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The correlation between CL(2) multiwavelet shrinkage and second-order nonlinear diffusion equation in multi-level

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Abstract
In many investigations, multi-wavelets outperformed scalar wavelets in decreasing signal and image noise. Multi-wavelet conversion has also been used in signal processing to reduce noise. This is due to the fact that multi-wavelet has crucial qualities such as orthogonality, short support, and symmetry, whereas scalar wavelet does not have such properties at the same time. In addition, the multi-wavelet is a stunning and modern contribution to the wavelet theory. The goal of this work is to discover the relationship between CL(2) multiwavelet shrinkage and second-order nonlinear diffusion equation in multi-level. The results presented here can help to improve the use of wavelets in communication applications.

Keywords: CL(2) Multiwavelet Shrinkage, Wavelet Theory, Second-Order Nonlinear Diffusion Equation Communication Applications

1. Introduction
Wavelets are highly useful in image and signal processing [1, 2]. Several studies have found that multi-wavelets outperform scalar wavelets in terms of signal and image noise reduction [3, 4]. Before using the discrete multiple wavelets transform in multiple wavelets, data must be handled. This is one of the main differences between them and single wavelets [5]. Recent studies have shown that multivariate shrinkage on multi-wavelet transform coefficients further enhances the performance of conventional wavelet methods. This is due to the fact that the multiwavelet transforms, when properly initialized, gives a superior representation of signals, enabling a clear distinction between them and noise.
Nonlinear diffusion filtering, on the other hand, can efficiently perform signal and image. Recently, nonlinear diffusion filters have become a powerful and trustworthy technique for multiscale image processing. Actually, various nonlinear diffusion filters have been proposed since Perona and Malik [6] described the nonlinear diffusion [7, 8]. Alkhidhr [9] demonstrate that the multiwavelet shrinkages of the widely used CL(2) and DGHM multiwavelets are related to a second-order nonlinear diffusion equation. Further, nonlinear diffusion filtering has interesting theoretical properties, such as well-posedness results, average grey value invariance, the maximum-minimum principle, and the presence of Lyapunov functionals, which are associated with and support the excellent quality of the produced images [10].
In the ongoing work, we demonstrate the relationship between CL(2) multiwavelet shrinkage and second-order nonlinear diffusion equation in multi-level. Namely, we discover demonstrate that a second-order nonlinear diffusion equation is connected to the multiwavelet shrinkages of the widely used CL(2) and DGHM multiwavelets. In fact, this correspondence opens the door to several crucial and more suitable applications in computer vision software, like shape from shading [8]. Furthermore, the majority of the publications are concerned with modeling rather than numerical implementation.
This study is arranged as follows. Section 2 presents the CL(2) multiwavelet shrinkage in multi-level. Section 3 displays the second-order nonlinear diffusion equation in a multiwavelet shrinkage field in multi-level. Finally, the conclusions are introduced in Section 4.

2. Chui-Lian CL(2) multiwavelet shrinkage in multi-level
Consider H and G be the orthogonal multifilters with nonzero H_k, G_k given by [11]:

be the Chui-Lian \$CL(2) multifilter bank, and $E = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$ is an orthogonal matrix. Then,

$$A_{-1} = EH_{-1}E^{-1} = \frac{1}{40}\begin{bmatrix} 6+\sqrt{7} & 12+2\sqrt{7} \\ 2-3\sqrt{7} & 4-6\sqrt{7} \end{bmatrix}, \quad A_0 = EH_0E^{-1} = \frac{1}{40}\begin{bmatrix} 19 & 3 \\ 3 & 11 \end{bmatrix}$$

$$A_1 = \frac{1}{40}\begin{bmatrix} 12-2\sqrt{7} & -6+\sqrt{7} \\ 4+6\sqrt{7} & -2-3\sqrt{7} \end{bmatrix}$$

$$B_{-1} = \frac{1}{40}\begin{bmatrix} -7 & -14 \\ 1 & 2 \end{bmatrix}, \quad B_0 = \frac{1}{40}\begin{bmatrix} 18+\sqrt{7} & 6-3\sqrt{7} \\ 6-3\sqrt{7} & 2+9\sqrt{7} \end{bmatrix}$$

$$B_1 = \frac{1}{40}\begin{bmatrix} -10 & 5 \\ -10 & 5 \end{bmatrix}.$$

Consider a signal that is provided by $U_q = \begin{bmatrix} u_{2q} \\ u_{2q+1} \end{bmatrix}$. With Chui-Lian \$CL(2)\$ multiwavelet coefficients, the shift-invariant multi-level multiwavelet decomposition algorithms are given by:

$$M_n^j \begin{bmatrix} M_{1,n}^j \\ M_{2,n}^j \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 6+\sqrt{7} & 12+2\sqrt{7} \\ 2-3\sqrt{7} & 4-6\sqrt{7} \end{bmatrix}\begin{bmatrix} u_{2n-2} \\ u_{2n-1} \end{bmatrix} + \frac{1}{40}\begin{bmatrix} 19 & 3 \\ 3 & 11 \end{bmatrix}\begin{bmatrix} u_{2n} \\ u_{2n+1} \end{bmatrix} + \frac{1}{40}\begin{bmatrix} 12-2\sqrt{7} & -6+\sqrt{7} \\ 4+6\sqrt{7} & -2-3\sqrt{7} \end{bmatrix}\begin{bmatrix} u_{2n+2} \\ u_{2n+3} \end{bmatrix}$$

$$\begin{bmatrix} M_{1,n}^j \\ M_{2,n}^j \end{bmatrix} = \frac{1}{40}\begin{bmatrix} (6+\sqrt{7})u_{2n-2} + (12+2\sqrt{7})u_{2n-1} + 19u_{2n} \\ +3u_{2n+1} + (12-2\sqrt{7})u_{2n+2} + (-6+\sqrt{7})u_{2n+3} \\ (2-3\sqrt{7})u_{2n-2} + (4-6\sqrt{7})u_{2n-1} + 3u_{2n} \\ +11u_{2n+1} + (4+6\sqrt{7})u_{2n+2} + (-2-3\sqrt{7})u_{2n+3} \end{bmatrix}$$

$$N_n^j = \begin{bmatrix} N_{1,n}^j \\ N_{2,n}^j \end{bmatrix} = \frac{1}{40}\begin{bmatrix} -7 & -14 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} u_{2n-2} \\ u_{2n-1} \end{bmatrix} + \frac{1}{40}\begin{bmatrix} 18+\sqrt{7} & 6-3\sqrt{7} \\ 6-3\sqrt{7} & 2+9\sqrt{7} \end{bmatrix}\begin{bmatrix} u_{2n} \\ u_{2n+1} \end{bmatrix}$$

