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0-Edge Magic Labeling of Some Graphs

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Abstract
In this paper, we prove that 0-edge magic labeling of shadowgraph of bistar and comb and splitting graph of comb, $S'(K_{2, m})$ and $S'(K_{1, 4, 4})$ graph and Herschel graph.

Keywords: Graph Labeling, Magic Labeling, 0-Edge Magic Labeling

1. Introduction

The concept of graph labeling was introduced by Rosa in 1967 [5]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions labeled graphs serve as useful models for a broad range of applications such as coding theory, X-ray, crystallography, radar, astronomy, circuit design, communication network adding the database management. Hence in the intervening years various labeling of graphs such as graceful labeling, harmonious labelings, magic labeling antimagic labeling, bimagic labeling, prime labeling etc. have been studied in over 1800 papers [1]. Various results on magic graphs have been studied in the literature [1, 4, 6, 7, 8]. The concept of 0-edge magic labeling was introduced in [2]. They proved that paths, grid graphs, cycles, wheels, binary trees and some flower graphs are 0-edge magic. This concept was generalized as n-edge magic graphs in [3] and also 0 edge magic labeling was introduced in [10] splitting graph, spl(P_n), spl(C_n), spl($K_{1, n}$), spl($B_{m, n}$) and also on 0-edge magic labeling of some graph was introduced in C11 Cartesian graphs. $P_m \times P_n$ and $C_m \times C_n$ and generalized Petersen graphs P(m, n) and also on 0 edge magic labeling in certain graphs introduced [12] in the complete n-array pseudo tree, two-dimensional cylindrical meshes $P_m \times C_n$; $n \equiv 0 \pmod 2$, n-dimensional hyper cube Q_n the graph obtained by attaching C_m to $mK_{1, n}$ ($m \equiv 0 \pmod 2$)) the circular ladder graph; the friendship graph (C_m) and the graph $P_m \times P_m \times P_m$ are 0-edge magic graphs.

In this paper we prove that 0-edge magic labeling of shadow graph of bistar and comb and splitting graph of comb and $S'(K_{2, m})$, $S'(K_{1, 4, 4})$ graph and Herschel graph.

2. Preliminaries

In this section we give the basic notions relevant to this paper. Let G = G (V, E) be finite simple and undirected graph with p vertices and q edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers called labels.

In this paper we deal with labeling with domain either the set of all vertices or the set of all edges or the set of all vertices and edges. We call these labeling as the vertex labeling or the edge labeling or the total labeling respectively.

2.1 Definition

Let G = (V, E) be a graph where $V = \{v_i;\ 1 \le i \le n\}$ and $E = \{v_i, v_{i+1};\ 1 \le i \le n\}$. Let $f: V \to \{-1, 1\}$ and $f^*: E \to \{0\}$ such that all $uv \in E$ $f^*(u, v) = f(u) + f(v) = 0$ then the labeling is said to be 0-edge magic labeling.

2.2 Definition

A comb is a caterpillar in which each vertex in the path is joined to exactly one pendent vertex.

2.3 Definition

The splitting graph of G, S(G) is obtained from G by adding to each vertex V of G a new vertex V′ so that V′ is adjacent to every vertex that is adjacent to V in G.

2.4 Definition
The shadowgraph D2(G) of a connected graph G is obtained by taking two copies of G say G′ and G′′ then joining each vertex u′ in G′ to neighbours of the corresponding vertex u′′ in G′′.

2.5 Definition
A bistar in the graph is obtained by joining the apex vertex of two copies of star $K_{1, n}$ by an edge.

3. Results on 0-Edge Magic Labeling
3.1 Theorem
The graph $D_2(B_{n, n})$ has 0-Edge magic labeling.
Proof:
Consider two copies of $B_{n, n}$. Let $\{u, v, u_i, v_i: 1 \le i \le n\}$ and $\{u', v', u'_i, v'_i: 1 \le i \le n\}$ be the corresponding vertex sets of each copy of $B_{n, n}$. Let G be the graph $D_2(B_{n, n})$ then $|V(G)| = 4(n+1)$ and $|E(G)| = 4(2n+1)$.

