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Multi-Level Marketing in the Context of Dynamical System

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Abstract
Multi-Level Marketing has got an international worth estimated to be around 180 billion dollars in the marketing industry. It is also known as network marketing or aggregate selling or referral marketing. Multi-Level Marketing is a type of trades approach where goods are offered directly to customers without passing through a mediator. It follows a dynamical system. This can be measured mathematically with the help of famous K-M SI model. Here, general customers are termed as susceptible persons for being agent of the market and existing agents of the market are termed as infective individuals. In this paper, the magic agent number for the growth of the model is obtained by the concept of reproduction number of the SI model.

Keywords: Dynamical System, Multi Level Marketing, Mathematical Modelling, Magic Agent Number

Introduction

Multi-Level Marketing (MLM) generates an average of 32.5 billion dollars in sales annually, and the worldwide network marketing industry is estimated to be worth around 180 billion dollars.

Multi-level marketing is also known as network marketing or aggregate selling or referral marketing [1]. It is a type of trades approach where goods are offered directly to customers without passing through a mediator like a retail store. A business strategy known as MLM entails hierarchical, unpaid sales teams selling goods directly to customers while also hiring new firm sales representatives. Referral marketing and network marketing are other names for multi-level marketing. The reason it's called multi-level is because a contractor can hire additional people and get paid based on how well they perform. Multi-level marketing now enjoys recognition on a global scale. Anyone can engage in multi-level marketing at any stage in their life and start earning money from it. Through the use of a dynamical system, we have coupled multi-level marketing to mathematical modelling here.

In an effort to find better decision models to handle issues like new-product development, media selection, retail inventory control, and sales force size, mathematicians and marketing executives have teamed up. In this article, author P. Kotler describes and provides examples of the operations research models that show the most promise for the marketing field [2].

A marketing strategy is an effort to promote a product so that word of mouth spreads about it. Customers were recruited to help promote the company's products as part of the company's combined marketing strategy. The referral marketing strategy is the name of this marketing approach. A pandemic state of a disease can be compared to the spread of viral information within a group. In this study, the epidemiology model and the marketing model with a referral method are compared. This study uses a literature review to present the findings of a marketing model that incorporates a referral strategy. Four groups—Unaware, Potential Broadcaster, Broadcaster, and Inert—are identified in marketing modelling with a referral plan based on the research. The SEIR model of epidemiology is appropriate for this illness. Because to the Unaware's direct transition to the Inert due to the Unaware's lack of faith in marketing materials, modifications to the SEIR Model are required. The simulation results demonstrate that the customer network and the incubation period for the information's virality are key characteristics that influence how quickly information spreads [3].

The article offers the findings of investigations on the mathematical elements of viral marketing. A prefractal graph was created as a mathematical representation of the spread of a marketing virus. The development of a viral information distribution algorithm [4].

To analyse the changes in the proportion of repeat and referred consumers in a certain organisation, a compartmental model is taken into consideration. The parameters used to model compartment transitions rely on the social network and the company's marketing strategy. In a few specific cases, we get some data regarding the asymptotic number of repeat customers and referral customers. Additionally, we provide a simulation that depicts the behaviour of the model and talk about its relevance [5].

To keep their distribution costs down, manufacturers prefer direct marketing channels to traditional marketing channels. Network marketing, also known as multilevel marketing, is a strategy used by businesses to hire and pay part-time salespeople to promote their goods on a compensation basis. There is much discussion on whether an MLM is really less expensive and actually provides the channel members with the returns they were promised. In the current study, an effort has been made to mathematically explain network behaviour and to forecast the likelihood that networks would succeed at different levels [6].

A real-life component is described via mathematical modelling. It involves creating a mathematical model of a real-world problem. The model is then resolved, examined, and put to the test against the real problem [7, 8].

Materials and Methods

Base Model

Here famous KerMack-McKendrick Susceptible-Infective model [9, 10] is incoroporated as the base model to study the growth pattern of the multi level marketing in the form of agents. The base model is:

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI$$

Where S is the susceptible human being, I is the infective population and β is the rate of contact of susceptible and infective population.

State Variables of the Proposed Model

C(t) is the general individual at time t,
A(t) is the number of agents of the business,
(N+1)(t) is the total population at time t.

Proposed Model

$$\left.\begin{aligned} \frac{dC}{dt} &= \gamma - \beta CA - \alpha C \\ \frac{dA}{dt} &= \beta CA - \theta \end{aligned}\right\} \qquad (1)$$

Where:

γ is immigrants
α is death rate of the general individual
β is the rate of interaction between general individual and agent
θ is the constant rate of agent who leaves the profession.

Procedure

1. Check the uniqueness of the model.
2. Positivity of thee model.
3. Boundedness of the model.
4. Equilibrium point.
5. Stability
6. Obtain the Magic Agent Number (MAN).
7. Sensitivity of the model.
8. Simulation

Results and Discussion

Uniqueness

It is assumed that (C_o, A_o) Є Θ, the space. Therefore, the coefficient of the equation (2.2.1) is Lipchitz continuous. Hence, for any given initial conditions (C_o, A_o) Є Θ, there exists a unique local solution (C(t), A(t)) for all t Є [0, T), where T is final time.

