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Mathematical Overview of the Volatility of Stock Market with Special Reference to the Closing Prices of Bandhan Bank, Vodafone-Idea and GMR Airport

[1] Pratyakshi Sarma, [2] Bapan Kalita
[1, 2] The Assam Royal Global University Guwahati, Assam, India

Corresponding Author: **Bapan Kalita**

Abstract
Price of stock market is volatile. This leads to the difficulty to find the growth rate of stock price in the market. Here, we collected prices of three stocks viz., Bandhan Bank, Vodafone-Idea and GMR Airport for a period of 28 days right from 3 April 2023 to 11 May 2023. First, we prepare a mathematical model on this data and check the stability of the model. Then we develop a data-driven second-degree parabola model on this data and verified the goodness of fit of the model. A drift-diffusion model is fitted on the collected data and the same is compared with the data obtained from the fitted model by using a Lorenz curve. Price of Bandhan Bank shows most deviation whereas the other two prices are almost similar. The price of Vodafone Idea is most consistent.

Keywords: Volatility, Stock Market, Mathematical Modelling, Data-Driven Model, Drift-Diffusion Model

Introduction

As of today, Bandhan Bank has 39.99%, Vodafone-Idea has 10.91% and GMR Airport has 59% shareholding as promoters in the capital market of India.

A stock market is a venue where buyers and sellers can exchange stocks and other types of assets. On this stock market, shares of publicly traded companies can be bought and sold [1]. Companies issue shares or stocks on the stock market to raise capital for a range of purposes, including business expansion, funding brand-new projects, or debt repayment. After then, anyone can purchase these shares. Investors can buy and sell these shares both individually and collectively on the stock market. Through the stock market, investors can take part in the ownership and potential profits of publicly traded enterprises. Investors that buy stock in a corporation are known as shareholders or stockholders and hold a stake in the business. Shareholders may profit from the company's performance through capital appreciation (an increase in the value of the stock) and dividends (a distribution of a portion of the company's revenues to shareholders).

Stock markets have a big impact on the economy because they make it easier for investors and companies to transfer money. Investors can put their money in them with the chance of earning returns, and businesses can utilise them to get financing for growth and expansion. The stock market also serves as a barometer of the state of the economy because the performance of stock indices is commonly used as a reflection of overall market sentiment and economic circumstances. Stock markets are managed by exchanges, which are centralised trading platforms. Several well-known stock exchanges are the New York Stock Exchange (NYSE), the London Stock Exchange (LSE), and the Tokyo Stock Exchange (TSE). Traditional exchanges, electronic trading platforms, and alternative trading techniques all help to make stock trading easier.

Bandhan Bank

A well-known private sector bank in India is called Bandhan Bank. It was established in 2014, and Kolkata, West Bengal, serves as its headquarters. Bandhan Bank's main focus is on providing banking and financial services to underserved and unbanked segments of society, such as low-income individuals, micro and small companies, and self-help organisations. The bank's initial customer was a microfinance organisation (MFI) called Bandhan Financial Services Limited in 2001 [2]. As a non-banking financial institution (NBFI), it provided microloans to support women's economic independence and fight poverty. The company eventually obtained a banking licence from the Reserve Bank of India (RBI) in 2014, and as a result of the gradual growth of its operations, it changed its name to Bandhan Bank. Bandhan Bank offers a wide range of banking services to its clients, including savings and current accounts, fixed deposits, loans (including microfinance loans), debit and credit cards, insurance products, and other financial services. The bank can operate thanks to a network of ATMs and branches scattered throughout numerous Indian states. In addition to extending its business beyond microfinance, Bandhan Bank

recently diversified its lending portfolio to include small business loans, affordable housing loans, personal loans, and other retail banking products. The bank has expanded tremendously since its inception and has become a dominant force in India's financial sector.

Vodafone-Idea

Vodafone Idea Limited, often known as Vodafone Idea or Vi, is one of the major telecom companies in India. It is a collaboration between the British multinational telecoms company Vodafone Group Plc and the Indian behemoth Aditya Birla Group [3].