Let $f: V \to \{1, -1\}$ such that $f(u) = f(u') = -1; 1 \le i \le n$
$f(v_i) = f(v'_i) = -1; 1 \le i \le n$
$f(v) = f(v') = 1; 1 \le i \le n$
$f(u_i) = f(u'_i) = 1; 1 \le i \le n$
$E(G) = \{uu_i : 1 \le i \le n\} \cup \{uu'_i : 1 \le i \le n\} \cup \{u'u_i ; 1 \le i \le n\} \cup \{u'u'_i ; 1 \le i \le n\} \cup$
$\{vv_i ; 1 \le i \le n\} \cup \{v'v'_i ; 1 \le i \le n\} \cup \{v'v_i ; 1 \le i \le n\} \cup \{v'v'_i ; 1 \le i \le n\} \cup$
$\{uv\} \cup \{vv'\} \cup \{vu'\} \cup \{u'v\}$.

The edge weight are calculated as follows
for $1 \le i \le n$; $f(u) + f(u_i) = 0$
for $1 \le i \le n$; $f(u) + f(u'_i) = 0$
for $1 \le i \le n$; $f(u') + f(u_i) = 0$
for $1 \le i \le n$; $f(u') + f(u'_i) = 0$
for $1 \le i \le n$; $f(v) + f(v_i) = 0$
for $1 \le i \le n$; $f(v) + f(v'_i) = 0$
for $1 \le i \le n$; $f(v') + f(v_i) = 0$
for $1 \le i \le n$; $f(v') + f(v'_i) = 0$

Also, $f(u) + f(v) = 0$
$f(u) + f(v') = 0$
$f(v) + f(u') = 0$
$f(v') + f(u') = 0$
Thus, all edges receive value 0. Hence the graph $D_2(B_{n, n})$ admits 0-Edge magic labeling.

3.2 Example
0-Edge magic labeling for $D_2(B_{5, 5})$

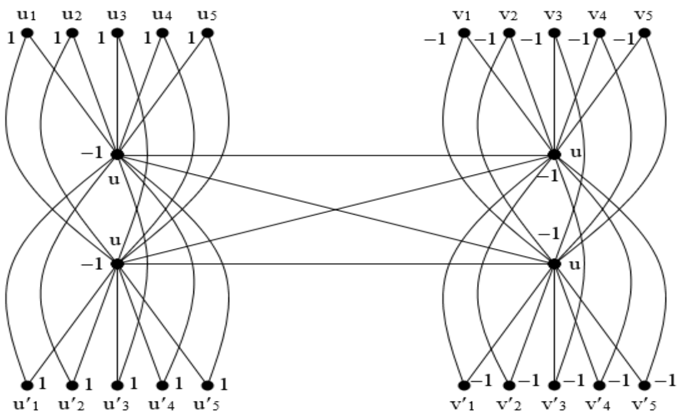

Fig 1: $D_2(B_{5, 5})$ and its 0-Edge magic labeling

3.3 Theorem
The split graph of comb has 0-edge Magic labeling.
Proof:
Let $\{v_i: 1 \le i \le n\}$ and $\{v'_i: 1 \le i \le n\}$ be the vertices of comb in which $\{v'_i; 1 \le i \le n\}$ are the pendent vertices.
Let $\{u_i : 1 \le i \le n\}$ and $\{u'_I : 1 \le i \le n\}$ be the newly added vertices.

Let $f : V \to \{1, -1\}$ such that $f(u_i) = f(v_i) = -1$; if i = odd
$f(u_i) = f(v_i) = 1$ if i = even
$f(u'_i) = f(v'_i) = 1$ if i = odd
$f(u'_i) = f(v'_i) = -1$ if i = even
$E(G) = \{v_iv'_i : 1 \leq i \leq n\} \cup \{v_iv_{i+1} : 1 \leq i \leq n\} \cup$
$\{v_iu'_i ; 1 \leq i \leq n\} \cup \{v_iu_{i+1} ; 1 \leq i \leq n\} \cup \{u_iv_{i+1} ; 1 \leq i \leq n\} \cup \{u_1v'_1\}$

The edge weight is calculated as follows
for $1 \leq i \leq n$; $f(v_i) + f(v'_i) = 0$
for $1 \leq i \leq n$; $f(v_i) + f(v_{i+1}) = 0$
for $1 \leq i \leq n$; $f(v_i) + f(u'_i) = 0$
for $1 \leq i \leq n$; $f(v_i) + f(u_{i+1}) = 0$
for $1 \leq i \leq n$; $f(u_i) + f(v_{i+1}) = 0$

Thus, all edges receive value 0. Hence the split graph of comb admits 0-Edge magic labeling.

