pendulum is derived by decomposing the plant into two distinct planes. In the first instance, we have the horizontal XY plane, which stands in for the fulcrum. The second plane is the XZ vertical plane, which represents the rotational movement of the pendulum [15]. Newton's laws of motion in the X and Z axes, and Euler's equations of rotational motion at the axes G and P, are used to derive the equations of motion for the mechanical system. The model's dynamics are described using equations that are very non-linear.

$$\begin{cases} \sum M_O = J_O \cdot \ddot{\theta} \\ \sum M_G = J_G \cdot \ddot{\alpha} \\ \sum F_z = m \cdot \ddot{z}_G \\ \sum F_x = m \cdot \ddot{x}_G \end{cases}$$
$$\Rightarrow \begin{cases} \left(J_{eq} + mr^2\right)\ddot{\theta} - mlr\cos(\alpha)\ddot{\alpha} + mlr\sin(\alpha)\dot{\alpha}^2 + B_{eq}\dot{\theta} = T \\ -mlr\cos(\alpha)\ddot{\theta} + \frac{4}{3}ml^2\alpha^2 - mgl\sin(\alpha) = 0 \end{cases} \quad (1)$$

The dynamic model (1) incorporates commonly acknowledged assumptions, such as the lack of friction and the existence of rigid objects. Furthermore, it provides a representation of the velocity of point *G*, which represents the center of gravity of the pendulum, with respect to point *P*. In addition, the model offers a depiction of the precise velocity of point G located on the pendulum. The equation denoted as (1) incorporates the variables J_O, J_G, M_O, M_G, F_z, and F_x which represent the forces, torques, and moments of inertia. The symbols B_{eq} and J_{eq} denote the equivalent inertia and viscous friction of the arm and pendulum, correspondingly. The specific values applied to B_{eq} and J_{eq} are 0.0034 kg. m^2 and 0.005, respectively. The variable *g* represents the gravitational acceleration, while *T* defines the input torque.

The mechanical system derives its kinetic energy from the rotational motion of the arm. The rotational movement of the arm is powered by a direct current (DC) motor, and the magnitude of the applied torque may be determined by analyzing the analogous model of a DC motor. The direct current (DC) motor has certain electromechanical properties, including the motor resistance (R_m) of 13Ω, negligible motor inductance (L_m), motor torque constant (K_m) and electromotive constant (K_t) of 0.007N.m/Amp and 0.007V/rad/sec respectively, moment of inertia of the motor (J_m) of $4.3e^{-7}$kg.m^2, and the voltage delivered to the DC motor (u_m).

$$T = \frac{K_t}{R_m}\cdot u_m - \frac{K_t \cdot K_m}{R_m}\cdot\dot{\theta} - J_m\cdot\ddot{\theta} \quad (2)$$

The Inverted Rotating Pendulum as a Linearized Model

Using the small angle formula in the model (3), we get a linear approximation to the non-linear system equations (1) and (2).

$$\begin{cases} k_1\cdot\ddot{\theta} - k_2\cdot\cos(\alpha)\cdot\ddot{\alpha} + k_2\cdot\sin(\alpha)\cdot\dot{\alpha}^2 + k_5\cdot\dot{\theta} = k_6\cdot V_m \\ -k_2\cdot\cos(\alpha)\cdot\ddot{\theta} + k_3\cdot\ddot{\alpha} - k_4\cdot\sin(\alpha) = 0 \end{cases}$$
$$\Rightarrow \begin{cases} k_1\cdot\ddot{\theta} - k_2\cdot\ddot{\alpha} + k_5\cdot\dot{\theta} = k_6\cdot u_m \\ -k_2\cdot\ddot{\theta} + k_3\cdot\ddot{\alpha} - k_4\cdot\alpha = 0 \end{cases} \quad (3)$$

Several new notations are included in (3).

$$k_1 = J_{eq} + m\cdot r^2 + J_m, k_2 = m\cdot l\cdot r, k_3 = \frac{4}{3}\cdot m\cdot l^2,$$
$$k_4 = m\cdot g\cdot l, k_5 = B_{eq} + \frac{K_t\cdot K_m}{R_m}, k_6 = \frac{K_t}{R_m}$$

Assuming that the starting circumstances are zero, the plant's transfer function defines the relationship between the amount by which the pendulum deviates from its ideal position and the input voltage delivered to the motor. Since obtaining the pendulum angle α as the necessary output from the plant is the primary goal of the linearized system (3), the pivot arm position θ was first eliminated. The Laplace transform was then applied to the data. Both the pole and the zero at the origin of the transfer function have been removed.

