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Magnetic Field Effect on the Nonlinear Propagation of Hollow Gaussian Laser Beam Inside Relativistic Plasma

[1] Abdulkareem Ali Nasser, [2] Munther B Hassan
[1, 2] Department of Physics, Faculty of Science, University of Kufa, Najaf, Iraq

Corresponding Author: **Munther B Hassan**

Abstract
In this manuscript the electrons in the collision less plasma will earn a high relativistic motion caused by the high intensity Hollow Gaussian laser beam (HGLB) thus the dielectric constant of plasma will be modified and the plasma will manifest a nonlinear a self-focusing phenomenon.
The final differential equation of (HGLB) self-focusing has been derived then it is solved numerically by designing an appropriate Matlab program.
This technique has prove that, in presence of an external longitudinal magnetic field, the self-focusing of (HGLB) will enhance thus the initial laser intensity will raise to ten times or more.

Keywords: Relativistic Nonlinearity, Hollow Gaussian Laser Beam (HGLB), Self-Focusing Phenomenon, Longitudinal External Magnetic Field

1. Introduction

Recently, research has grown in the nonlinear interactions between a hollow Gaussian laser beam with high-intensity(HGLB) and magnetized plasma in theoretical and practical ways [1-4]. Because of the diffraction behavior of the laser beam in the magnetic plasma, attention has been paid to generating the phenomenon of self-focusing in the vicinity [5-6] and that all nonlinear optical phenomena occupy a unique place, especially the phenomenon of self-focusing because of its importance in many modern applications such as in the medical field, communications, and others. In the paraxial ray approximation, when the relativistic nonlinearity is operating, the hollow Gaussian laser beam via magnetised plasma will be self-focused [7]. Theoretical analysis and numerical calculations demonstrate that in both situations (longitudinal and transverse magnetic fields), increasing the values of the external magnetic field will increase and enhance the nonlinear self-focusing of the Hollow Gaussian laser beam [8-10].

The paper organization will be as following: in section (2) the final formula of relativistic nonlinear dielectric tensor is calculated. (3) The self-focusing equations for the paraxial Hollow Gaussian laser beam are established in section (4). The numerical data are presented together with extensive discussion of the key findings and section (5) respectively.

2. The Relativistic Dielectric Constant

Imagine a uniform magnetized plasma of equilibrium electron density n_0 immersed in static magnetic field $\vec{B}_0$ aligned along z-direction. The electric field vector $\vec{E}_{0+}$ of a right circularly polarized electromagnetic wave propagating along z-direction through the magnetized plasma may be given as following [11]:

$$\vec{E}_{0+} = \vec{A}_{0+} \exp i(\omega_0 t - k_{0+} z) \quad (1)$$

Where $A_{0+} = \vec{E}_x + i\vec{E}_y$ is the electric field amplitude, ω_0 and k_{0+} are the angular frequency and wave propagation vector respectively.

The wave propagation vector k_{0+} and the dielectric constant ε_{0+} are related together by the following dispersion relation;

$$k_{0+}^2 = \frac{\varepsilon_{0+}\omega_0^2}{c^2},$$

Where c is the light velocity in the vacuum.

The general motion equation of an electron in electromagnetic field inside plasma is written as;

$$m_0 \frac{\partial}{\partial t}\vec{v} = -e\vec{E} - \frac{e}{c}\ (\vec{v} \times \vec{B}_0) \tag{2}$$

Where the oscillation velocities $(\vec{v})$ induced by the laser beam and external magnetic field $(\vec{B}_0)$, respectively and relativistic factor (γ) [12].

It is suitable to rewrite the aforementioned equations for velocity, current density, and related conductivity as for an electron oscillating in a right-handed circularly polarised electromagnetic wave [13–16].