3.4 Example

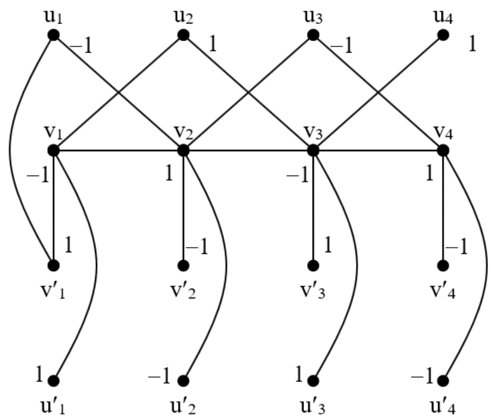

Fig 2: 0-Edge magic labelling for split graph

3.5 Theorem
D_2(comb) admits 0-Edge magic labeling.
Proof:
Consider two copies of comb G_1 and G_2. Let $\{v_i : 1 \leq i \leq n\}$ and $\{v'_i : 1 \leq i \leq n\}$ be the vertices of comb G_1.
Let $\{u_i : 1 \leq i \leq n\}$ and $\{u'_i : 1 \leq i \leq n\}$ be the vertices of comb G_2.
Let $f : V \to \{1, -1\}$ such that $f(u'_i) = f(v'_i) = 1$; if i = odd
$f(u'_i) = f(v'_i) = -1$ if i is even
$f(v_i) = f(u_i) = -1$ if i is odd
$f(v_i) = f(u_i) = 1$ if i is even
$E(G) = \{v_iv_{i+1} : 1 \leq i \leq n\} \cup \{v_iv'_i : 1 \leq i \leq n\}$
$\cup \{v_iu'_i ; 1 \leq i \leq n\} \cup \{v_iu_{i+1} ; 1 \leq i \leq n\}$
$\cup \{u_iv_{i+1} ; 1 \leq i \leq n\} \cup \{u_iv_{i+1} ; 1 \leq i \leq n\}$
$\cup \{u_iu'_i ; 1 \leq i \leq n\} \cup \{v'_iu_i ; 1 \leq i \leq n\}$

The edge weight are calculated as follows
for $1 \leq i \leq n$; $f(v_iv_{i+1}) = 0$
for $1 \leq i \leq n$; $f(v_iv'_i) = 0$
for $1 \leq i \leq n$; $f(v_iu'_i) = 0$
for $1 \leq i \leq n$; $f(v_iu_{i+1}) = 0$
for $1 \leq i \leq n$; $f(u_iv_{i+1}) = 0$
for $1 \leq i \leq n$; $f(u_iu_{i+1}) = 0$
for $1 \leq i \leq n$; $f(u_iu'_i) = 0$
for $1 \leq i \leq n$; $f(v'_iu_i) = 0$

Thus, all edges receive value 0. Hence the graph D_2(comb) admits 0-Edge magic labeling.

3.6 Example

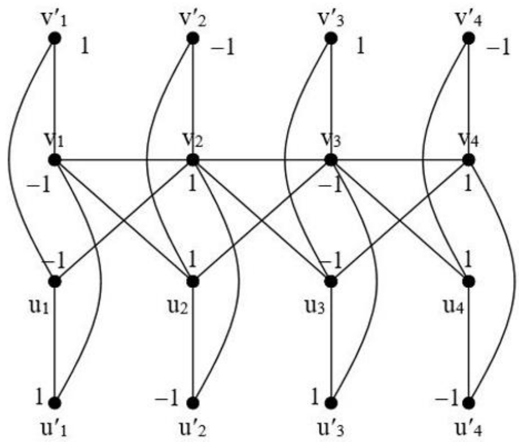

Fig 3: 0-Edge magic labeling for D2(comb)

3.7 Theorem

The graph $S'(K_{2,m})$ is 0-Edge magic labeling.

Proof:

Let $x_1, x_2, v_1, v_2, \ldots, v_m$ be the vertices of $K_{2,m}$

then $x_1, x_2, v_1, v_2, \ldots, v_m, x'_1, x'_2, v'_1, v'_2, \ldots, v'_m$ are the vertices of $S'(K_{2,m})$

and $|V(G)| = 2m+4$ and $|E(G)| = 6m$.