$$H_{RIP}(s) = \frac{A(s)}{U_m(s)} = \frac{k_2\cdot k_6\cdot s}{(k_1\cdot k_3 - k_2^2)\cdot s^3 + k_3\cdot k_5\cdot s^2 - k_1\cdot k_4\cdot s - k_4\cdot k_5} \quad (4)$$

Response in an Open Loop

Both the non-linear and linearized models were run in Matlab/Simulink, a modeling and simulation environment. Fig 2 displays a comparison of the two systems' open-loop responses. Before releasing the pendulum into free fall, we start by giving it a little nudge of 0.0001 radians by holding it upright. The motor receives no power at all throughout this whole duration. The simulation results show that the linear model accurately represents the pendulum's motion up to an angle of 20 degrees, which occurs during the first 1.2 seconds. Subsequently, the model deviates from the observed motion. Additionally, the system exhibits instability and non-linearity.

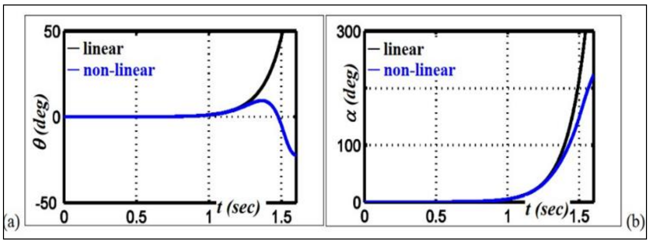

Fig 2: It is the sum of two angles: (a) the arc of the pivot arm, and (b) the arc of the inverted pendulum

3. Controlling the Inverted Pendulum's Swing Up and Keeping it Balanced

As stated in the introduction of the article, the major objective of the global control problem is to facilitate the transition of the pendulum from its stable downward position to the precarious upright position, and afterwards maintain its equilibrium in that state. These jobs have the capability to be completed either via a single controller or by using many systems. From this particular standpoint, the primary concern may be separated into two separate control subproblems: swing-up control and balance/stabilizing control. An additional crucial subject to contemplate is the determination of the suitable criteria for switching between these two control strategies.

The majority of scientific literature references the involvement of a third controller, often referred to as the switching/catching controller, in the switching process. The whole control process exhibits greater resilience when subjected to disturbances if the catching controller is able to effectively transition between swing up control and balancing control. Certain writers have used an energy-based mode switching control strategy in order to enhance the performance of a system. This is accomplished by monitoring the angle of a pendulum and comparing the system's energy with a predetermined threshold value, denoted as E_0. Several writers use a fusion approach, mixing dynamic programming with control strategies based on reinforcement learning. However, dynamic programming may need a prohibitive amount of processing power.

It was determined that the phrase "switching criterion" was more suitable than the previous work "switching/catching controller" for the purposes of this investigation. The results of the simulations and tests show that the RIP system is rather robust. This is shown by the fact that its performance is unaffected by the switch mode's implementation, even at the fixed pendulum angle of 20 degrees. If the specified condition holds true, the switching criteria will prioritize the balancing control; otherwise, the controller will continue to function in the swing-up mode. In the event that the Restoring Inverted Pendulum (RIP) system experiences a disturbance that disrupts its equilibrium and causes the pendulum to deviate from its vertical position by an amount beyond the predetermined threshold value, it is necessary for the criteria to activate the swing up control mechanism in order to restore stability.

The Implementation of Swing-up Control with Traditional Proportional-Derivative (PD) Controllers

The arm may be swung away from its fixed downward posture like a pendulum thanks to the swing-up controller. As the DC motor is supplied with electricity, and a sufficient force is applied to the arm to produce its reciprocal motion, energy is gradually transferred to the rotating inverted pendulum (RIP) system. Consequently, the pendulum may be elevated to the unstable state by swinging motion.

Many different control algorithms may be used to the problem of swing up control. Trajectory tracking, energy-based methods, the rectangular reference input swing-up type, and adaptive or intelligent processes are all examples. Due to its straightforward design, potent performance, and simple tuning process, a positive feedback proportional-integral-derivative (PID) controller is advocated for use in this paper. As can be seen in Fig 3, the block control diagram consists of two loops. In order to attain equilibrium, the pendulum must swing back and forth, with the inner loop responsible for controlling the arm's location and the outer loop determining the appropriate trajectory for the arm's angle.