By Using Eq. (2), we calculate the electron oscillating velocity ($\vec{v}_{0+}$ in a right − handed circularly polarised is written as;

$$\vec{v}_{0+} = \vec{v}_x + i\vec{v}_y = ie\frac{\vec{E}_{0+}}{m_0\gamma\omega_0(1-\frac{\omega_{ce}}{\gamma\omega_0})} \tag{3}$$

Where $\omega_{ce} = \frac{eB_0}{m_0 c}$ the cyclotron frequency, $-e$ and m_0 are the charge and rest mass of electron respectively. The nonlinearity appears here in mass rise in plasma frequency Where the γ relativistic factor can write as:-

$$\gamma = \left(1 - \frac{v_{0+}^2}{c^2}\right)^{-\frac{1}{2}}$$

Suggesting($1<\gamma<2$) [17] the relativistic factor γ will be;

$$\gamma \cong 1 + \frac{1}{2}\frac{e^2}{m_e^2\, c^2\, \omega_0^2}\frac{\vec{A}_{0+}\vec{A}_{0+}^*}{\left(1-\frac{\omega_{ce}}{\omega_0}\right)^2} = 1 + \alpha_+\vec{A}_{0+}.\vec{A}_{0+}^* \tag{4}$$

Where $\gamma = 1$ the relativistic nonlinearity factor can write as

$\alpha_+ = 1 + \frac{1}{2}\frac{e^2}{m_e^2\, c^2\, \omega_0^2}\frac{\vec{A}_{0+}\vec{A}_{0+}^*}{\left(1-\frac{\omega_{ce}}{\omega_0}\right)^2}$ will become zero at non-relativistic system (i.e., $\gamma = 1$).

The components of the tensor dielectric constants $\underline{\underline{\varepsilon}}$ in the plasma medium will be defined as follows due to the appearance of relativistic nonlinearity.

$$\varepsilon_{xx} = \varepsilon_{yy} = 1 - \frac{\omega_{pe}^2}{\omega_0^2\left(1-\frac{\omega_{ce}^2}{\omega_0^2}\right)}$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = \frac{-i\left(\frac{\omega_{pe}^2}{\omega_0^2}\right)\left(\frac{\omega_{ce}}{\omega_0}\right)}{\left(1-\frac{\omega_{ce}^2}{\omega_0^2}\right)}$$

$$\varepsilon_{zz} = 1 - \frac{\omega_{pe}^2}{\omega_0^2}$$

$$\varepsilon_{xz} = \varepsilon_{yz} = \varepsilon_{zx} = \varepsilon_{zy} = 0$$

And the right circular polarised laser beam's effective dielectric constant ε_+ will be represented by the following formula.

$$\varepsilon_+ = \varepsilon_{xx} - i\varepsilon_{xy} = 1 - \frac{\frac{\omega_p^2}{\omega_0^2\gamma}}{\left(1 - \frac{\omega_{Ce}}{\omega_0\gamma}\right)}$$

Where $\omega_p = \sqrt{\left(\frac{4\pi n_0 e^2}{m_0}\right)}$ is the electron plasma frequency

By using Eq (4) the effective dielectric constant ε_+ may be expressed as follows.

$$\varepsilon_+ = 1 - \frac{\left(\frac{\omega_p}{\omega_0}\right)^2}{\left(1 - \frac{\omega_{Ce}}{\omega_0}\right)} + \frac{\left(\frac{\omega_p}{\omega_0}\right)^2}{\left(1 - \frac{\omega_{Ce}}{\omega_0}\right)} \alpha_+ \vec{A}_{0+} \vec{A}^*_{0+} \quad (5)$$

According to Eq. (5), the effective dielectric constant ε_+ is made up of a linear component (ε_{0+}), as well as a nonlinear part $\emptyset_+(\vec{A}_{0+}\vec{A}^*_{0+})$, which is emerging as the relativistic electron mass increases. The effective dielectric constant ε_+ can be expressed as having two parts [18]:

$$\varepsilon_{0+} = 1 - \frac{\left(\frac{\omega_p}{\omega_0}\right)^2}{\left(1 - \frac{\omega_{ce}}{\omega_0}\right)} \quad (6)$$

$$\phi_+ = \varepsilon_{2+} \vec{A}_{0+} \vec{A}^*_{0+} \quad (7)$$

where ε_{2+} is determined by

$$\varepsilon_{2+} = \frac{1}{2}\left(\frac{e}{m_0 c\, \omega_0}\right)^2 \frac{\left(\frac{\omega_p}{\omega_0}\right)^2}{\left(1 - \frac{\omega_{Ce}}{\omega_0}\right)^4} \quad (8)$$