Vodafone India and Idea Cellular merged to form Vodafone Idea in August 2018. The merger aimed to create a stronger organisation in the fiercely competitive Indian telecom industry. The company operates in all 22 telecom circles in India and provides a broad range of services, including mobile phone, wireless internet, and enterprise solutions. Customers of Vodafone Idea can choose between prepaid and post-paid mobile services. It provides voice and data services, value-added services, and a range of price alternatives to cater to various customer segments. The company has a huge customer base and competes with other significant Indian telecom carriers. Like other cellular firms, Vodafone Idea has faced challenges and severe competition in the Indian telecom industry. The industry has experienced substantial changes and disruptions due to the entry of new competitors, changes in legislation, and pricing wars. The sales and profitability of telecom providers have been under pressure as a result of these factors.

GMR Airport

GMR Airport is a reference to the GMR Group's involvement in the airport sector. The infrastructure firm GMR Group, which has its corporate headquarters in India [5], represents a wide range of industries, including airports, energy, transportation, and urban infrastructure [4]. The GMR Group holds a significant amount of market share in the airport sector through its subsidiary, GMR Airports Limited. The company specialised in building, running, and managing airports. The modernization and expansion of airport infrastructure have been considerably aided by the efforts of GMR Airports on numerous airport projects in India and abroad. The GMR Group has experience in building and managing airports, including passenger terminals, runways, airside and landside infrastructure, retail and commercial areas and other airport facilities. The company is dedicated to offering smooth, customer-focused airport services while upholding the highest standards of effectiveness, security, and safety worldwide.

Bae *et al*. provide a mathematical model for stock market volatility flocking is presented here. In our suggested model, Cucker-Smale (C-S) flocking and regime switching mechanisms are combined with geometric Brownian movements with time-varying volatilities. They show that the common volatility emerges asymptotically and describe its financial applications for all-to-all interactions where we assume that all assets' volatilities are tied to each other with a constant interaction weight. They also offer a number of numerical simulations and contrast them with earlier analytical findings [5].

In the study Amadi *et al*.[6], the Mean Square Error (MSE) was used as a selection criterion to compare several approaches for estimating the Weibull distribution's parameters. Other methods couldn't compare to The Method of Moments. In the same situation, the estimated findings were logically expanded to construct a matrix by investigating the characteristics of the fundamental matrix solution, where we were able to anticipate stock prices and asset returns for a 12-month period. Finally, a theorem derived from the fundamental matrix system was demonstrated to show various levels of changes that affect the stock market in the short- and long-term, respectively.

M. Alisen and H. Merdan use a new excess demand function established by Caginalp for a market including more information on demand and supply for a stock than their values at a particular price, a system of ordinary differential equations is derived and used to study asset price movements. In addition to investment methods that are based on both price momentum (trend) and valuation considerations, derivation is based on the finiteness of assets (rather than assuming unbounded arbitrage). Numerical simulations are used to compare the time evolutions of asset prices for this novel model and the earlier models that Caginalp and Balenovich retrieved using the traditional excess demand function [7].

Caginalp and Ermentrout, G.B. propose a model that has been put forth to describe the distinctive and psychological features of financial markets in a pure setting is numerically computed. These computations show overreactions, volatility, and convergence to actual values. One can get a spectrum of patterns, from efficient to chaotic markets, by adjusting parameters related to either emotional or intellectual incentives [8].

Caginalp and Ermentrout, G.B. arrive to a set of equations that serve as a straightforward representation of investor behaviour in a hypothetical financial market. The model takes into account the emotional component of investor sentiment as well as the exponentially decaying recall of past price movements. According to this hypothesis, the body of investors will buy when prices have recently increased and sell when prices have recently decreased. With the potential for some inertia in taking action, the rational reasons are centred on capitalising on the discrepancy between the price and intrinsic value. Fluctuations and instability are based on these two antagonistic effects [9].