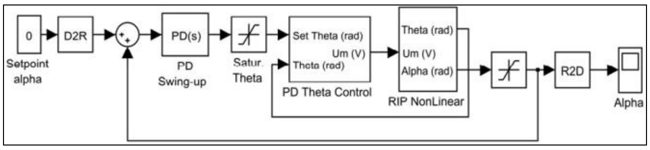

Fig 3: The RIP swing-up control as a block diagram in Simulink

A method for control based on proportional-derivative (PD) control is developed so that the servo arm can precisely track the desired position.

$$u_m(t) = k_{p\theta} \cdot \varepsilon_\theta + k_{d\theta} \cdot \frac{d\varepsilon_\theta}{dt} \qquad (5)$$

The PD controller provides the motor with a voltage input that enables the arm angle θ to follow the desired position θ_d. The tuning parameters $k_{p\theta}$ and $k_{d\theta}$ for the typical PD controller have been established at 100 and 2.5, respectively, in order to get the necessary performance levels inside its internal feedback loop.

A positive feedback proportional-derivative (PD) control loop was utilized to increase the amplitude of the pendulum's swing in the outer loop. The presence of a positive feedback loop in this system leads to an amplification of energy supplied to the pendulum, resulting in a destabilizing effect. This occurs due to the inherent stability of the pendulum's downward position. The tuning parameters $k_{p\alpha}$ and $k_{d\alpha}$ significantly influence the smooth operation of the pendulum. The parameters, namely the proportional and derivative constants, may be adjusted to modify the level of "positive damping" inside the system. Furthermore, it is essential to restrict the orders produced by the swing-up controller, namely the target angle of the pivot arm, within a range that includes both positive and negative 180 degrees. The incorporation of this preventive measure is considered crucial in order to minimize the potential hazards arising from potential collisions between the arm and other hardware components.

Equation (6) represents the swing-up proportional-derivative (PD) control law.

$$\theta_d(t) = k_{p\alpha} \cdot \varepsilon_\alpha + k_{d\alpha} \cdot \frac{d\varepsilon_\alpha}{dt} \qquad (6)$$

The constants of proportion and differentiation have been set as 0.15 and 0.04, respectively. To optimize the performance of the derivative constant $k_{d\alpha}$, it is necessary to strike a balance between enhancing the response time and minimizing the amplification of noise.

The Use of a Fuzzy-PD Controller for Balance and Stabilization Control

The stabilizing controller's job is to keep the pendulum from falling as it swings closer and closer to a vertical position. State feedback, linear quadratic based on the linearized plant model, and pole placement are just a few of the algorithms that may be utilized in this situation. A fuzzy-PD controller was chosen as the ideal method to handle the stabilization problem because of the Rotary Inverted Pendulum's (RIP) unique properties, such as its unstable upright position, non-linearities, and modeling mistakes.

Fig 4 displays the block diagram representing the swing-up control.

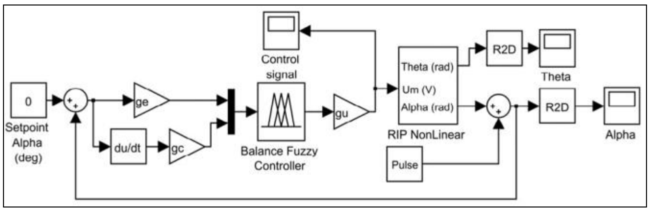

Fig 4: The RIP stability and balance control system as shown in a Simulink block diagram

The construction of a fuzzy controller for the stabilization issue involves the creation of four key components: the rule base, the inference engine, the fuzzification interface, and the defuzzification interface. These components together contribute to the functionality and operation of the fuzzy controller.

The variables used by the fuzzy system are the angle error $c\varepsilon_\alpha$, which is measured with respect to the upright position as the reference, and the change in angle error ε_α. The variable u_m denotes the output of the fuzzy system. The range of the variables was extended to include the interval [-1, 1], and the process of normalization was accomplished by applying scaling gains (g_e, g_c, g_u). The use of a consistent strategy for the membership functions will enhance the seamless integration of the controller in future endeavors. A set of 53 rules was developed by using a technique that included the use of five membership functions for each of the three fuzzy variables, resulting in a total of 26 combinations. The membership functions were designed to be symmetric and triangular in shape, with a 50% overlap (as depicted in Fig 5 (b). Consequently, it is possible for a maximum of four rules (i.e., 2^2) to be active simultaneously. A set of rules in the form of IF-THEN statements is used as the basis for the fuzzy controller's decision-making. The aforementioned criteria were established by a heuristic approach, utilizing the existing information pertaining to the plant. The rule table obtained is shown in Fig 5 (a).