3. The self-Focusing of Hollow Gaussian Laser Beam in Relativistic Plasma

The general wave equation of the laser electric field through magnetized plasma is governed by;

$$\nabla^2\vec{E} - \nabla(\nabla\cdot\vec{E}) + \frac{\omega_0^2}{c^2}\underline{\underline{\varepsilon}}.\vec{E} = 0 \quad (9)$$

Since the electromagnetic wave's field varies along the external magnetic field (i.e., the z-direction) more than it varies along the wave front plane (i.e., the x-y plane) [19], no space charge occurs and the electromagnetic wave is thus considered to be a transverse wave.

$$\vec{\nabla}\cdot\vec{D} = \vec{\nabla}\cdot(\varepsilon\vec{E}) = 0 \quad (10)$$

Using Eq. (10) with components of dielectric tensor, can be written as;

$$\frac{\partial E_z}{\partial z} \cong -\frac{1}{\varepsilon_{zz}}\left[\varepsilon_{xx}\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}\right) + \varepsilon_{xy}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\right] \quad (11)$$

Using the zero order approximation and Eq.(11) in Eq.(8), we were able to get the differential equation for the circularly polarised electric field's amplitude A_{0+} as

$$\frac{\partial^2 A_{0+}}{\partial z^2} + \frac{1}{2}\left(1 + \frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)A_{0+} + \frac{\omega_0^2}{c^2}(\varepsilon_{0+} + \varepsilon_{2+} A_{0+} A^*_{0+})A_{0+} = 0 \quad (12)$$

Where the product of the nonlinear component with $\frac{\partial^2 A_{0+}}{\partial x^2}$ or $\frac{\partial^2 A_{0+}}{\partial y^2}$ has been disregarded [20].

Using the assumption that $A'_{0+} = A_{0+} e^{i(\omega_0 t - k_{0+} z)}$, and inserting its value in Eq. (12), one may obtain;

$$2ik_{0+}\frac{\partial A'_{0+}}{\partial z} = \frac{1}{2}\left(1 + \frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)A'_{0+} + \frac{\omega_0^2}{c^2}(\varepsilon_{2+} A'_{0+} A'^*_{0+})A_{0+} \quad (13)$$

The complex amplitude of the (HGLB) is represented by (A'_{0+}) in this instance, and its value at z = 0 is provided as follows.

$$(A'_{0+})_{z=0} = E_{00}\left(\frac{x^2}{2x_0^2}\right)^n e^{-\left(\frac{x^2}{2x_0^2}\right)} \tag{14}$$

Whereas for GB, n=0, and for HGLB, n=1, 2, etc.

By adding an eikonal $A'_{0+} = A^0_{0+}\, e^{(i\, k_{0+} S_+)}$ in a two-dimensional Gaussian beam ($\frac{\partial}{\partial y} = 0$), where ($A^0_{0+}$) is the real functions and (S_+) is the phase for the laser beam within magnetic, Eq. (13) may be divided into real and imaginary components, as shown below [21];

$$2\frac{\partial S_+}{\partial z} + \frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)\left(\frac{\partial S_+}{\partial z}\right)^2 + \frac{1}{2\,k_{0+}^2\, A^0_{0+}}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)\frac{\partial^2 A^0_{0+}}{\partial x^2} - \frac{\varepsilon_{2+}}{\varepsilon_{0+}}(A^0_{0+})^2 = 0 \tag{15a}$$

$$\frac{\partial (A^0_{0+})^2}{\partial z} + \frac{1}{2}(A^0_{0+})^2\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)\frac{\partial^2 S_+}{\partial x^2} + \frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)\frac{\partial S_+}{\partial x}\frac{\partial (A^0_{0+})^2}{\partial x} = 0 \tag{15b}$$

In the paraxial ray approximation, S may be extended to $S_+ = \frac{1}{2}x^2\beta_+(z) + \varphi_+(z)$, where β_+^{-1} can be interpreted as the laser beam's curvature radius and φ_+ is a constant that is independent of x. the initial beam radius x_0 and the initial hollow Gaussian beam for (z>0), as;

$$(A^0_{0+})^2 = \frac{E_{00}^2}{2^{2n} f_{0+}^2}\left(\frac{x}{r_0 f_{0+}}\right)^{4n} e^{-\left(\frac{x}{r_0 f_{0+}}\right)^2} \tag{16}$$

By adding S_+ to Eq. (15), z will obtain the following expression [22].