$$\begin{bmatrix} N_{1,n}^j \\ N_{2,n}^j \end{bmatrix} = \frac{1}{40}\begin{bmatrix} -7u_{2n-2} - 14u_{2n-1} + (18+\sqrt{7})u_{2n} \\ +(6-3\sqrt{7})u_{2n+1} - 10u_{2n+2} + 5u_{2n+3} \\ u_{2n-2} + 2u_{2n-1} + (6-3\sqrt{7})u_{2n} \\ +(2+9\sqrt{7})u_{2n+1} - 10u_{2n+2} + 5u_{2n+3} \end{bmatrix}$$

and the Shrunk data id defined as:

$$\begin{bmatrix} c_{1,q} \\ c_{2,q} \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 6+\sqrt{7} & 2-3\sqrt{7} \\ 12+2\sqrt{7} & 4-6\sqrt{7} \end{bmatrix}\begin{bmatrix} M_{1,q-n}^j \\ M_{2,q-n}^j \end{bmatrix} + \frac{1}{40}\begin{bmatrix} 19 & 3 \\ 3 & 11 \end{bmatrix}\begin{bmatrix} M_{1,q-n}^j \\ M_{2,q-n}^j \end{bmatrix}$$
$$+ \frac{1}{40}\begin{bmatrix} 12-2\sqrt{7} & 4+6\sqrt{7} \\ -6+\sqrt{7} & -2-3\sqrt{7} \end{bmatrix}\begin{bmatrix} M_{1,q-n}^j \\ M_{2,q-n}^j \end{bmatrix}$$
$$+ \frac{1}{40}\begin{bmatrix} -7 & 1 \\ -14 & 2 \end{bmatrix}\begin{bmatrix} D_{11}\left(N_{1,q-n}^j\right) + D_{21}\left(N_{2,q-n}^j\right) \\ D_{12}\left(N_{1,q-n}^j\right) + D_{22}^j\left(N_{2,q-n}^j\right) \end{bmatrix}$$
$$+ \frac{1}{40}\begin{bmatrix} 18+\sqrt{7} & 6-3\sqrt{7} \\ 6-3\sqrt{7} & 2+9\sqrt{7} \end{bmatrix}\begin{bmatrix} D_{11}\left(N_{1,q-n}^j\right) + D_{21}\left(N_{2,q-n}^j\right) \\ D_{21}\left(N_{1,q-n}^j\right) + D_{22}\left(N_{2,q-n}^j\right) \end{bmatrix}$$
$$+ \frac{1}{40}\begin{bmatrix} -10 & -10 \\ 5 & 5 \end{bmatrix}\begin{bmatrix} D_{11}\left(N_{1,q-n}^j + D_{21}\left(N_{2,q-n}^j\right)\right. \\ D_{12}\left(N_{1,q-n}^j\right) + D_{22}\left(N_{2,q-n}^j\right) \end{bmatrix}.$$

$$\begin{bmatrix} c_{1,q} \\ c_{2,q} \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 6+\sqrt{7} & 2-3\sqrt{7} \\ 12+2\sqrt{7} & 4-6\sqrt{7} \end{bmatrix}\begin{bmatrix} M_{1,q-n}^j \\ M_{2,q-n}^j \end{bmatrix} + \frac{1}{40}\begin{bmatrix} 19 & 3 \\ 3 & 11 \end{bmatrix}\begin{bmatrix} M_{1,q-n}^j \\ M_{2,q-n}^j \end{bmatrix}$$
$$+ \frac{1}{40}\begin{bmatrix} 12-2\sqrt{7} & 4+6\sqrt{7} \\ -6+\sqrt{7} & -2-3\sqrt{7} \end{bmatrix}\begin{bmatrix} M_{1,q-n}^j \\ M_{2,q-n}^j \end{bmatrix}$$

$$
+\frac{1}{40}\begin{bmatrix}-7 & 1\\ -14 & 2\end{bmatrix}\begin{bmatrix}D_{11}\left(N_{1,q-n}^{j}\right)+D_{21}\left(N_{2,q-n}^{j}\right)\\ D_{12}\left(N_{1,q-n}^{j}\right)+D_{22}^{j}\left(N_{2,q-n}^{j}\right)\end{bmatrix}
+\frac{1}{40}\begin{bmatrix}18+\sqrt{7} & 6-3\sqrt{7}\\ 6-3\sqrt{7} & 2+9\sqrt{7}\end{bmatrix}\begin{bmatrix}D_{11}\left(N_{1,q-n}^{j}\right)+D_{21}\left(N_{2,q-n}^{j}\right)\\ D_{21}\left(N_{1,q-n}^{j}\right)+D_{22}\left(N_{2,q-n}^{j}\right)\end{bmatrix}
+\frac{1}{40}\begin{bmatrix}-10 & -10\\ 5 & 5\end{bmatrix}\begin{bmatrix}D_{11}\left(N_{1,q-n}^{j}+D_{21}\left(N_{2,q-n}^{j}\right)\right.\\ D_{12}\left(N_{1,q-n}^{j}\right)+D_{22}\left(N_{2,q-n}^{j}\right)\end{bmatrix}.
$$

Hence,

$$
\begin{bmatrix}c_{1,q}\\ c_{2,q}\end{bmatrix}=\frac{1}{40}\begin{bmatrix}(6+\sqrt{7})M_{1,q+1}^{j}+(2-3\sqrt{7})M_{2,q+1}^{j}+19M_{1,q}^{j}\\ +3M_{2,q}^{j}+(12-2\sqrt{7})M_{1,q-1}^{j}+(4+6\sqrt{7})M_{2,q-1}^{j}\\ \\ (12+2\sqrt{7})M_{1,q+1}^{j}+(4-6\sqrt{7})M_{2,q+1}^{j}+3M_{1,q}^{j}\\ +11M_{2,q}^{j}+(-6+\sqrt{7})M_{1,q-1}^{j}+(-2-3\sqrt{7})M_{2,q-1}^{j}\end{bmatrix}
+\frac{1}{40}\begin{bmatrix}-7D_{11}\left(N_{1,q+1}^{j}\right)+D_{21}\left(N_{2,q+1}^{j}\right)+(18+\sqrt{7})D_{11}\left(N_{1,q}^{j}\right)\\ +(6-3\sqrt{7})D_{21}\left(N_{2,q}^{j}\right)-10D_{11}\left(N_{1,q-1}^{j}\right)-10D_{21}\left(N_{2,q-1}^{j}\right)\\ \\ -14D_{12}\left(N_{1,q+1}^{j}\right)+2D_{22}\left(N_{2,q+1}^{j}\right)+(6-3\sqrt{7})D_{12}\left(N_{1,q}^{j}\right)+5D_{12}\left(N_{1,q-1}^{j}\right)\\ +5D_{22}\left(N_{2,q-1}^{j}\right)+(2+9\sqrt{7})D_{22}\left(N_{2,q}^{j}\right)+5D_{12}\left(N_{1,q-1}^{j}\right)+5D_{22}\left(N_{2,q-1}^{j}\right)\end{bmatrix}.
$$