Materials and methods

Materials

To access a mathematical overview of the volatility of the stock market prices of the three stocks, we have collected the closing prices for a period of 28 days from 3 April 2023 to 11 May 2023. These prices are visualized in Fig 1, Fig 2 and Fig 3 respectively for Bandhan Bank, Vodafone-Idea and GMR Airport.

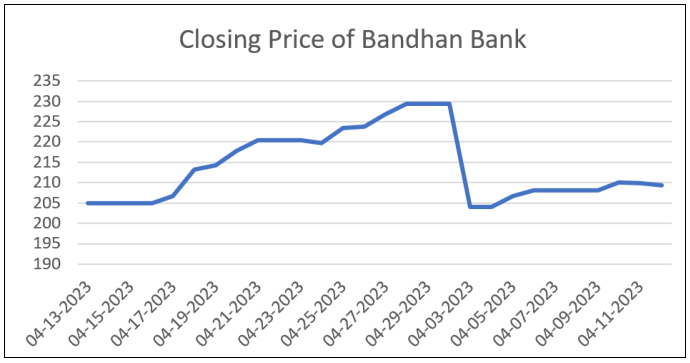

Fig 1: Closing Price of Bandhan Bank

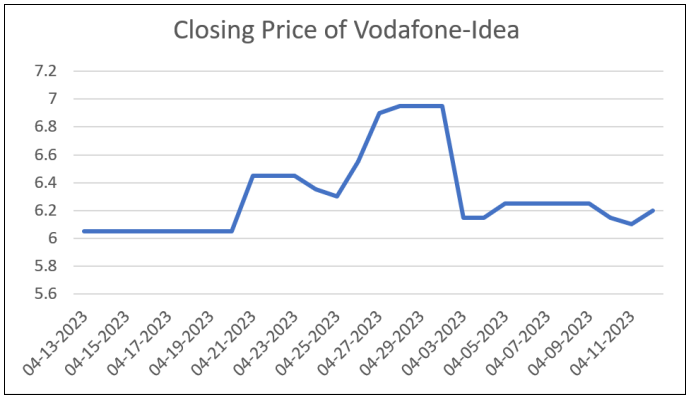

Fig 2: Closing price of Vodafone-Idea

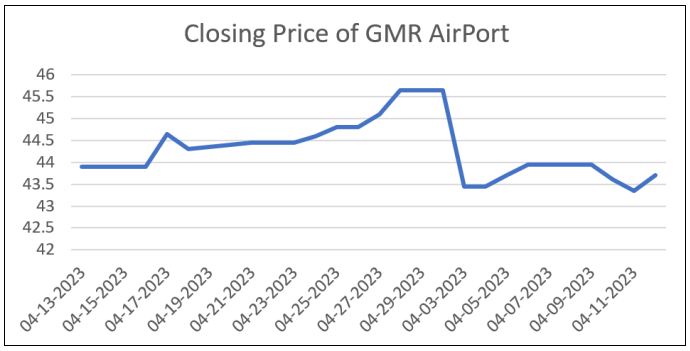

Fig 3: Closing Price of GMR Airport

Methods

The methods incorporated here are:

1. Mathematical modelling
2. Data-driven model
3. Goodness of fit
4. Drift-diffusion model
5. Lorenz curve

Mathematical Model

The mathematical model [10] incoroporated here is:

$$\left.\begin{aligned}\frac{dx_1}{dt} &= ax_1 - \theta \\ \frac{dx_2}{dt} &= bx_2 - \gamma \\ \frac{dx_3}{dt} &= cx_3 - \delta\end{aligned}\right\} \quad (1)$$

Where:

a is rate of change of price of Bandhan Bank per unit time
b is rate of change of price of Vodafone idea per unit time
c is rate of change of price of GMR Airport per unit time

θ, γ and δ are constant deduction of prices of Bandhan Bank, Vodafone idea and GMR Airport.
x_1 (t) is the state variable of closing price of Bandhan Bank
x_2(t) is the state variable of closing price of Vodafone-Idea
x_3(t) is the state variable of closing price of GMR Airport

Data-Driven Model
Using the Data Driven Modelling technique, the configurator model components are dynamically fed into the model based on data gathered from other systems such as catalogue systems, Customer Relationship Management (CRM), Watson, and others. Here, a second degree parabola is configured out from the data collected in the study [11].