$\beta_+(z) = 2\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{zz}}\right)^{-1}\frac{1}{f_{0+}}\frac{df_{0+}}{dz}$ where f_{0+} represents the beam width parameter

$$\frac{d\beta_+}{dz} = \frac{2r_0^2}{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)}\left(\frac{df_{0+}}{dz}\right)^2 + \frac{2r_0^2 f_{0+}}{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)}\frac{d^2 f_{0+}}{dz^2} \tag{17b}$$

Also

$$\therefore \frac{2(\sqrt{2n}+\eta)^2 r_0^2 f_{0+}}{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)}\frac{d^2 f_{0+}}{dz^2} = \frac{1}{2}\frac{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)}{k_{0+}^2 r_0^2 f_{0+}^2}\left[\frac{4n^2}{(\sqrt{2n}+\eta)^2} - \frac{4n(\sqrt{2n}+\eta)}{(\sqrt{2n}+\eta)} + 2n + 2\eta\sqrt{2n} + \eta^2 + \frac{2n}{(\sqrt{2n}+\eta)^2} - 1\right] +$$

$$\frac{\varepsilon_{+2}}{\varepsilon_{0+}}\frac{A_{0+}^2}{\left(1-\frac{\omega_c}{\omega}\right)^2}\frac{E_{00+}^2}{2^{2n} f_{0+}^2}\left(\sqrt{2n}+\eta\right)^{4n} e^{-(\sqrt{2n}+\eta)^2} \tag{17b}$$

taking only the terms will lead to η^2, so

$$\frac{2\eta^2 r_0^2 f_{0+}}{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)}\frac{d^2 f_{0+}}{dz^2} = \frac{1}{2}\frac{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)\eta^2}{k_{0+}^2 r_0^2 f_{0+}^2} + \frac{\varepsilon_{+2}}{\varepsilon_{0+}}\frac{1}{\left(1-\frac{\omega_c}{\omega}\right)^2}\frac{E_{00+}^2}{2^{2n} f_{0+}^2}\left(\sqrt{2n}+\eta\right)^{4n} e^{-(\sqrt{2n}+\eta)^2} \tag{18}$$

$$\frac{2r_0^2 f_{0+}}{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)}\frac{d^2 f_{0+}}{dz^2} = \frac{1}{2}\frac{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)}{k_{0+}^2 r_0^2 f_{0+}^2} - 4\frac{\varepsilon_{+2}}{\varepsilon_{0+}}\frac{1}{\left(1-\frac{\omega_c}{\omega}\right)^2}\frac{E_{00+}^2}{2^{2n} f_{0+}^2}\left[-2(2n)^{2n} e^{-2n}\right](2)^{2n} \tag{19}$$

$$\therefore \frac{d^2 f_{0+}}{dz^2} = \frac{1}{4}\frac{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)^2}{k_{0+}^2 r_0^4 f_{0+}^3} - \frac{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)}{\left(1-\frac{\omega_c}{\omega}\right)^2}\left[\frac{\varepsilon_{+2}E_{00+}^2}{\varepsilon_{0+}}\right]\frac{1}{r_0 f_{0+}^2}\left[(n)^{2n} e^{-2n}\right] \tag{20}$$

Where r_0 is the beam's starting radius, n the order of(HGLB) and $R_{d+} = k_0 r_0^2$ indicates the diffraction length.

The final equation may be recast as follows in terms of normalisation distance of propagation $\xi = {}^{z}/_{R_{d+}}$ to make it more acceptable for computing programmes.

$$\frac{d^2 f_{0+}}{d\xi^2} = \frac{1}{4}\frac{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)^2}{f_{0+}^3} - \frac{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{ozz}}\right)}{\left(1-\frac{\omega_c}{\omega}\right)^2}\left[\frac{\varepsilon_{+2}E_{00+}^2}{\varepsilon_{0+}}\right]\frac{R_{d+}^2}{r_0^2 f_{0+}^2}\left[(n)^{2n} e^{-2n}\right] \tag{21}$$

The final equation (21) indicates the fluctuation in Hollow Gaussian laser beam profile spot size caused by a disagreement between diffraction and self-focusing terms (first and second terms on the right-hand side of Eq.(21) respectively), it can be solved numerically for several external magnetic fields.