Define $f: V \to \{-1, 1\}$ such that

$f(x_1) = -1, f(x_2) = -1, f(x'_1) = -1, f(x'_2) = -1$

$f(v_i) = 1; 1 \le i \le m; f(v'_i) = 1; 1 \le i \le m$

$V(G) = \{x_1, x'_1, x_2, x'_2, v_i, v'_i \,/\, 1 \le i \le m, 1 \le i \le m'\}$

$E(G) = \{x_i v_i : 1 \le i \le n\} \cup \{v_i x_2 : 1 \le i \le n\} \cup \{v_i x'_i ; 1 \le i \le n\} \cup \{x'_2 v_i ; 1 \le i \le n\}$
$\cup \{x_1 v'_i ; 1 \le i \le n\} \cup \{v'_i x'_2 ; 1 \le i \le n\}$

The edge weight are calculated as follows

for $1 \le i \le n$; $f(x_1) + f(v_i) = 0$
for $1 \le i \le n$; $f(x_2) + f(v_i) = 0$
for $1 \le i \le n$; $f(x'_1) + f(v_i) = 0$
for $1 \le i \le n$; $f(x_2) + f(v_i) = 0$
for $1 \le i \le n$; $f(x_1) + f(v'_i) = 0$
for $1 \le i \le n$; $f(x'_2) + f(v'_i) = 0$

3.8 Example

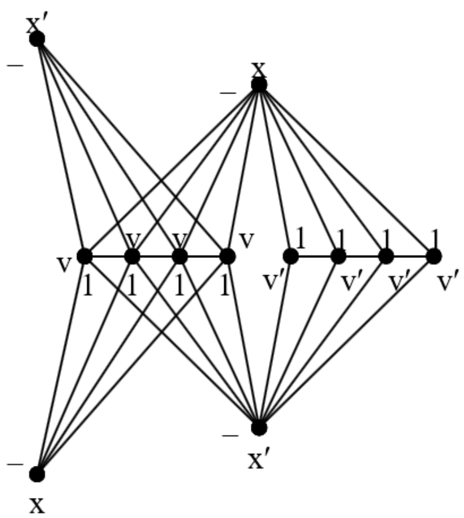

Fig 4: S′(2, 4) graph admits 0-Edge magic labeling

3.9 Theorem

The graph $S'(K_{1, n, n})$ is 0-edge magic labeling.

Proof:

Let $x, v_1, v_2, \ldots, v_n, v_{n+1}, v_{n+2}, \ldots, v_{2n}$ be the vertices of $K_{1, n, n}$ then $x, v_1, v_2, \ldots, v_n, v_{n+1}, v_{n+2}, \ldots, v_{2n}, x', v'_1, v'_2, \ldots, v'_n, v'_{n+1}, v'_{n+2}, \ldots, v'_{2n}$ are the vertices of $S'(K_{1, n, n})$ then $|V(G)| = 4n+2$ and $|E(G)| = 6n$.

Define $f : V(G) \to \{-1, 1\}$ such that
$f(x) = -1$, $f(x') = -1$,
$f(v_i) = 1, 1 \le i \le n$, $f(v'_i) = 1, 1 \le i \le n$,
$f(v_{n+i}) = -1; 1 \le i \le n$; $f(v'_{n+i}) = -1; 1 \le i \le n$
$V(G) = \{x, x', v_i, v'_i, v'_{n+I} / 1 \le i \le n\}$
$E(G) = \{xv'_i : 1 \le i \le n\} \cup \{xv_i : 1 \le i \le n\}$
$\cup \{x'v'_i ; 1 \le i \le n\} \cup \{v'_{n+i}v_i ; 1 \le i \le n\}$
$\cup \{v_{n+i}v'_i ; 1 \le i \le n\} \cup \{v_{n+i}v_i ; 1 \le i \le n\}$

The edge weight are calculated as follows

for $1 \le i \le n$; $f(x) + f(v'_i) = 0$
for $1 \le i \le n$; $f(x) + f(v_i) = 0$
for $1 \le i \le n$; $f(x') + f(v'_i) = 0$
for $1 \le i \le n$; $f(v'_{n+i}) + f(v'_i) = 0$
for $1 \le i \le n$; $f(v'_{n+i}) + f(v'_i) = 0$
for $1 \le i \le n$; $f(v_{n+i}) + f(v_i) = 0$