If there is no shrinkage apply, then

$$
\begin{aligned}
\begin{bmatrix}c_{1,q}\\ c_{2,q}\end{bmatrix}=\frac{1}{1600}&\begin{bmatrix}(320+30\sqrt{7})u_{2q-1}+(360-10\sqrt{7})u_{2q-2}-120u_{2q-3}-60u_{2q-4}+920u_{2q}\\ +90u_{2q+1}+(390-10\sqrt{7})u_{2q+2}+(-110-20\sqrt{7})u_{2q+3}+60u_{2q+4}+30u_{2q+5}\\ \\ (40-90)u_{2q-1}+(-80-20\sqrt{7})u_{2q-2}+60u_{2q-3}+30u_{2q-4}+90u_{2q}\\ +680u_{2q+1}+(320+80)u_{2q+2}+(40-9\sqrt{7})u_{2q+3}-120u_{2q+4}+60u_{2q+5}\end{bmatrix}\\
+\frac{1}{1600}&\begin{bmatrix}-7\left(-7u_{2q}-14u_{2q+1}+(18+\sqrt{7})u_{2q+2}+(6-3\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)\\ \\ -14\left(-7u_{2q}-14u_{2q+1}+(18+\sqrt{7})u_{2q+2}+(6-3\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)\end{bmatrix}\\
+\frac{1}{1600}&\begin{bmatrix}\left(u_{2q}+2u_{2q+1}+(6-3\sqrt{7})u_{2q+2}+(2+9\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)\\ \\ 2\left(u_{2q}+2u_{2q+1}+(6-3\sqrt{7})u_{2q+2}+(2+9\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)\end{bmatrix}\\
+\frac{1}{1600}&\begin{bmatrix}(18+\sqrt{7})\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)\\ \\ (6-3\sqrt{7})\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)\end{bmatrix}\\
+\frac{1}{1600}&\begin{bmatrix}(6-3\sqrt{7})\left(u_{2q-2}+2u_{2q-1}+(6-3\sqrt{7})u_{2q}+(2+9\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)\\ \\ (2+9\sqrt{7})\left(u_{2q-2}+2u_{2q-1}+(6-3\sqrt{7})u_{2q}+(2+9\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)\end{bmatrix}\\
+\frac{1}{1600}&\begin{bmatrix}-10\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)\\ \\ 5\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)\end{bmatrix}\\
+\frac{1}{1600}&\begin{bmatrix}-10\left(u_{2q-4}+2u_{2q-3}+(6-3\sqrt{7})u_{2q-2}+(2+9\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right)\\ \\ 5\left(u_{2q-4}+2u_{2q-3}+(6-3\sqrt{7})u_{2q-2}+(2+9\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right)\end{bmatrix}
\end{aligned}
$$

$$
\text{i.e.,}\quad \begin{bmatrix}c_{1,q}\\ c_{2,q}\end{bmatrix}=\begin{bmatrix}u_{2q}\\ u_{2q+1}\end{bmatrix}
$$

3. The second-order nonlinear diffusion equation in a multiwavelet shrinkage field in multi-level

The second-order non linear diffusion formula of *c(x, t)* with noise is given by:

$$
c_t=\frac{\sigma}{\partial x}\left\{g\left(c_x^2\right)c_x\right\}
$$

with original condition *c(x, 0) = f(x)*. The approximation $\frac{\partial}{\partial x}$ c(x, t) at (2 q s, r τ) is:

$$
c_x\approx\frac{-7c_{2q}^{r}-14c_{2q+1}^{r}+(18+\sqrt{7})c_{2q+2}^{r}+(6-3\sqrt{7})c_{2q+3}^{r}-10c_{2q+4}^{r}+5c_{2q+5}^{r}}{(28+\sqrt{x})s}
$$

and the approximation of $\frac{\partial}{\partial x}$ c(x, t) at (2 [q+1] s, r τ) is:

$$c_x \approx \frac{c^r_{2q} + 2c^r_{2q+1} + (6-3\sqrt{7})c^r_{2q+2} + (2+9\sqrt{7})c^r_{2q+3} - 10c^r_{2q+4} + 5c^r_{2q+5}}{(1-3\sqrt{7})s}.$$

Then the value of $\frac{\partial}{\partial x}$ c(x, t) at $U_q = \begin{bmatrix} (2qs, r\tau) \\ ([2q+1]2, r\tau) \end{bmatrix}$ by using high-pass filters can be written as:

$$\begin{aligned}
\frac{\partial}{\partial t}\begin{bmatrix} c(2qs, r\tau) \\ c([2q+1]s, r\tau) \end{bmatrix} &\approx \begin{bmatrix} \frac{40}{(28+\sqrt{7})s} & 0 \\ 0 & \frac{40}{(1-3\sqrt{7})s} \end{bmatrix} \sum B_n \begin{bmatrix} c(2(q+n)s, r\tau) \\ c([2(q+n)+1]s, r\tau) \end{bmatrix} \\
&= \begin{bmatrix} \frac{40}{(28+\sqrt{7})s} & 0 \\ 0 & \frac{40}{(1-3\sqrt{7})s} \end{bmatrix} \sum B_n \begin{bmatrix} c^r_{2(q+n)} \\ c^r_{2(q+n)+1} \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{(28+\sqrt{7})s} & 0 \\ 0 & \frac{1}{(1-3\sqrt{7})s} \end{bmatrix} \left\{ \begin{bmatrix} -7 & -14 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} c^r_{2q} \\ c^r_{2q+1} \end{bmatrix} \right. \\
&\quad \left. + \begin{bmatrix} 18+\sqrt{7} & 6-3\sqrt{7} \\ 6-3\sqrt{7} & 2+9\sqrt{7} \end{bmatrix} \begin{bmatrix} c^r_{2q+2} \\ c^r_{2q+3} \end{bmatrix} + \begin{bmatrix} -10 & 5 \\ -10 & 5 \end{bmatrix} \begin{bmatrix} c^r_{2q+4} \\ c^r_{2q+5} \end{bmatrix} \right\}
\end{aligned}$$

and the approximating partial derivatives of F(x, t)=g($(c_x)^2$)c_x at $\begin{bmatrix} (2qs, r\tau) \\ ([2q+1]2, r\tau) \end{bmatrix}$ is defined as:

$$\frac{\partial}{\partial x}\begin{bmatrix} F(2qs, r\tau) \\ F([2q+1]s, r\tau) \end{bmatrix} \approx \begin{bmatrix} \frac{40}{(16-3\sqrt{7})s} & 0 \\ 0 & \frac{40}{(-3+9\sqrt{7})s} \end{bmatrix} \sum B^T_m \begin{bmatrix} F(2(q,-m)s, r\tau) \\ F([2(q-m)+1]s, r\tau) \end{bmatrix}$$