4. The Numerical Results and Discussions

The numerical calculations of (HGLB) self-focusing, due to nonlinear interaction between the pump wave from a carbon dioxide (CO2) pulsed laser's Hollow Gaussian mode and the hydrogen magnetized plasma, have been achieved. The last equations (21) have been numerically solved by designing a suitable Matlab program and introducing the following set of experimentally determined parameters:

- The orders of laser beam are (n=0) for Gaussian laser beam (GLB) and (n=1, 2 and 3) for hollow Gaussian laser beam (HGLB).
- The angular frequency of Carbon Dioxide (CO_2) pulsed laser $(\omega_0 = 1.778\times10^{14}\,rad/\sec)$ corresponding to the wavelength ($\lambda = 10.6\,\mu m$).
- The initial laser beam intensity ($I = 10^{17}\ w/_{cm^2}$) which is corresponded to the laser strength parameter ($\alpha_0 = \frac{e E_{00}}{m_e \omega_0 c} = 0.9$).
- The initial laser beam diameter $(x_0 = 30\mu m)$.
- The plasma density $(n_0 = 0.895\times10^{18} cm^{-3})$ which is corresponded to plasma frequency $(\omega_{pe} = 0.533\times10^{14}\,rad.s^{-1})$.
- The applied magnetic fields $(B_0 = (50,150 \text{ and } 250)\,kG)$ which may be written in term of the cyclotron frequencies as follows $\left(\frac{\omega_{ce}}{\omega_0} = 0.005, \frac{\omega_{ce}}{\omega_0} = 0.015 \text{ and } \frac{\omega_{ce}}{\omega_0} = 0.025\right)$.

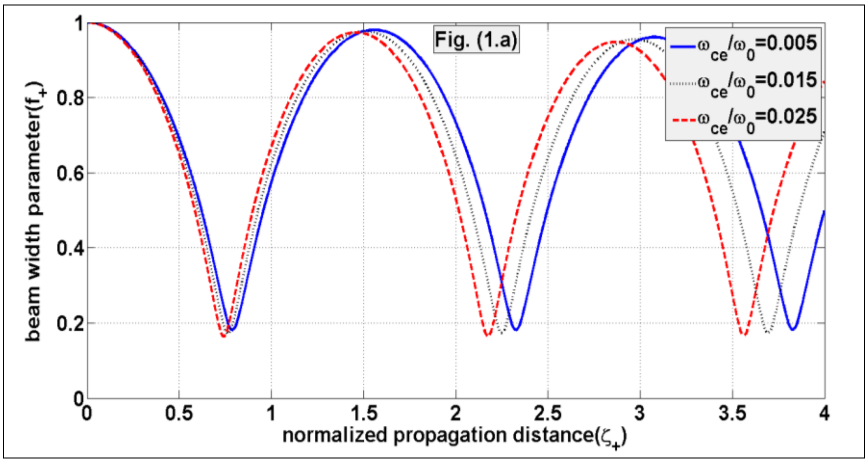

Fig 1(a): (Color online) Variation of laser beam width parameter f_{0+} with normalized distance $\xi_{0+} = \frac{z}{k_{0+} x_0^2}$ of the Gaussian laser beam of order (n=0) at different values of magnetic field $\left(\frac{\omega_{ce}}{\omega_0} = 0.005, \frac{\omega_{ce}}{\omega_0} = 0.015 \text{ and } \frac{\omega_{ce}}{\omega_0} = 0.025\right)$