The min-max inference engine was selected for its ability to analyze premises using the maximum operator for logical OR and the minimum operator for logical AND. Each rule's conclusion, indicated by the keyword "THEN," is likewise determined by some kind of minimal requirement. The ultimate decision on the set of active rules is made by selecting the highest value from the several fuzzy sets under consideration.

The center of gravity (COG) defuzzification approach is used in order to get a precise result. The crisp value represents the output of the controller.

(a)

e(kT) / c(kT)	NB	NS	Z	PS	PB
NB	PB	PB	PB	PS	Z
NS	PB	PB	PS	Z	NS
Z	PB	PS	Z	NS	NB
PS	PS	Z	NS	NB	NB
PB	Z	NS	NB	NB	NB

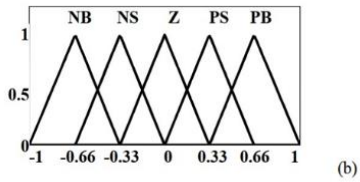

Fig 5: (a) Table of fuzzy rules for the controller; (b). Variable fuzzy sets

4. The Outcomes of the Simulation

In order to investigate the control of the rotating inverted pendulum via simulation, the systems were mathematically represented and implemented using Matlab/Simulink. The outcomes of the simulation are visually shown in Fig 6.

The inner loop of the swing-up control system, represented by the pivot position controller, was the first subject of analysis. Fig 6 (a) is a schematic showing the responses of the closed-loop system to a 10° step reference. The measured response shows an overrun of 4%, with the first peak appearing after 0.1 seconds. The position controller works as expected within the specified limits.

Fig 6 (b) shows that the intended goal of the swing-up control system has been met. Proof of this may be seen in the upward motion of the pendulum, as shown by the arc of the angle over the -20° mark, which happens after around 4.5 seconds. As was previously indicated, an impulsive control mechanism may shorten the swing-up time and increase the system's overall energy.

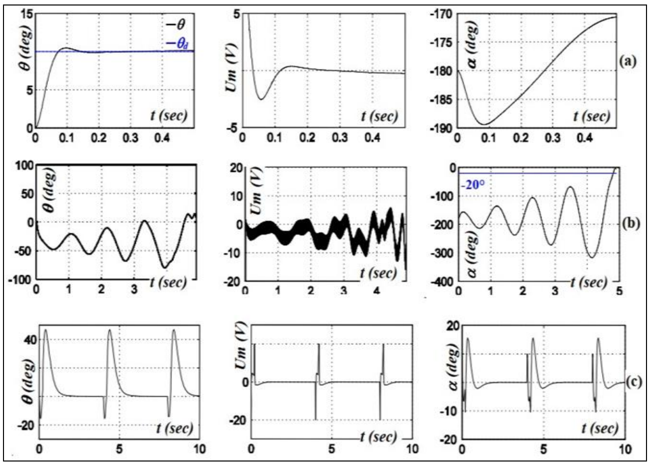

Fig 6: (a) signals associated with arm position control, (b) signals associated with swing-up control, and (c) signals connected with balance/stabilizing control are all examples of the kind of signals that may be received by the various control modules

5. Conclusions

This article gives a scholarly investigation into the control mechanisms used in managing the rotating inverted pendulum (RIP). The task of managing RIP control may be further broken into many subtasks. Initially, the pendulum must be elevated from its state of equilibrium, sometimes referred to as the "downward position." Two regular proportional-derivative (PD) controllers were implemented as part of a cascade control strategy for the experiment. Although the outside loop uses a swing-up controller, the inner loop uses a proportional-derivative (PD) position controller for the pivot arm. An approximate time of 4.3 seconds is all that is needed to complete the activity. The implementation of the balance/stabilizing control is initiated by the fuzzy-PD system at this point. The implementation of the present system was designed with careful consideration of the distinct attributes of the RIP system, including non-linearities, modeling mistakes, and instability. The system was thoroughly examined, even in scenarios where minor impulse perturbations were present. To attain global stability for the control process as a whole, an endeavor was undertaken to include a simplified switching criteria that is contingent upon a predetermined threshold value for the angle of the pendulum. The control techniques shown in this work have the potential to be applied to other complex systems.

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