Goodness of fit
Using the observed data and the data expected from the data-driven model are tested fro goodness of fit with help of Chi-square test [12].

Drift-Diffusion Model
To study the volatility of the prices, a drift-diffusion model [13] from the concept of Brownian motion is typically adopted as:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \quad (2)$$

Here, S(t) represents the price of the variable at time t, μ represents the drift rate, σ represents the volatility, dt represents an infinitesimal time increment, and dW(t) represents a Wiener process (Brownian motion) increment.

Lorenz Curve
Using the observed data and the expected data obtained from Data Driven Modelling technique, Lorenz curve [14] is drwan to see the deviations of observed and the expected data.

Coefficient of Variation
To understand the consistency of the price in both observed and the expected environment, the coefficient of variation(C.V.) [15] is implemented as:

$$\text{C.V.} = \frac{\text{Volatility rate}}{\text{Drift rate}} \times 100\%$$

Results and Discussion
Positivity
Solving the differential equations of system (1), we get x_1 (t), x_2 (t), x_3 (t), > 0

Equilibrium Point
Equating the equations of the system (1), we get

The equilibrium point as: $E(\frac{\theta}{a}, \frac{\gamma}{b}, \frac{\delta}{c})$

Local Stability
Jacobian of system (1) is

$$J = \begin{vmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{vmatrix} = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix}$$

Now,

$$|J - \lambda I| = 0$$

$$= \begin{vmatrix} a-\lambda & 0 & 0 \\ 0 & b-\lambda & 0 \\ 0 & 0 & c-\lambda \end{vmatrix} = 0$$

$= \lambda = a, b, c$

Hence, the model is locally stable under the condition a, b, c < 0.

Global Stability

Let us assume a Lyapunov function

$$L(W) = \frac{1}{2} w_1 x_1^2 + \frac{1}{2} w_2 x_2^2 + \frac{1}{2} w_3 x_3^2 \text{ where, } w_1, w_2 \text{ and } w_3 \text{ are non} - \text{negative numbers}$$

Now,

$$\frac{dL}{dt} = \frac{\partial L}{\partial x_1}.\frac{dx_1}{dt} + \frac{\partial L}{\partial x_2}.\frac{dx_2}{dt} + \frac{\partial L}{\partial x_3}.\frac{dx_3}{dt}$$

$$= w_1 x_1(a^{x_1} - \theta) + w_2 x_2(b^{x_2} - \gamma) + w_3 x_3(c^{x_3} - \delta)$$

$$= w_1 \frac{\theta}{a}(a^{\frac{\theta}{a}} - \theta) + w_2 \frac{\gamma}{b}(b^{\frac{\gamma}{b}} - \gamma) + w_3 \frac{\delta}{c}(c^{\frac{\delta}{c}} - \delta)$$

$$= 0$$

The model is therefore globally unstable at the equilibrium point.

Data-Driven Model

Assuming a second-degree polynomial we fit the data for the closing price of Bandhan bank, Vodafone idea and GMR Airport in Matlab 2016a. The fitted data driven model are:

Second degree polynomial for closing price of Bandhan Bank is

$$y(X_1) = C_1 X_1^2 + B_1 X_1 + A_1$$

Coefficients (with 95% confidence bounds):

C_1 = 0.04552 (0.0266, 0.06444),

B_1 = -0.2794 (-0.8082, 0.2493),

A_1 = 206.5 (203.4, 209.6)