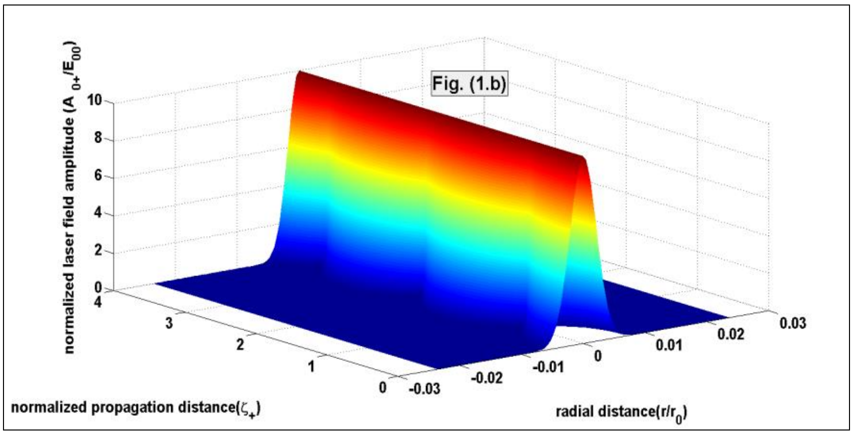

Fig 1(b): (Color online) Variation of normalized laser beam amplitude with normalized distance $\xi_{0+} = \frac{z}{k_{0+} x_0^2}$ and radial distance (r/r_0) in paraxial case of the Gaussian laser beam (n=0)

Fig (1.a) is showing that, for propagation of the Gaussian laser beam (GLB) along normalized propagation distance, the (ξ_{0+}) beamwidth parameter (f_{0+}) is decreasing, when the external magnetic field is increasing. Variation of normalized Gaussian laser beam amplitude $\left(A_{0+}/E_{00}\right)$ with normalized propagation distance $\left(\xi_{0+}\right)$ and radial distance (r/r$_0$) in paraxial case by operating the relativistic nonlinearity is explained in Fig (1.b).

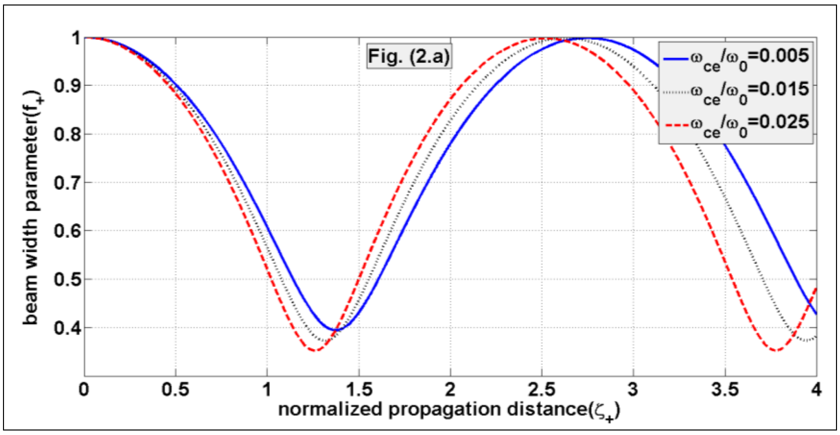

Fig 2(a): (Color online) Variation of laser beam width parameter f_{0+} with normalized distance $\xi_{0+}=\frac{z}{k_{0+}x_0^2}$ of the hollow Gaussian laser beam of order (n=1) at different values of magnetic field $\left(\frac{\omega_{ce}}{\omega_0}=0.005, \frac{\omega_{ce}}{\omega_0}=0.015 \text{ and } \frac{\omega_{ce}}{\omega_0}=0.025\right)$

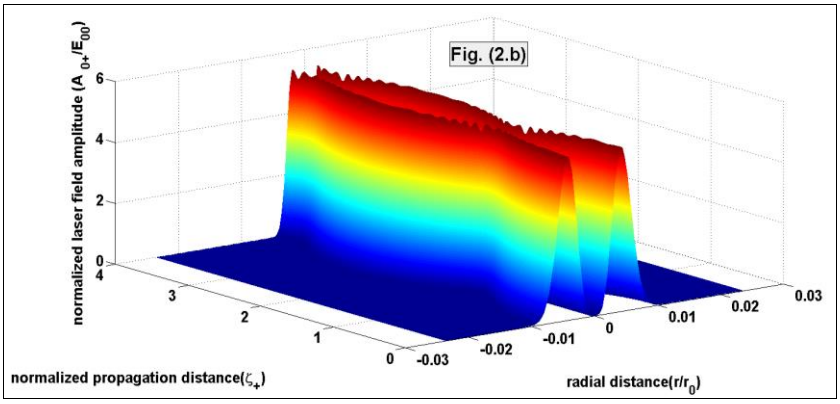

Fig 2(b): (Color online) Variation of normalized laser beam amplitude with normalized distance $\xi_{0+}=\frac{z}{k_{0+}x_0^2}$ and radial distance (r/r$_0$) in paraxial case of the hollow Gaussian laser beam (n=1)