Positivity

Solutions of $\frac{dC}{dt} = \gamma - \beta CA - \alpha C$ gives the result as:

$$=> \frac{dC}{dt} = \gamma - C(\beta A + \alpha)$$

$$=> \frac{dC}{dt} + C(\beta A + \alpha) = \gamma$$

Solving this differential equation, we get C(t) > 0
Similarly, we can prove that A(t) > 0.

Boundness
It can be deduced that C(t) + A(t) ≤ AT for all t Є [0, T).

$$\frac{d(N+1)}{dt} \leq \frac{dC}{dt} + \frac{dA}{dt} = \gamma - \alpha - \theta - \alpha C$$

$$=> dN \leq \gamma\, dt$$

$$=> N \leq \gamma t + \text{constant}$$

$$=> \lim_{t \to \infty} \sup(N) \leq \gamma T \text{ for all } t \in [0, T).$$

Which shows the boundedness of the system.

Equilibrium Point
Equating the equations of the system (1), we get

$$C = \frac{\gamma - \theta}{\alpha} \text{ and } A = \frac{\theta\alpha}{\beta(\gamma - \theta)}$$

Therefore, the equilibrium point is: $E(C^*, A^*) = E\left(\frac{\gamma - \theta}{\alpha}, \frac{\theta\alpha}{\beta(\gamma - \theta)}\right)$

Local Stability
Jacobian of system (1) is

$$J = \begin{vmatrix} \frac{\partial f_1}{\partial C} & \frac{\partial f_1}{\partial A} \\ \frac{\partial f_2}{\partial C} & \frac{\partial f_2}{\partial A} \end{vmatrix}$$

$$= \begin{vmatrix} -\beta A - \alpha & -\beta C \\ \beta A & \beta C \end{vmatrix}$$

Now,

$$| J - \lambda I | = 0$$

$$=> \begin{vmatrix} -\beta A - \alpha - \lambda & -\beta C \\ \beta A & \beta C - \lambda \end{vmatrix} = 0$$

$$=> (-\beta A - \alpha - \lambda)(\beta C - \lambda) + (\beta C)(\beta A) = 0$$

$$=> -\beta^2 AC + \beta A\lambda - \alpha\beta C + \alpha\lambda - \lambda\beta C + \lambda^2 + \beta^2 CA = 0$$

$$=> \beta A\lambda - \alpha\beta C + \alpha\lambda - \lambda\beta C + \lambda^2 = 0$$

$$=> \lambda^2 + \lambda(\beta A + \alpha - \beta C) - \alpha\beta C = 0$$

$$=> \lambda = \frac{-(\beta A + \alpha - \beta C) \pm \sqrt{(\beta A + \alpha - \beta C)2 + 4\alpha\beta C}}{2}$$

λ will be negative if

$$-(\beta A + \alpha - \beta C) \pm \sqrt{(\beta A + \alpha - \beta C)2 + 4\alpha\beta C} < 0$$

$$=> \pm \sqrt{(\beta A + \alpha - \beta C)2 + 4\alpha\beta C} < (\beta A + \alpha - \beta C)$$

$$=> (\beta A + \alpha - \beta C)^2 + 4\alpha\beta C < (\beta A + \alpha - \beta C)^2$$

$=> 4\alpha\beta C < 0$

$=> \alpha\beta C < 0$

$=> \alpha\beta\left(\frac{\gamma-\theta}{\alpha}\right) < 0$

$=> \beta(\gamma - \theta) < 0$

$=> \gamma - \theta < 0$

$=> \gamma < \theta$

Hence, the model is locally stable under the condition $\gamma < \theta$.

Global Stability

Let us assume a Lyapunov function $L(w) = \frac{1}{2}w_1C^2 + \frac{1}{2}w_2A^2$ where, w_1 and w_2 are non-negative numbers.

Now,

$$\frac{dL}{dt} = \frac{\partial L}{\partial C}\frac{dC}{dt} + \frac{\partial L}{\partial A}\frac{dA}{dt}$$

$$=> \frac{dC}{dt} = w_1C(\gamma - \beta CA - \alpha C) + w_2A(\beta CA - \theta)$$

$$=> \frac{dC}{dt} = 0$$

The model is therefore globally unstable at the equilibrium point.

Magic Agent Number (MAN)

Recruitment process of agent will be going on if

$\frac{dA}{dt} > 0$

$=> \beta CA - \theta > 0$

$=> \beta CA > \theta$

$=> \frac{\beta CA}{\theta} > 1$

Therefore, MAN is: $M_o = \frac{\beta CA}{\theta}$

Sensitivity Analysis

The sensitivity of the MAN with respect to the state variables and the parameters can be studied by using a sensitivity index as [9, 10]:

$$\frac{\partial M_0}{\partial P}\frac{P}{M_0}$$

1. $\frac{\partial M_0}{\partial \beta}\frac{\beta\theta}{\beta CA} = \frac{CA}{\theta}\frac{\beta\theta}{\beta CA} = 1$ indicates that MAN is directly proportionate to the change of the contact rate

2. $\frac{\partial M_0}{\partial \theta}\frac{\theta 2}{\beta CA} = -\frac{\beta CA}{\theta 2}\frac{\theta 2}{\beta CA} = -1$ indicates that MAN is inversely proportionate to the change of the leaving rate
3. $\frac{\partial M_0}{\partial C}\frac{C\theta}{\beta CA} = \frac{\beta A}{\theta}\frac{C\theta}{\beta CA} = 1$ indicates that MAN is directly proportionate to the change of the general people
4. $\frac{\partial M_0}{\partial A}\frac{A\theta}{\beta CA} = 1$ indicates that MAN is directly proportionate to the change of the number of agents.