The second-degree polynomial for closing price of Vodafone Idea is:

$$y(X_2) = C_2 X_2^2 + B_2 X_2 + A_2$$

Coefficients (with 95% confidence bounds):

C_2 = 0.003085 (0.002252, 0.003919),

B_2 = -0.065 (-0.08991, -0.0401),

A_2 = 6.403 (6.247, 6.56)

The second-degree polynomial for closing price of GMR Airport is:

$$y(X_3) = C_3 X_3^2 + B_3 X_3 + A_3$$

Coefficients (with 95% confidence bounds):

C_3 = 0.003217 (0.001723, 0.004712),

B_3 = -0.02178 (-0.06643, 0.02288),

A_3 = 43.71 (43.43, 43.99)

Goodness of Fit

Applying Chi-square test of goodness of fit in Matlab 2016a, we have h = 0 for each of the curves. This shows that the Chi-square tests supports the goodness of fit of the data-driven model.

Drift-Diffusion Model

For observed closing price of:

Bandhan Bank: Drift rate (μ) = 0.0043, Volatility rate = 0.009358.

ii) Vodafone Idea: Drift rate (μ) = 0.0097, Volatility rate = 0.016257

iii) GMR Airport: Drift rate (μ) = 0.0018, Volatility rate = 0.005192

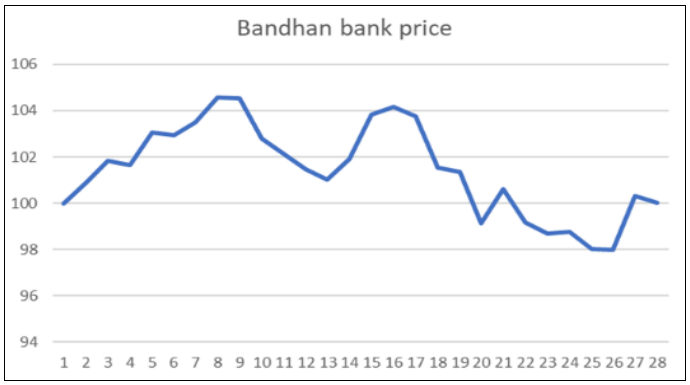

Fig 4: Bandhan Bank (O)

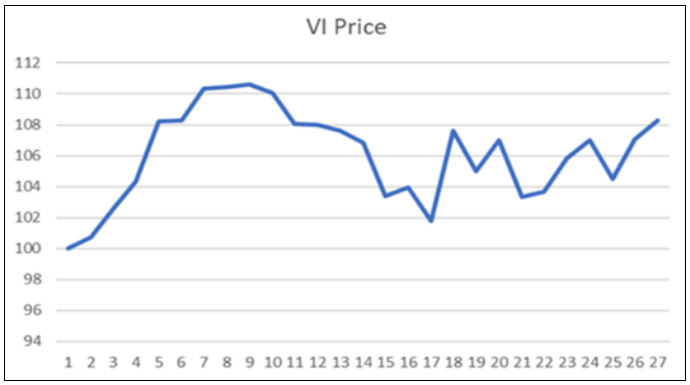

Fig 5: Vodafone-Idea (O)

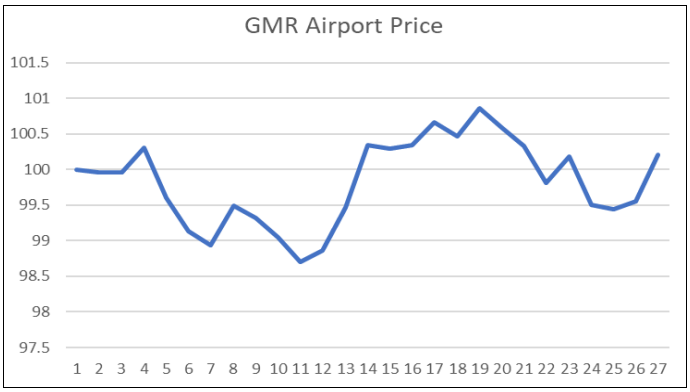

Fig 6: GMR Airport (O)