Then the second-order nonlinear diffusion equation can be descretized as:

$$\begin{bmatrix} c^{r+1}_{2q} \\ c^{r+1}_{2q+1} \end{bmatrix} = \begin{bmatrix} c^r_{2q} \\ c^r_{2q+1} \end{bmatrix} + \begin{bmatrix} \frac{40\tau}{(16-3\sqrt{7})s} & 0 \\ 0 & \frac{40\tau}{(-3+9\sqrt{7})s} \end{bmatrix} \sum B^T_m$$

$$\times \begin{bmatrix} \frac{g}{(28+\sqrt{7})s}\left[\frac{\left(-7c^r_{2(q-m)} - 14c^r_{2(q-m)+1} + (18+\sqrt{7})c^r_{2(q-m)+2} + (6-3\sqrt{7})c^r_{2(q-m)+3} - 10c^r_{2(q-m)+4} + 5c_{2(q-m)+5}\right)^2}{(28+\sqrt{7})^2 s^2}\right] \\ \left(-7c^r_{2(q-m)} - 14c^r_{2(q-m)+1} + (18+\sqrt{7})c^r_{2(q-m)+2} + (6-3\sqrt{7})c^r_{2(q-m)+3} - 10c^r_{2(q-m)+4} + 5c_{2(q-m)+5}\right) \\ \frac{g}{(1-3\sqrt{7})}\left[\frac{\left(c^r_{2(q-m)} + 2c^r_{2(q-m)+1} + (6-3\sqrt{7})c^r_{2(q-m)+2} + (2+9\sqrt{7})c^r_{2(q-m)+3} - 10c^r_{2(q-m)+4} + 5c_{2(q-m)+5}\right)^2}{(1-3\sqrt{7})^2 s^2}\right] \\ \left(c^r_{2(q-m)} + 2c^r_{2(q-m)+1} + (6-3\sqrt{7})c^r_{2(q-m)+2} + (2+9\sqrt{7})c^r_{2(q-m)+3} - 10c^r_{2(q-m)+4} + 5c_{2(q-m)+5}\right) \end{bmatrix}$$

By applying Chui-Lian *cL*(2) coefficients:

$$\begin{bmatrix} c^{r+1}_{2q} \\ c^{r+1}_{2q+1} \end{bmatrix} = \begin{bmatrix} c^r_{2q} \\ c^r_{2q+1} \end{bmatrix} + \begin{bmatrix} \frac{\tau}{(16-3\sqrt{7})s} & 0 \\ 0 & \frac{\tau}{(-3+9\sqrt{7})s} \end{bmatrix} \left\{ \begin{bmatrix} -7 & 1 \\ -14 & 2 \end{bmatrix} \right.$$

$$\times \begin{bmatrix} \frac{g}{(28+\sqrt{7})s}\left[\frac{\left(-7c^r_{2q} - 14c^r_{2q+1} + (18+\sqrt{7})c^r_{2q+2} + (6-3\sqrt{7})c^r_{2q+3} - 10c^r_{2q+4} + 5c_{2q+5}\right)^2}{(28+\sqrt{7})^2 s^2}\right] \\ \left(-7c^r_{2q} - 14c^r_{2q+1} + (18+\sqrt{7})c^r_{2q+2} + (6-3\sqrt{7})c^r_{2q+3} - 10c^r_{2q+4} + 5c_{2q+5}\right) \\ \frac{g}{(1-3\sqrt{7})}\left[\frac{\left(c^r_{2q} + 2c^r_{2q+1} + (6-3\sqrt{7})c^r_{2q+2} + (2+9\sqrt{7})c^r_{2q+3} - 10c^r_{2q+4} + 5c_{2q+5}\right)^2}{(1-3\sqrt{7})^2 s^2}\right] \\ \left(c^r_{2q} + 2c^r_{2q+1} + (6-3\sqrt{7})c^r_{2q+2} + (2+9\sqrt{7})c^r_{2q+3} - 10c^r_{2q+4} + 5c_{2q+5}\right) \end{bmatrix}$$

$$+ \begin{bmatrix} 18+\sqrt{7} & 6-3\sqrt{7} \\ 6-3\sqrt{7} & 2+9\sqrt{7} \end{bmatrix}$$

$$\times\begin{bmatrix} \frac{g}{(28+\sqrt{7})s}\left[\frac{\left(-7c^r_{2q-2}-14c^r_{2q-1}+(18+\sqrt{7})c^r_{2q}+(6-3\sqrt{7})c^r_{2q+1}-10c^r_{2q+2}+5c_{2q+3}\right)^2}{(28+\sqrt{7})^2s^2}\right] \\ \left(-7c^r_{2q-2}-14c^r_{2q-1}+(18+\sqrt{7})c^r_{2q}+(6-3\sqrt{7})c^r_{2q+1}-10c^r_{2q+2}+5c_{2q+3}\right) \\ \frac{g}{(1-3\sqrt{7})}\left[\frac{\left(c^r_{2q-2}+2c^r_{2q-1}+(6-3\sqrt{7})c^r_{2q}+(2+9\sqrt{7})c^r_{2q+1}-10c^r_{2q+2}+5c_{2q+3}\right)^2}{(1-3\sqrt{7})^2s^2}\right] \\ \left(c^r_{2q-2}+2c^r_{2q-1}+(6-3\sqrt{7})c^r_{2q}+(2+9\sqrt{7})c^r_{2q+1}-10c^r_{2q+2}+5c_{2q+3}\right) \end{bmatrix}$$