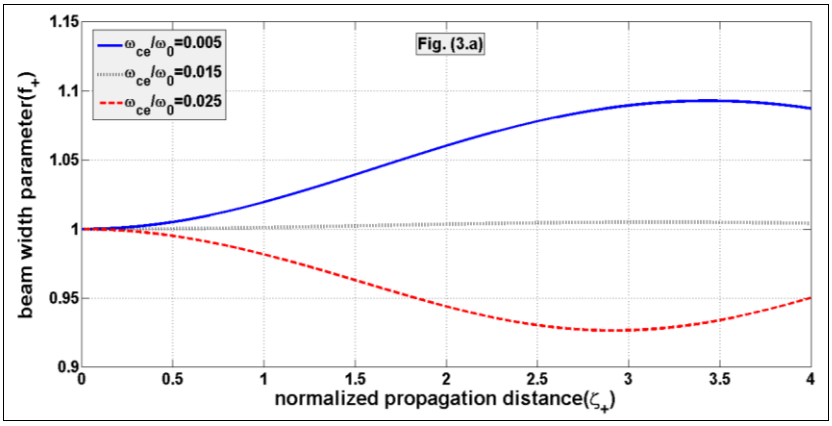

Fig 3(a): (Color online) Variation of laser beam width parameter f_{0+} with normalized distance $\xi_{0+}=\frac{z}{k_{0+}x_0^2}$ of the hollow Gaussian laser beam of order (n=2) at different values of magnetic field $\left(\frac{\omega_{ce}}{\omega_0}=0.005, \frac{\omega_{ce}}{\omega_0}=0.015 \text{ and } \frac{\omega_{ce}}{\omega_0}=0.025\right)$

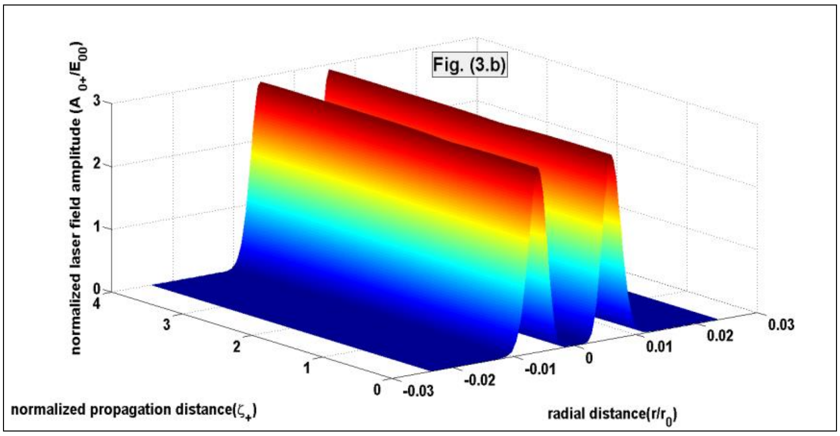

Fig 3(b): (Color online) Variation of normalized laser beam amplitude with normalized distance $\xi_{0+} = \frac{z}{k_{0+}x_0^2}$ and radial distance (r/r_0) in paraxial case of the hollow Gaussian laser beam (n=2)