For expected closing price of

1. Bandhan Bank: Drift rate (μ) = 0.00778, Volatility rate = 0.016804.
2. Vodafone Idea: Drift rate (μ) = 0.00008949, Volatility rate = 0.00038314
3. GMR Airport: Drift rate (μ) = 0.000447, Volatility rate = 0.001263

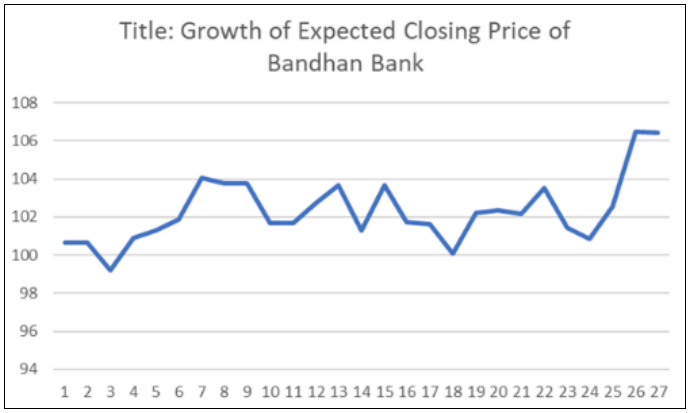

Fig 7: Bandhan Bank (O)

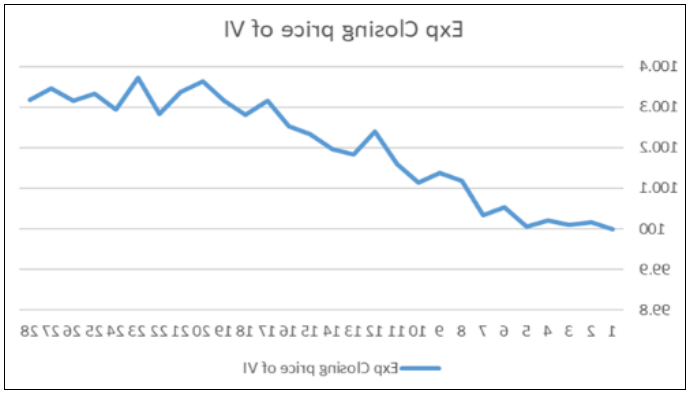

Fig 8: Vodafone-Idea (O)

Fig 9: GMR Airport (O)

Lorenz Curve

The deviations of the various observed growth rates of the prices vs expected growth prices extracted out of the data-driven model are studied with the Lorenz curve and these are depicted in the following figures.

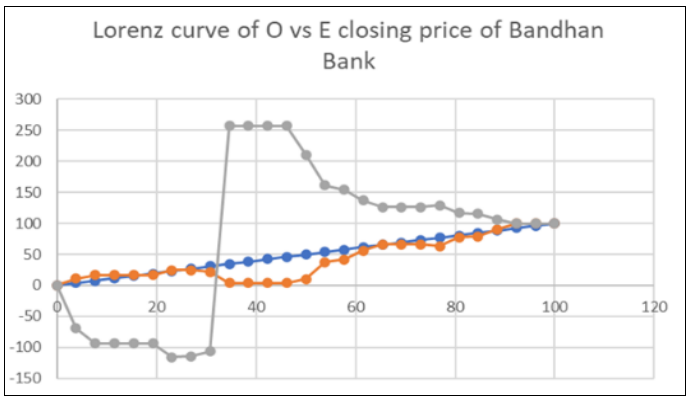

Fig10: Bandhan Bank (O vs E)