$$+\begin{bmatrix}-10 & -10\\ 5 & 5\end{bmatrix}$$

$$\times\begin{bmatrix} \frac{g}{(28+\sqrt{7})s}\left[\frac{\left(-7c^r_{2q-4}-14c^r_{2q-3}+(18+\sqrt{7})c^r_{2q-2}+(6-3\sqrt{7})c^r_{2q-1}-10c^r_{2q}+5c_{2q+1}\right)^2}{(28+\sqrt{7})^2s^2}\right] \\ \left(-7c^r_{2q-4}-14c^r_{2q-3}+(18+\sqrt{7})c^r_{2q-2}+(6-3\sqrt{7})c^r_{2q-1}-10c^r_{2q}+5c_{2q+1}\right) \\ \frac{g}{(1-3\sqrt{7})}\left[\frac{\left(c^r_{2q-4}+2c^r_{2q-3}+(6-3\sqrt{7})c^r_{2q-2}+(2+9\sqrt{7})c^r_{2q-1}-10c^r_{2q}+5c_{2q+1}\right)^2}{(1-3\sqrt{7})^2s^2}\right] \\ \left(c^r_{2q-4}+2c^r_{2q-3}+(6-3\sqrt{7})c^r_{2q-2}+(2+9\sqrt{7})c^r_{2q-1}-10c^r_{2q}+5c_{2q+1}\right) \end{bmatrix}$$

Then the second-order nonlinear diffusion equation can be written as:

$$\begin{bmatrix}c^{r+1}_{2q}\\ c^{r+1}_{2q+1}\end{bmatrix}=\begin{bmatrix}c^r_{2q}\\ c^r_{2q+1}\end{bmatrix}+\begin{bmatrix}\frac{\tau}{(16-3\sqrt{7})s} & 0\\ 0 & \frac{\tau}{(-3+9\sqrt{7})s}\end{bmatrix}$$

$$\times\left\{\begin{bmatrix} \frac{-7g}{(28+\sqrt{7})s}\left[\frac{\left(-7c^r_{2q}-14c^r_{2q+1}+(18+\sqrt{7})c^r_{2q+2}+(6-3\sqrt{7})c^r_{2q+3}-10c^r_{2q+4}+5c_{2q+5}\right)^2}{(28+\sqrt{7})^2s^2}\right] \\ \left(-7c^r_{2q}-14c^r_{2q+1}+(18+\sqrt{7})c^r_{2q+2}+(6-3\sqrt{7})c^r_{2q+3}-10c^r_{2q+4}+5c_{2q+5}\right) \\ \frac{-14g}{(28+\sqrt{7})s}\left[\frac{\left(-7c^r_{2q}-14c^r_{2q+1}+(18+\sqrt{7})c^r_{2q+2}+(6-3\sqrt{7})c^r_{2q+3}-10c^r_{2q+4}+5c_{2q+5}\right)^2}{(28+\sqrt{7})^2s^2}\right] \\ \left(-7c^r_{2q}-14c^r_{2q+1}+(18+\sqrt{7})c^r_{2q+2}+(6-3\sqrt{7})c^r_{2q+3}-10c^r_{2q+4}+5c_{2q+5}\right) \end{bmatrix}\right.$$

$$+\begin{bmatrix} \frac{g}{(1-3\sqrt{7})s}\left[\frac{\left(c^r_{2q}+2c^r_{2q+1}+(6-3\sqrt{7})c^r_{2q+2}+(2+9\sqrt{7})c^r_{2q+3}-10c^r_{2q+4}+5c_{2q+5}\right)^2}{(1-3\sqrt{7})^2s^2}\right] \\ \left(c^r_{2q}+2c^r_{2q+1}+(6-3\sqrt{7})c^r_{2q+2}+(2+9\sqrt{7})c^r_{2q+3}-10c^r_{2q+4}+5c_{2q+5}\right) \\ \frac{2g}{(1-3\sqrt{7})s}\left[\frac{\left(c^r_{2q}+2c^r_{2q+1}+(6-3\sqrt{7})c^r_{2q+2}+(2+9\sqrt{7})c^r_{2q+3}-10c^r_{2q+4}+5c_{2q+5}\right)^2}{(1-3\sqrt{7})^2s^2}\right] \\ \left(c^r_{2q}+2c^r_{2q+1}+(6-3\sqrt{7})c^r_{2q+2}+(2+9\sqrt{7})c^r_{2q+3}-10c^r_{2q+4}+5c_{2q+5}\right) \end{bmatrix}$$

$$+\begin{bmatrix} \frac{(18+\sqrt{7})g}{(28+\sqrt{7})s}\left[\frac{\left(-7c^r_{2q-2}-14c^r_{2q-1}+(18+\sqrt{7})c^r_{2q}+(6-3\sqrt{7})c^r_{2q+1}-10c^r_{2q+2}+5c_{2q+3}\right)^2}{(28+\sqrt{7})^2s^2}\right] \\ \left(-7c^r_{2q-2}-14c^r_{2q-1}+(18+\sqrt{7})c^r_{2q}+(6-3\sqrt{7})c^r_{2q+1}-10c^r_{2q+2}+5c_{2q+3}\right) \\ \frac{(6-3\sqrt{7})g}{(28+\sqrt{7})s}\left[\frac{\left(-7c^r_{2q-2}-14c^r_{2q-1}+(18+\sqrt{7})c^r_{2q}+(6-3\sqrt{7})c^r_{2q+1}-10c^r_{2q+2}+5c_{2q+3}\right)^2}{(28+\sqrt{7})^2s^2}\right] \\ \left(-7c^r_{2q-2}-14c^r_{2q-1}+(18+\sqrt{7})c^r_{2q}+(6-3\sqrt{7})c^r_{2q+1}-10c^r_{2q+2}+5c_{2q+3}\right) \end{bmatrix}$$

$$+\begin{bmatrix} \frac{(6-3\sqrt{7})g}{(1-3\sqrt{7})s}\left[\frac{\left(c^r_{2q-2}+2c^r_{2q-1}+(6-3\sqrt{7})c^r_{2q}+(2+9\sqrt{7})c^r_{2q+1}-10c^r_{2q+2}+5c_{2q+3}\right)^2}{(1-3\sqrt{7})^2s^2}\right] \\ \left(c^r_{2q-2}+2c^r_{2q-1}+(6-3\sqrt{7})c^r_{2q}+(2+9\sqrt{7})c^r_{2q+1}-10c^r_{2q+2}+5c_{2q+3}\right) \\ \frac{(2+9\sqrt{7})g}{(1-3\sqrt{7})s}\left[\frac{\left(c^r_{2q-2}+2c^r_{2q-1}+(6-3\sqrt{7})c^r_{2q}+(2+9\sqrt{7})c^r_{2q+1}-10c^r_{2q+2}+5c_{2q+3}\right)^2}{(1-3\sqrt{7})^2s^2}\right] \\ \left(c^r_{2q-2}+2c^r_{2q-1}+(6-3\sqrt{7})c^r_{2q}+(2+9\sqrt{7})c^r_{2q+1}-10c^r_{2q+2}+5c_{2q+3}\right) \end{bmatrix}$$