Simulation

Using least square method for the proposed system (1) in Python, we get the parameters as: γ = 1.13972039, β = 0.74613238, α = 1.44803635, θ = 0.80162429. Taking initial general population as 500 and number of agents as 2 at time t = 0 in Python, we get the solution of the system as:

t = 0.010081608050310192, C = 542.1857249222156, A = 9.7137623802023
t = 0.019909321492515, C = 546.7477394536326, A = 53.78898937423729
t = 0.02959277595007541, C = 400.8567935129905, A = 247.61440598745315
t = 0.03929221335463588, C = 137.57055582356782, A = 558.9589131207713
t = 0.043261156815266336, C = 77.83408627761493, A = 638.3730232777103
t = 0.04723010027589679, C = 46.744300411633105, A = 689.1439713289428

and so on. This can be depicted in Fig 1 as follows:

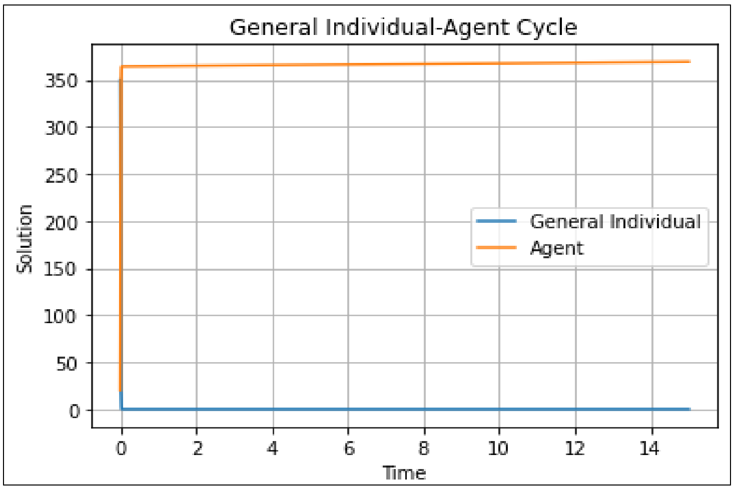

Fig 1: Cycle of General Individual and Agent

Fig 1 shows that in case of the proposed model, almost all the general individual in the present scenario become the agent of the business and as a result, the number of general individuals becomes nearing to zero.

The stability of the model is depicted with the help of a phase portrait in Figure 2 as follows:

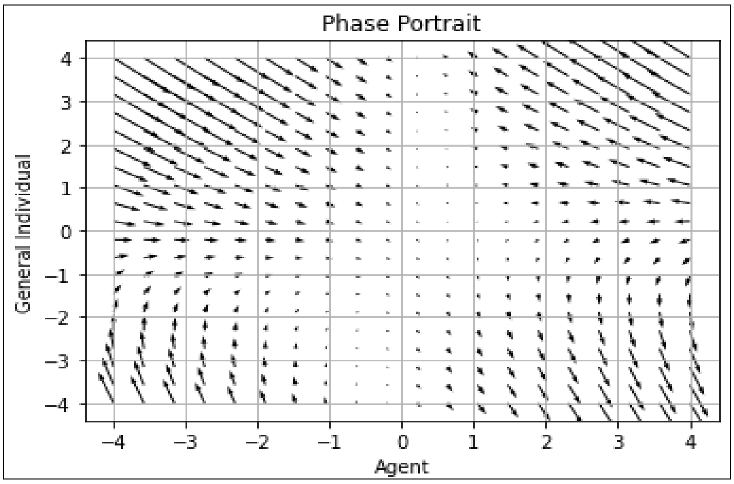

Fig 2: Depicts that the model is asymptotically stable towards the equilibrium point

Conclusions

In this paper, a mathematical modelling of multi-level marketing is proposed under the platform of famous SI model coined by Kermack-McKendrick in the year 1927. With the incorporation of least square method in Python, parameters are obtained as: γ = 1.13972039, β = 0.74613238, α = 1.44803635, θ = 0.80162429. The equilibrium point is: E (0.23348592, 4.60194547). Magic Agent Number (MAN) at the equilibrium is M_o = 1.0001. This shows the business is growing well. Local stability condition of the system (1) around the equilibrium point is found to be γ < θ. The proposed model in not globally stable around the equilibrium point. Sensitivity analysis claims that the MAN is directly proportionate to growth rate, number of General Individuals and Agent and inversely proportionate to the constant rate that the agents leave the profession. Phase portrait indicates asymptotic stability.

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