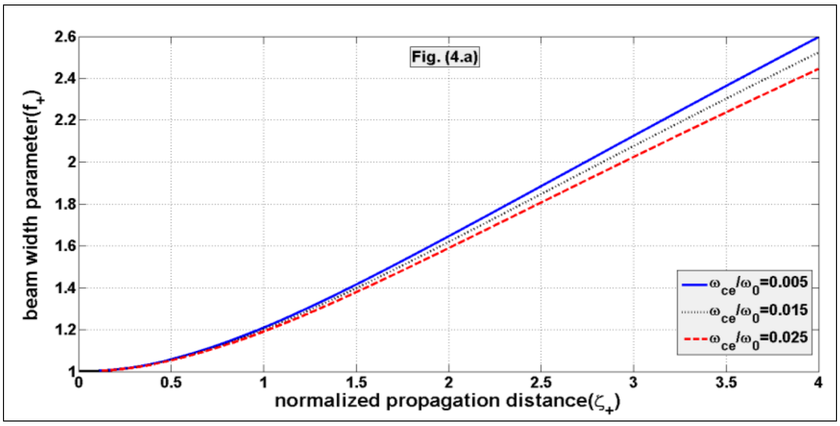

Fig 4(a): (Color online) Variation of laser beam width parameter f_{0+} with normalized distance $\xi_{0+} = \frac{z}{k_{0+}x_0^2}$ of the hollow Gaussian laser beam of order **(n=3)** at different values of magnetic field $\left(\frac{\omega_{ce}}{\omega_0} = 0.005, \frac{\omega_{ce}}{\omega_0} = 0.015 \text{ and } \frac{\omega_{ce}}{\omega_0} = 0.025\right)$

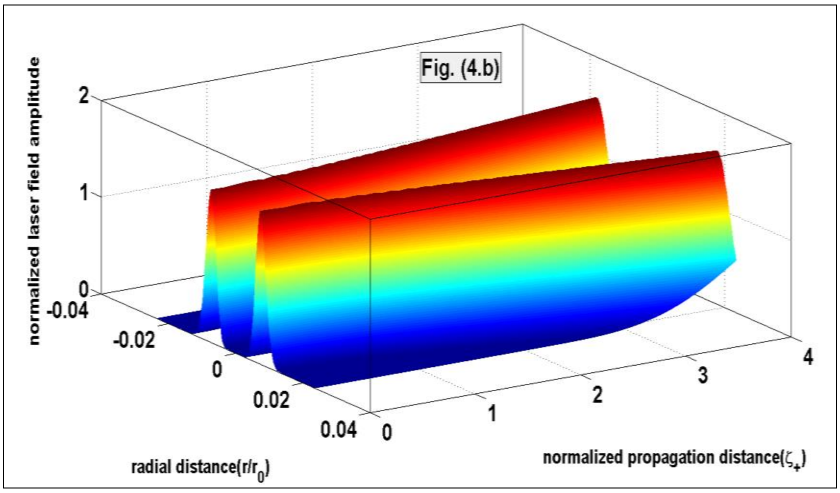

Fig 4(b): (Color online) Variation of normalized laser beam amplitude with normalized distance $\xi_{0+} = \frac{z}{k_{0+}x_0^2}$ and radial distance (r/r_0) in paraxial case of the hollow Gaussian laser beam (n=3)

For (HGLB) at order (n=1, 2 and 3), the same effect of external magnetic field have been recorded on the behavior of laser beam self-focusing but with less influence comparing with (GLB) case (see Figures 2, 3 and 4). In Figures (2.b, 3.b and 4.b), the variation of normalized hollow Gaussian laser beam amplitude $\left(A_{0+}/E_{00}\right)$ with normalized propagation distance $\left(\xi_{0+}\right)$ and radial distance (r/r_0) are illustrated taken in our consideration the paraxial approximation case and operating the relativistic nonlinearity.

One may note that, in the Fig (3), the influence of the external magnetic field on the nonlinear propagation of (HGLB) where (n=2), is very clear. Three distinguished cases namely, defocusing, equilibrium and self-focusing have been obtained by raising the magnetic field values $\left(\frac{\omega_{ce}}{\omega_0}=0.005, \frac{\omega_{ce}}{\omega_0}=0.015 \text{ and } \frac{\omega_{ce}}{\omega_0}=0.025\right)$.

In Fig (4), it is important to mention that the (HGLB), at (n=3), will undergo a defocusing state which it refers to that the natural diffraction effect (first term on the right-hand side of Eq. (21)) will overcome the self-focusing effect (second term on the right-hand side of Eq. (21)).

5. Conclusions

One may conclude that the external magnetic field play very important role in enhancement the nonlinear propagation of laser beam through magnetized plasma. The controlling in magnetic field leads to the controlling the diameter laser beam thus the laser intensity will be controlled.

At the same parameters of plasma, laser and external magnetic field, it may be deduced that the (GLB) will manifest a strong and sharp self-focusing comparing with (HGLB) modes which can be understood as follows: since more electrons in center of (GLB) comparing with (HGLB) thus more electrons will contribute in relativistic nonlinearity thus in self-focusing effect.

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