$$
+\left[\begin{array}{l}
\dfrac{-10g}{(28+\sqrt{7})s}\left[\dfrac{\left(-7c^{r}_{2q-4}-14c^{r}_{2q-3}+(18+\sqrt{7})c^{r}_{2q-2}+(6-3\sqrt{7})c^{r}_{2q-1}-10c^{r}_{2q}+5c_{2q+1}\right)^{2}}{(28+\sqrt{7})^{2}s^{2}}\right]\\
\left(-7c^{r}_{2q-4}-14c^{r}_{2q-3}+(18+\sqrt{7})c^{r}_{2q-2}+(6-3\sqrt{7})c^{r}_{2q-1}-10c^{r}_{2q}+5c_{2q+1}\right)\\
\dfrac{5g}{(28+\sqrt{7})s}\left[\dfrac{\left(-7c^{r}_{2q-4}-14c^{r}_{2q-3}+(18+\sqrt{7})c^{r}_{2q-2}+(6-3\sqrt{7})c^{r}_{2q-1}-10c^{r}_{2q}+5c_{2q+1}\right)^{2}}{(28+\sqrt{7})^{2}s^{2}}\right]\\
\left(-7c^{r}_{2q-4}-14c^{r}_{2q-3}+(18+\sqrt{7})c^{r}_{2q-2}+(6-3\sqrt{7})c^{r}_{2q-1}-10c^{r}_{2q}+5c_{2q+1}\right)
\end{array}\right]
$$

$$
\left.+\left[\begin{array}{l}
\dfrac{-10g}{(1-3\sqrt{7})s}\left[\dfrac{\left(c^{r}_{2q-4}+2c^{r}_{2q-3}+(6-3\sqrt{7})c^{r}_{2q-2}+(2+9\sqrt{7})c^{r}_{2q-1}-10c^{r}_{2q}+5c_{2q+1}\right)^{2}}{(1-3\sqrt{7})^{2}s^{2}}\right]\\
\left(c^{r}_{2q-4}+2c^{r}_{2q-3}+(6-3\sqrt{7})c^{r}_{2q-2}+(2+9\sqrt{7})c^{r}_{2q-1}-10c^{r}_{2q}+5c_{2q+1}\right)\\
\dfrac{5g}{(1-3\sqrt{7})s}\left[\dfrac{\left(c^{r}_{2q-4}+2c^{r}_{2q-3}+(6-3\sqrt{7})c^{r}_{2q-2}+(2+9\sqrt{7})c^{r}_{2q-1}-10c^{r}_{2q}+5c_{2q+1}\right)^{2}}{(1-3\sqrt{7})^{2}s^{2}}\right]\\
\left(c^{r}_{2q-4}+2c^{r}_{2q-3}+(6-3\sqrt{7})c^{r}_{2q-2}+(2+9\sqrt{7})c^{r}_{2q-1}-10c^{r}_{2q}+5c_{2q+1}\right)
\end{array}\right]\right]\right\}
$$

When $r = 0$ with $\begin{bmatrix} c^{0}_{2q} \\ c^{0}_{2q+1} \end{bmatrix} = \begin{bmatrix} u_{2q} \\ u_{2q+1} \end{bmatrix}$. We have

$$
\begin{bmatrix} c^{1}_{2q} \\ c^{1}_{2q+1} \end{bmatrix} = \begin{bmatrix} u_{2q} \\ u_{2q+1} \end{bmatrix} +
$$

$$
\left\{\left[\begin{array}{l}
\dfrac{-7\tau g}{(16-3\sqrt{7})(28+\sqrt{7})s^{2}}\left[\dfrac{\left(-7u_{2q}-14u_{2q+1}+(18+\sqrt{7})u_{2q+2}+(6-3\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)^{2}}{(28+\sqrt{7})^{2}s^{2}}\right]\\
\left(-7u_{2q}-14u_{2q+1}+(18+\sqrt{7})u_{2q+2}+(6-3\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)\\
\dfrac{-14\tau g}{(-3+9\sqrt{7})(28+\sqrt{7})s^{2}}\left[\dfrac{\left(-7u_{2q}-14u_{2q+1}+(18+\sqrt{7})u_{2q+2}+(6-3\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)^{2}}{(28+\sqrt{7})^{2}s^{2}}\right]\\
\left(-7u_{2q}-14u_{2q+1}+(18+\sqrt{7})u_{2q+2}+(6-3\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)
\end{array}\right]\right.
$$

$$
+\left[\begin{array}{l}
\dfrac{\tau g}{(16-3\sqrt{7})(1-3\sqrt{7})s^{2}}\left[\dfrac{\left(u_{2q}+2u_{2q+1}+(6-3\sqrt{7})u_{2q+2}+(2+9\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)^{2}}{(1-3\sqrt{7})^{2}s^{2}}\right]\\
\left(u_{2q}+2u_{2q+1}+(6-3\sqrt{7})u_{2q+2}+(2+9\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)\\
\dfrac{2\tau g}{(-3+9\sqrt{7})(1-3\sqrt{7})s^{2}}\left[\dfrac{\left(u_{2q}+2u_{2q+1}+(6-3\sqrt{7})u_{2q+2}+(2+9\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)^{2}}{(1-3\sqrt{7})^{2}s^{2}}\right]\\
\left(u_{2q}+2u_{2q+1}+(6-3\sqrt{7})u_{2q+2}+(2+9\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)
\end{array}\right]
$$

$$
+\left[\begin{array}{l}
\dfrac{(18+\sqrt{7})\tau g}{(16-3\sqrt{7})(28+\sqrt{7})s^{2}}\left[\dfrac{\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)^{2}}{(28+\sqrt{7})^{2}s^{2}}\right]\\
\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)\\
\dfrac{(6-3\sqrt{7})\tau g}{(-3+9\sqrt{7})(28+\sqrt{7})s^{2}}\left[\dfrac{\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)^{2}}{(28+\sqrt{7})^{2}s^{2}}\right]\\
\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)
\end{array}\right]
$$