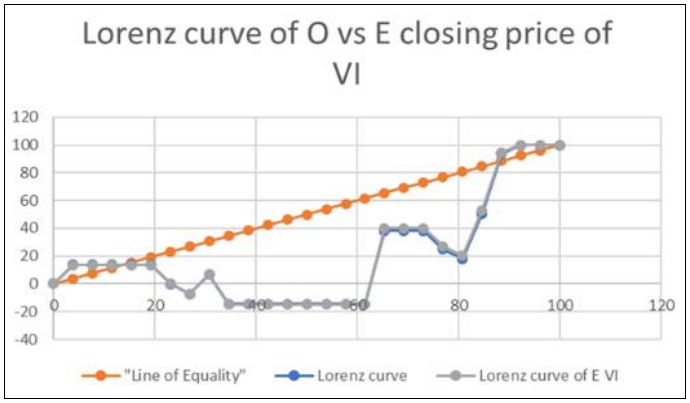

Fig 11: Vodafone-Idea (O vs E)

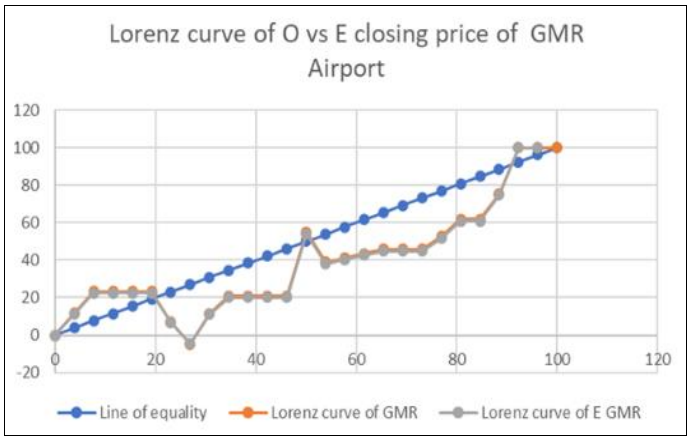

Fig 12: GMR Airport (O vs E)

Fig 10 shows that the observed growth rate of the prices of Bandhan Bank is nearing to the line of equality, whereas Fig 11 and Fig 12 show that observed and expected growth rates of Vodafone-Idea and GMR Airport are similar in nature.

Comparison of consistency:

- C.V.(Bandhan Bank) = 219.66%; C.V.(VI) =166.79%; C.V.(GMR Airport) =286.20%;
- C.V.(E-Bandhan Bank) = 215.99%; C.V.(E-VI) = 428.11%; C.V.(E-GMR Airport) = 282.69%.

It is found that during the period of data collection, the closing price of Vodafone Idea is most consistent whereas the closing price of GMR Airport is most deviated. From the C.V of the expected closing prices of Bandhan bank, Vodafone Idea and GMR Airport, it is found that during the period of data collection, the closing price of Bandhan Bank is most consistent whereas the closing price of Vodafone Idea is most deviated.

Conclusions

In this study, we tried to compare the volatility of stock market prices of three companies. Here, we collected prices of three stocks viz., Bandhan Bank, Vodafone-Idea and GMR Airport for a period of 28 days right from 3 April 2023 to 11 May 2023. First, we prepared a mathematical model on this data and check the stability of the model. Positivity of the model is checked. Equilibrium point is: $E(\frac{\theta}{a}, \frac{\gamma}{b}, \frac{\delta}{c})$. The model is found locally stable under the condition a, b, c < 0. This model is globally unstable. A data-driven model is sought and a second-degree polynomial for each of the prices are derived. Data-driven model is:

$$y(X_1) = 0.04552\,X_1^2 - 0.2794\,X_1 + 206.5$$

$$Y(X_2) = 0.003085\,X_2^2 - 0.065\,X_2 + 6.403$$

$$Y(X_3) = 0.003217\,X_3^2 - 0.02178\,X_3 + 43.71$$

Chi-square test conducted reflects the goodness of fit of each of the data-driven model. A drift-diffusion model is fitted on the collected data and the same is compared with the data obtained from the fitted model by using a Lorenz curve. Price of Bandhan Bank shows most deviation whereas the other two prices are almost similar. The price of Vodafone Idea is most consistent

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