3.10 Example

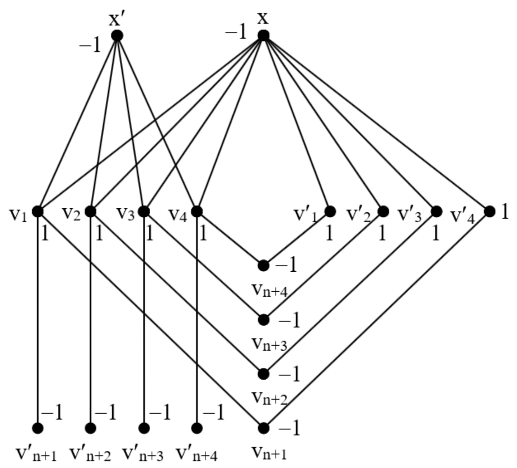

Fig 5: $S'(K\_(1,4,4))$) graph admits 0-Edge magic labeling

3.11 Definition

The Herschel graph H_S is a bipartite undirected graph with 11 vertices and 18 edges.

3.12 Example

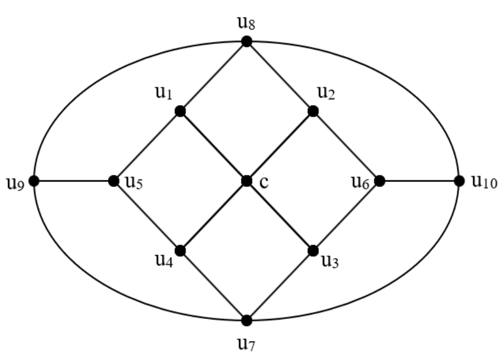

Fig 6: Herschel graph

3.13 Theorem
The Herschel graph H_S is 0-Edge magic labeling.
Proof:
Let H_S be the Herschel graph with 11 vertices and 18 edges and let C be the centre of the Herschel graph then $|V(H_S)| = 11$ and $|E(H_S)| = 18$

If $f(c) = -1$ and $f(u_i) = 1$ for $1 \leq i \leq 4$ where u_i's are adjacent to c.
i.e., $f(u_1) = f(u_2) = f(u_3) = f(u_4) = 1$

Next, we label the vertex u_5 which is adjacent to u_1 and u_4 having label values 1. $\therefore$ $f(u_5) = -1$
We label the vertex u6 which is adjacent to u_2 and u_3 having the same label value 1. $\therefore$ $f(u6) = -1$
Similarly, u_7 is adjacent to u_3 and u_4 both having label 1. $\therefore$ $f(u_7) = -1$ and u_8 is adjacent to u_1 and u_2 and having label value 1. $\therefore$ $f(u_8) = -1$.
Finally, $f(u_9) = 1$ and $f(u_{10}) = 1$. Hence for each $e = cu_i \in H_S$. $f(c, u_i) = 0$ $\forall$ $1 \leq i \leq 4$ and for the edge $e = u_iu_j \in H_S$ such that $f(u_iu_j) = 0$.
Hence HS admits 0-Edge magic labeling.

3.14 Examples or Illustrations

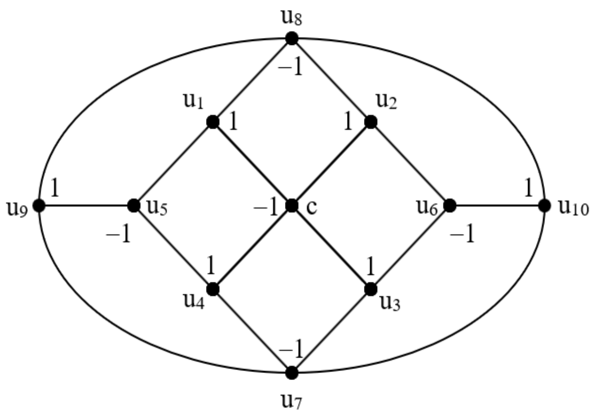

Fig 7: H_S - 0-Edge magic labeling

4. Conclusion
In this paper we have shown that the graph such as shadow graph of bistar and comb and splitting graph of comb and $S'(K_{2, m})$ and $S'(K_{1, 4, 4})$ graph and Herschel graph.

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