$$
+\left[\begin{array}{l}
\dfrac{(6-3\sqrt{7})\tau g}{(16-3\sqrt{7})(1-3\sqrt{7})s^{2}}\left[\dfrac{\left(u_{2q-2}+2u_{2q-1}+(6-3\sqrt{7})u_{2q}+(2+9\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)^{2}}{(1-3\sqrt{7})^{2}s^{2}}\right]\\
\left(u_{2q-2}+2u_{2q-1}+(6-3\sqrt{7})u_{2q}+(2+9\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)\\
\dfrac{(2+9\sqrt{7})\tau g}{(-3+9\sqrt{7})(1-3\sqrt{7})s^{2}}\left[\dfrac{\left(u_{2q-2}+2u_{2q-1}+(6-3\sqrt{7})u_{2q}+(2+9\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)^{2}}{(1-3\sqrt{7})^{2}s^{2}}\right]\\
\left(u_{2q-2}+2u_{2q-1}+(6-3\sqrt{7})u_{2q}+(2+9\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)
\end{array}\right]
$$

$$
+\begin{bmatrix}
\dfrac{-10\tau g}{(16-3\sqrt{7})(28+\sqrt{7})s^2}\left[\dfrac{\left(-7u_{2q-4}-14u_{2q-3}+(18+\sqrt{7})u_{2q-2}+(6-3\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right)^2}{(28+\sqrt{7})^2s^2}\right] \\
\left(-7u_{2q-4}-14u_{2q-3}+(18+\sqrt{7})u_{2q-2}+(6-3\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right) \\
\dfrac{5\tau g}{(-3+9\sqrt{7})(28+\sqrt{7})s^2}\left[\dfrac{\left(-7u_{2q-4}-14u_{2q-3}+(18+\sqrt{7})u_{2q-2}+(6-3\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right)^2}{(28+\sqrt{7})^2s^2}\right] \\
\left(-7u_{2q-4}-14u_{2q-3}+(18+\sqrt{7})u_{2q-2}+(6-3\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right)
\end{bmatrix}
$$

$$
\left.+\begin{bmatrix}
\dfrac{-10\tau g}{(16-3\sqrt{7})(1-3\sqrt{7})s^2}\left[\dfrac{\left(u_{2q-4}+2u_{2q-3}+(6-3\sqrt{7})u_{2q-2}+(2+9\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right)^2}{(1-3\sqrt{7})^2s^2}\right] \\
\left(u_{2q-4}+2u_{2q-3}+(6-3\sqrt{7})u_{2q-2}+(2+9\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right) \\
\dfrac{5\tau g}{(-3+9\sqrt{7})(1-3\sqrt{7})s^2}\left[\dfrac{\left(u_{2q-4}+2u_{2q-3}+(6-3\sqrt{7})u_{2q-2}+(2+9\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right)^2}{(1-3\sqrt{7})^2s^2}\right] \\
\left(u_{2q-4}+2u_{2q-3}+(6-3\sqrt{7})u_{2q-2}+(2+9\sqrt{7})u_{2q}{}^{9}{}_{-1}-10u_{2q}+5u_{2q+1}\right)
\end{bmatrix}\right]\right\}
$$

Write $\begin{bmatrix} u_{2q} \\ u_{2q+1} \end{bmatrix}$ as

$$
\begin{bmatrix} c_{1,q} \\ c_{2,q} \end{bmatrix}=\frac{1}{1600}\begin{bmatrix}
(320+30\sqrt{7})u_{2q-1}+(360-10\sqrt{7})u_{2q-2}-120u_{2q-3}-60u_{2q-4}+920u_{2q} \\
+90u_{2q+1}+(390-10\sqrt{7})u_{2q+2}+(-110-20\sqrt{7})u_{2q+3}+60u_{2q+4}+30u_{2q+5} \\
(40-90)u_{2q-1}+(-80-20\sqrt{7})u_{2q-2}+60u_{2q-3}+30u_{2q-4}+90u_{2q} \\
+680u_{2q+1}+(320+80)u_{2q+2}+(40-9\sqrt{7})u_{2q+3}-120u_{2q+4}+60u_{2q+5}
\end{bmatrix}
$$

$$
+\frac{1}{1600}\begin{bmatrix}
-7\left(-7u_{2q}-14u_{2q+1}+(18+\sqrt{7})u_{2q+2}+(6-3\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right) \\
-14\left(-7u_{2q}-14u_{2q+1}+(18+\sqrt{7})u_{2q+2}+(6-3\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)
\end{bmatrix}
$$

$$
+\frac{1}{1600}\begin{bmatrix}
\left(u_{2q}+2u_{2q+1}+(6-3\sqrt{7})u_{2q+2}+(2+9\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right) \\
2\left(u_{2q}+2u_{2q+1}+(6-3\sqrt{7})u_{2q+2}+(2+9\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)
\end{bmatrix}
$$

$$
+\frac{1}{1600}\begin{bmatrix}
(18+\sqrt{7})\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right) \\
(6-3\sqrt{7})\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)
\end{bmatrix}
$$

$$
+\frac{1}{1600}\begin{bmatrix}
(6-3\sqrt{7})\left(u_{2q-2}+2u_{2q-1}+(6-3\sqrt{7})u_{2q}+(2+9\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right) \\
(2+9\sqrt{7})\left(u_{2q-2}+2u_{2q-1}+(6-3\sqrt{7})u_{2q}+(2+9\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)
\end{bmatrix}
$$

$$
+\frac{1}{1600}\begin{bmatrix}
-10\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right) \\
5\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)
\end{bmatrix}
$$

$$
+\frac{1}{1600}\begin{bmatrix}
-10\left(u_{2q-4}+2u_{2q-3}+(6-3\sqrt{7})u_{2q-2}+(2+9\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right) \\
5\left(u_{2q-4}+2u_{2q-3}+(6-3\sqrt{7})u_{2q-2}+(2+9\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right)
\end{bmatrix}
$$

After one-step diffusion, the second-order nonlinear diffusion equation looks like this:

$$
\begin{bmatrix} c_{2q}^{1} \\ c_{2q+1}^{1} \end{bmatrix}=\frac{1}{1600}\begin{bmatrix}
(320+30\sqrt{7})u_{2q-1}+(360-10\sqrt{7})u_{2q-2}-120u_{2q-3}-60u_{2q-4}+920u_{2q} \\
+90u_{2q+1}+(390-10\sqrt{7})u_{2q+2}+(-110-20\sqrt{7})u_{2q+3}+60u_{2q+4}+30u_{2q+5} \\
(40-90)u_{2q-1}+(-80-20\sqrt{7})u_{2q-2}+60u_{2q-3}+30u_{2q-4}+90u_{2q} \\
+680u_{2q+1}+(320+80)u_{2q+2}+(40-9\sqrt{7})u_{2q+3}-120u_{2q+4}+60u_{2q+5}
\end{bmatrix}
$$

$$
+\begin{bmatrix}
\left\{\dfrac{-7}{1600}-\dfrac{-7\tau g}{(16-3\sqrt{7})(28+\sqrt{7})s^2}\left[\dfrac{\left(-7u_{2q}-14u_{2q+1}+(18+\sqrt{7})u_{2q+2}+(6-3\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)^2}{(28+\sqrt{7})^2s^2}\right]\right\} \\
\left(-7u_{2q}-14u_{2q+1}+(18+\sqrt{7})u_{2q+2}+(6-3\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right) \\
\left\{\dfrac{-14}{1600}-\dfrac{-14\tau g}{(-3+9\sqrt{7})(28+\sqrt{7})s^2}\left[\dfrac{\left(-7u_{2q}-14u_{2q+1}+(18+\sqrt{7})u_{2q+2}+(6-3\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)^2}{(28+\sqrt{7})^2s^2}\right]\right\} \\
\left(-7u_{2q}-14u_{2q+1}+(18+\sqrt{7})u_{2q+2}+(6-3\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)
\end{bmatrix}
$$

$$
+\left[\begin{array}{l}
\left\{\frac{1}{1600}-\frac{\tau g}{(16-3\sqrt{7})(1-3\sqrt{7})s^2}\left[\frac{\left(u_{2q}+2u_{2q+1}+(6-3\sqrt{7})u_{2q+2}+(2+9\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)^2}{(1-3\sqrt{7})^2s^2}\right]\right\} \\
\left(u_{2q}+2u_{2q+1}+(6-3\sqrt{7})u_{2q+2}+(2+9\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right) \\
\left\{\frac{2}{1600}-\frac{2\tau g}{(-3+9\sqrt{7})(1-3\sqrt{7})s^2}\left[\frac{\left(u_{2q}+2u_{2q+1}+(6-3\sqrt{7})u_{2q+2}+(2+9\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)^2}{(1-3\sqrt{7})^2s^2}\right]\right\} \\
\left(u_{2q}+2u_{2q+1}+(6-3\sqrt{7})u_{2q+2}+(2+9\sqrt{7})u_{2q+3}-10u_{2q+4}+5u_{2q+5}\right)
\end{array}\right]
$$

$$
+\left[\begin{array}{l}
\left\{\frac{(18+\sqrt{7})}{1600}-\frac{-(18+\sqrt{7})\tau g}{(16-3\sqrt{7})(28+\sqrt{7})s^2}\left[\frac{\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)^2}{(28+\sqrt{7})^2s^2}\right]\right. \\
\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right) \\
\left\{\frac{(6-3\sqrt{7})}{1600}-\frac{-(6-3\sqrt{7})\tau g}{(-3+9\sqrt{7})(28+\sqrt{7})s^2}\left[\frac{\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)^2}{(28+\sqrt{7})^2s^2}\right]\right. \\
\left(-7u_{2q-2}-14u_{2q-1}+(18+\sqrt{7})u_{2q}+(6-3\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)
\end{array}\right.
$$

$$
+\left[\begin{array}{l}
\left\{\frac{(6-3\sqrt{7})}{1600}-\frac{-(6-3\sqrt{7})\tau g}{(16-3\sqrt{7})(1-3\sqrt{7})s^2}\left[\frac{\left(u_{2q-2}+2u_{2q-1}+(6-3\sqrt{7})u_{2q}+(2+9\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)^2}{(1-3\sqrt{7})^2s^2}\right]\right\} \\
\left(u_{2q-2}+2u_{2q-1}+(6-3\sqrt{7})u_{2q}+(2+9\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right) \\
\left\{\frac{(2+9\sqrt{7})}{1600}-\frac{-(2+9\sqrt{7})\tau g}{(-3+9\sqrt{7})(1-3\sqrt{7})s^2}\left[\frac{\left(u_{2q-2}+2u_{2q-1}+(6-3\sqrt{7})u_{2q}+(2+9\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)^2}{(1-3\sqrt{7})^2s^2}\right]\right\} \\
\left(u_{2q-2}+2u_{2q-1}+(6-3\sqrt{7})u_{2q}+(2+9\sqrt{7})u_{2q+1}-10u_{2q+2}+5u_{2q+3}\right)
\end{array}\right.
$$

$$
+\left[\begin{array}{l}
\left\{\frac{-10}{1600}-\frac{10\tau g}{(16-3\sqrt{7})(28+\sqrt{7})s^2}\left[\frac{\left(-7u_{2q-4}-14u_{2q-3}+(18+\sqrt{7})u_{2q-2}+(6-3\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right)^2}{(28+\sqrt{7})^2s^2}\right]\right\} \\
\left(-7u_{2q-4}-14u_{2q-3}+(18+\sqrt{7})u_{2q-2}+(6-3\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right) \\
\left\{\frac{5}{1600}-\frac{-5\tau g}{(-3+9\sqrt{7})(28+\sqrt{7})s^2}\left[\frac{\left(-7u_{2q-4}-14u_{2q-3}+(18+\sqrt{7})u_{2q-2}+(6-3\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right)^2}{(28+\sqrt{7})^2s^2}\right]\right\} \\
\left(-7u_{2q-4}-14u_{2q-3}+(18+\sqrt{7})u_{2q-2}+(6-3\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right)
\end{array}\right]
$$

$$
+\left[\begin{array}{l}
\left\{\frac{-10}{1600}-\frac{-10\tau g}{(16-3\sqrt{7})(1-3\sqrt{7})s^2}\left[\frac{\left(u_{2q-4}+2u_{2q-3}+(6-3\sqrt{7})u_{2q-2}+(2+9\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right)^2}{(1-3\sqrt{7})^2s^2}\right]\right\} \\
\left(u_{2q-4}+2u_{2q-3}+(6-3\sqrt{7})u_{2q-2}+(2+9\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right) \\
\left\{\frac{5}{1600}-\frac{-5\tau g}{(-3+9\sqrt{7})(1-3\sqrt{7})s^2}\left[\frac{\left(u_{2q-4}+2u_{2q-3}+(6-3\sqrt{7})u_{2q-2}+(2+9\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right)^2}{(1-3\sqrt{7})^2s^2}\right]\right\} \\
\left(u_{2q-4}+2u_{2q-3}+(6-3\sqrt{7})u_{2q-2}+(2+9\sqrt{7})u_{2q-1}-10u_{2q}+5u_{2q+1}\right)
\end{array}\right]
$$

4. Conclusions

The correlation between CL(2) multiwavelet shrinkage and second-order nonlinear diffusion equation in multi-level has been examined in our research. In reality, this connection offers various nonlinear diffusion equation types. Multi-wavelet conversion has long been used to minimize noise in signal processing. The results of the current study may assist wavelets be used more effectively in communication applications.

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